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Review of L1 Adaptive Control for Solving the Wing Rock Problem

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Abstract: This article presents an L1 Adaptive Control technique to solve the wing rock stabilization problem which is described as a second order nonlinear system. The wing rock phenomenon is characterized by the appearance of a limit cycle in flight dynamics, caused by flow asymmetries that build nonlinear aerodynamic roll damping. The problem has been treated in the literature using L1 Adaptive technique, but several questions arise, especially in the way the prefilter is chosen such that the resulting augmented system is in fact L1 and guarantees closed loop stability under actuator and model dynamicsuncertainties. It is found in this article that L1 first order filters used in the simulations reproduce results of referenced authors but higher order L1 filters used, generate unstable response, even though the calculation of named L1 Norm Condition, considered in L1 adaptive control theory as necessary and sufficient condition is met which is an exception to the theory. It is shown also that the choice of a proper third order filter instead of an strictly proper render steady state response with stationary error. Two schemes for the same wing rock system dynamics are treated, reproducing results of referenced authors in this article but higher strictly proper L1 filters used, generate unstable response not reproducing others results.

INTRODUCTION

L1 adaptive control has been used in several aerodynamic control applications with success[1-7], although, there are authors[3, 4] that consider that PI or PID control may provide similar performance as the adaptive scheme proposed (at least in some scalar systems) or that L1 Control is even unnecessary[4]. Even though L1 control has proven to be efficient in dealing with plant-model mismatch, the appropriate choice of the prefilter needed for the synthesis and application of the resulting controller is not straight forward. Even more, we have detected some discrepancies with previous results presented for the wing rock problem and the satellite attitude control problem. The cause of the discrepancy is explored and all parameters and L1 conditions and bounds are being calculated to verify compliance with L1 Adaptive control theory. It has been found that, unlike the results by Hovakymian and Cao[6], the resulting controller does not produce tracking and only with a different filter tracks with stationary error.

MATERIALS AND METHODS

System dynamics: The general system dynamics are the following[6]:

$$\dot{x} = A_m x(t) + b\big(u(t) + f(t, x(t))\big) \tag{1}$$

where: $x(0) = x_0$; $y(t) = c^T x(t)$ (^{T}means Transpose); $x(t)\in R^n$, is the (measured) state vector; $A_m \in R^{n\times n}$ is a known Hurwitz matrix specifying the desired closed-loop dynamics; b $c^T \in R^n$ are known constant vectors; $u(t)\in R$, is the control input; f(t, x): $R\times R^n \rightarrow R$ is an unknown nonlinear map, continuous in its arguments; $y(t)\rightarrow R$ is the regulated output. The initial condition x_0 is assumed to be inside an arbitrarily large known set. $\|x_0\|_\infty \le \rho_0 < \infty$ with known $\rho_0 > 0$[6]. The following assumptions are made:

- Uniform boundedness of f(t, 0). There exists B>0 such that: $|f(t,0)| \le B, \forall t \ge 0$
- Semiglobal uniform boundedness of partial derivatives of f(t)

There exists $df_t(\delta)>0$ and $df_x(\delta)>0$ independent of time for arbitrary $\delta>0$ such that for arbitrary $\|x\|_\infty \le \delta$, the partial derivatives of f(t, x) are piecewise-continuous and bounded[6] $\|\partial f(t,x)/\partial x\|_1 \le d_{f_x}(\delta), |\partial f(t,x)/\partial t| \le d_{f_t}(\delta)$.

The control objective is to design a full state feedback adaptive controller system to ensure y(t) tracks a given bounded piecewise continuous reference signal r(t) with quantifiable performance bounds[6].

General L1 adaptive system architecture: The L1 adaptive Control Block systems are composed by:

State space System Representation (SR): The system can be written in state-space form by using (1):

$$\dot{x} = A_m x(t) + b\big(\omega u(t) + f(t, x(t))\big);\ x(0) = x_0;\ y(t) = c^T x(t)$$

State Predictor (SP): The state predictor can be considered as:

$$\dot{\hat{x}}(t) = A_m \hat{x}(t) + b\big(\hat{\omega}(t)u(t) + \hat{\theta}(t)\|x(t)\|_\infty + \hat{\sigma}(t)\big) \tag{2}$$

Where: $\hat{x}(0) = x_0;\ \hat{y}(t) = c^T \hat{x}(t);\ \hat{\omega}(t) \in R\ \hat{\theta}(t) \in R$ and $\hat{\sigma}(t) \in R$ are the adaptive estimates.

Adaptation Law (AL): By means of the approach proposed by Hovakymian and Cao[6], the adaptive laws are defined by the projection operators as follow:

$$\begin{aligned}
\dot{\hat{\theta}}(t) &= \Gamma \mathrm{Proj}\big(\hat{\theta}(t), -\tilde{x}^T(t)Pb\|x(t)\|_\infty\big), \hat{\theta}(0) = \hat{\theta}_0 \\
\dot{\hat{\sigma}}(t) &= \Gamma\, \mathrm{P\,roj}\big(\hat{\sigma}(t), -\tilde{x}^T(t)Pb\big), \hat{\sigma}(0) = \hat{\sigma}_0 \\
\dot{\hat{\omega}}(t) &= \Gamma\, \mathrm{P\,roj}\big(\hat{\omega}(t), -\tilde{x}^T Pbu(t)\big), \hat{\omega}(0) = \hat{\omega}_0
\end{aligned} \tag{3}$$

Where:

$\hat{\theta}(t)$	=	The semi linear adaptive estimate of the nonlinear function
$\hat{\omega}(t)$	=	The uncertain estimate input gain
$\hat{\sigma}(t)$	=	The semi linear independent term of the adaptive estimate time function
$\tilde{x}(t) = \hat{x}(t) - x(t)$	=	The state error
$\Gamma \in R^+$	=	The adaptive gain

Proj(v, z) = the projection operator of z towards v and $\|x(t)\|_\infty$ is the infinity norm of state vector x(t). The matrix P is the solution of algebraic Lyapunov equation:

$$A_m^T P + PA_m = -Q$$

where $P = P^T > 0$, and $Q = Q^T > 0$ (^{T} means Transpose), Hovakymian and Cao[6].

Control Law (CL): The Laplace transform of the control signal u(s) is generated as the output of the following (feedback) system:

$$u(s) = -kD(s)\big(\hat{\eta}(s) - k_g r(s)\big) \tag{4}$$

where ή(s) is the Laplace transform of ή(t) given as:

$$\hat{\eta}(t) = \hat{\omega}(t)u(t) + \hat{\theta}(t)\|x(t)\|_\infty + \hat{\sigma}(t) \tag{5}$$

where ή(t) is the input gain and nonlinear function estimate; $\hat{\theta}(t)\|x(t)\|_\infty + \hat{\sigma}(t)$ is the semi linear adaptive estimate of f(t, x); k is the L1 filter gain; D(s) is the Laplace transform of L1 filter; $k_g = -1/\left(c^T A_m^{-1} b\right)$ is the reference signal pre filter and r(s) is the Laplace transform of reference command signal[6]. Figure 1 shows the interconnection or relation between these blocks.

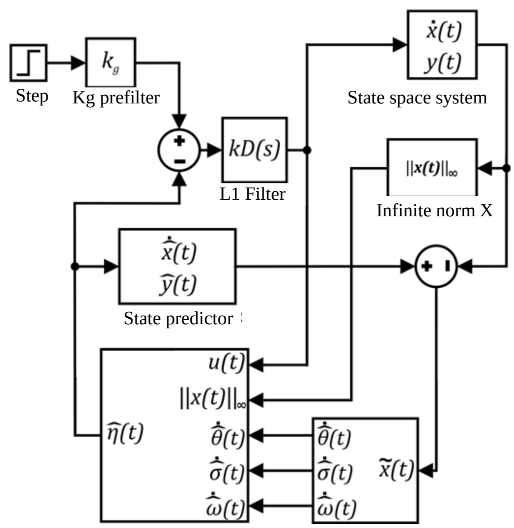

Fig. 1: L1 Adaptive control block system interconnection

L1 Requisites and conditions

Conditions relating L1 filter: The conditions relating to L1 filter design are:

- k>0, filter gain is positive
- D(s) must be a strictly proper transfer function which lead to C(s) transfer function given by: $C(s)=\omega kD(s)/(1+\omega kD(s))$
- C(s) must be a Strictly proper stable transfer function, for all $\omega \in [\omega_l\ \omega_u]$, known bounds on ω, low and upper
- DC gain must be C(0) = 1. Let $x_{in}(t)$ be the signal with its Laplace transform being $x_{in}(s)=(sI-A_m)^{-1}x_0$
- Since, A_m is Hurwitz, k and D(s) must ensure that for a given ρ_0, bound on $\|x_0 = x(0)\|_\infty \le \rho_0$, there exists $\rho_r > \rho_{in} = \left\| s(sI-A_m)^{-1} \right\|_{L_1} \rho_0$ such that d) is satisfied

Uniform boundedness of unknown parameters: $\theta \in \Theta$ (Known Convex Compact set). $|\sigma(t)| \le \Delta \in R^+$ known, $\forall t \ge 0$. Uniform boundedness of rate of variation of parameters:

$$\left\|\dot{\theta}(t)\right\| \le d_\theta < \infty, \forall t \ge 0$$
$$\left\|\dot{\sigma}(t)\right\| \le d_\sigma < \infty, \forall t \ge 0$$

L1 norm condition:

$$\left\|G(s)\right\|_{L_1} < \frac{\rho_r - \left\|H(s)C(s)k_g\right\|_{L_1}\left\|r\right\|_{L_\infty} - \rho_{in}}{L_{\rho_r}\rho_r + B} \tag{6}$$

where, $G(s)=H(s)(1-C(s)),\ H(s)=(sI-A_m)^{-1}b$ and $k_g=-1/(c^T A_m^{-1} b)$ is the feed-forward gain required for tracking of step reference command r(t) with zero steady-state error[6]. Further let:

Table 1: Coefficients for wing rock motion

α	a_0	a_1	a_2	a_3	a_4	W
27°	0.005	-0.01	0.2	-0.0025	0.025	0.9
35°	0.006	-0.012	0.2	-0.0075	0.04	1.2

$$L_\delta = \frac{\bar{\delta}(\delta)}{\delta} d_{f_x}\left(\bar{\delta}(\delta)\right), \quad \bar{\delta}(\delta) = \delta + \bar{\gamma}_1 \tag{7}$$

where, $\bar{\gamma}_1$ is an arbitrary small positive constant,? $\delta = \rho_r, L_\delta = L_{\rho_r}$ If (d) is satisfied and all other expressed assumptions, y(t) tracks r(t)[6].

Wing rock modeling and L1 adaptation control: Wing Rock dynamics can be described by a second-order nonlinear system[8-10]. An analytical model of wing rock is given by:

$$\begin{aligned}\ddot{\phi}(t) = &-\frac{a_0}{t_r^2}\phi(t) - \frac{a_1}{t_r}\dot{\phi}(t) - a_2\left|\dot{\phi}(t)\right|\dot{\phi}(t) - \frac{a_3}{t_r^2}\phi^3(t) \\ &-\frac{a_4}{t_r}\dot{\phi}(t)\phi^2(t) - \frac{\omega}{t_r^2}u(t) - d\left(t, \phi(t), \dot{\phi}(t)\right)\end{aligned} \tag{8}$$

where, $\Phi(t) \in R$ is the roll angle in degrees, Φ(t) is the roll angle speed in deg/s, Φ(t) is the roll angle acceleration in deg/s^2. Both the roll angle Φ(t) and its derivative Φ(t) are assumed to be available for feedback.

In the same way, $u(t) \in R$ is the anti-symmetric aileron deflection in degrees, $\omega \in R$ is the unknown control effectiveness; a is the angle of attack, $d(t, \phi(t), \dot{\phi}(t)) \in R$ models disturbances and unknown nonlinearities, t_r is the reference time conversion coefficient.

The aerodynamic coefficients a_0 to a_4 depend upon the angle of attack and are given in Table 1 for two fixed values of attack α[10]. For the air speed v_f = 30 m sec^{-1} and the wingspan b_ω = 169 mm, the reference time conversion coefficient $t_r = b_\omega/2v_f$ = 0.0028 sec[6].

Letting $x(t) = [\phi,\ \dot{\phi}]^T$ the state vector, it is then possible to write the equation dynamics in state space representation as:

$$\dot{x} = A_m x(t) + b\left(\omega u_{ad}(t) + g(t, x(t))\right) \tag{9}$$

With:

$$x(0) = x_0;\ \phi(t) = c^T x(t)$$

where: $A_m = A - bk_m^T,\ k_m \in R^2$ is the static feedback gain and $u_{ad}(t)$ is the adaptive control input. Considering the control law as follow:

$$u(t) = u_m(t) + u_{ad}(t); \quad u_m(t) = -k_m^T x(t);$$
$$A = \begin{bmatrix} 0 & 1 \\ -a_0/t_r^2 & -a_1/t_r \end{bmatrix}; b = \begin{bmatrix} 0 \\ 1/t_r^2 \end{bmatrix}; c = \begin{bmatrix} 1 \\ 0 \end{bmatrix};$$
$$g(t,x) = (1-\omega)k_m^T x + g_0(t,x)$$

letting $g_0(t, x)$ as follows:

$$g_0(t,x) = -t_r^2 a_2 \left|\dot{\phi}(t)\right| \dot{\phi}(t) - a_3\phi^3(t)$$
$$-t_r a_4 \dot{\phi}(t)\phi^2(t) - t_r^2 d\left(t, \phi(t), \dot{\phi}(t)\right)$$

As a result of $u_m(t)$ chosen value, A_m turns Hurwitz:

$$A_m = \left(A - bk_m^T\right) = \begin{bmatrix} 0 & 1 \\ -a_{m1} & -a_{m2} \end{bmatrix} \quad (10)$$

$$k_m = \begin{bmatrix} t_r^2 a_{m1} - a_0 \\ t_r^2 a_{m2} - t_r a_1 \end{bmatrix} \quad (11)$$

where a_{m1} and a_{m2} are design parameters specifying desired closed loop dynamics. The adaptation control signal should be according to L1 adaptive theory:

$$u_{ad}(s) = -kD(s)\left(\hat{\eta}(s) - k_g r(s)\right) \quad (12)$$

Where:

$$\hat{\eta}(t) = \hat{\omega}(t)u(t) + \hat{\theta}(t)\left\|x(t)\right\|_\infty + \hat{\sigma}(t)$$

The terms ή(s) and r(s) are the Laplace transforms of ή(t) and r(t), respectively.

RESULTS AND DISCUSSION

Previous and actual simulation results: Performance of the L1 control is measured according to the following objective:

The L1 controlled system should track the state $x_{ref}(t)$ of a reference system using a control input $u_{ref}(t)$ that compensates the uncertainties and nonlinearities of the real uncontrolled system, within the bandwidth of the C(s) filter:

$$\dot{x}_{ref}(t) = A_m x_{ref}(t) + b\left(\omega u_{ref}(t) + f\left(t, x_{ref}(t)\right)\right) \quad (13)$$

With: $x_{ref}(0) = x_0$ and the Laplace transforms of signals are given as:

$$u_{ref}(s) = -\frac{C(s)}{\omega}\left(\eta_{ref}(s) - k_g r(s)\right) \quad (14)$$

With:

$$\eta_{ref}(t) = f\left(t, x_{ref}(t)\right); \; y_{ref}(t) = c^T x_{ref}(t)$$

If the L1 norm condition (d) is satisfied (along with the assumptions (a)-c)), then ρ_r is a uniform bound for the reference state and ρ_{ur} a uniform bound for the reference control input where:

$$\left\|x_{ref}\right\|_{L_\infty} < \rho_r; \left\|u_{ref}\right\|_{L_\infty} < \rho_{ur}$$

Where:

$$\rho_{ur} = \left\|C(s)/\omega\right\|_{L_1}\left(L_{\rho_r}\rho_r + B + \left|k_g\right|\left\|r\right\|_{L_\infty}\right) \quad (15)$$

And let γ_1 be given by:

$$\gamma_1 = \frac{\left\|C(s)\right\|_{L_1}}{1 - \left\|G(s)\right\|_{L_1} \times L_{\rho r}}\gamma_0 + \beta \quad (16)$$

where γ_0 and β are arbitrary small positive constants. If the adaptive gain Γ is chosen such:

$$\Gamma \geq \frac{\theta_m(\rho, \rho_u)}{\lambda_{min}(P) \times \gamma_0^2} \quad (17)$$

Where:

$$\theta_m(\rho,\rho_u) = 4\theta_b^2 + 4\Delta^2 + \left(\omega_u - \omega_l\right)^2 + 4\frac{\lambda_{max}(P)}{\lambda_{min}(Q)} \times \left(\theta_b d_\theta(\rho,\rho_u) + \Delta d_\sigma(\rho,\rho_u)\right) \quad (18)$$

where, $\lambda_{max/min}$ is the max or min eigen value of matrix. $\rho = \rho_r + \overline{\gamma}_1$, $\rho_u = \rho_{ur} + \gamma_2$. It then follows that:

$$\left\|x_\tau\right\|_{L_\infty} < \rho, \left\|u_\tau\right\|_{L_\infty} < \rho_u \quad (19)$$

$$\left\|\tilde{x}(t) = \hat{x}(t) - x(t)\right\|_{L_\infty} \leq \gamma_0 \quad (20)$$

where, $\tilde{x}(t) = \hat{x}(t) - x(t)$, $\Gamma \in R^+$ is the adaptation gain. Then we have $\left\|\tilde{x}\right\|_{L_\infty} \leq \gamma_0$ and:

$$\left\|x_{ref} - x\right\|_{L_\infty} \leq \gamma_1 \quad (21)$$

$$\left\|u_{ref} - u\right\|_{L_\infty} \leq \gamma_2 \quad (22)$$

Where:

$$\gamma_2 = \left\|C(s)/\omega\right\|_{L_1} L_{\rho_r}\gamma_1 + \left\|H_1(s)/\omega\right\|_{L_1}\gamma_0 \quad (23)$$

Let:

$$H_1(s) = C(s)\frac{c_0^T}{c_0^T H(s)} \quad (24)$$

Table 2: Calculus for establishing if the L1 norm condition (d) is satisfied in the wing rock system

Expression	Results
$\left\|H(s)(1-C(s))\right\|_{L1} = \left\|G(s)\right\|_{L1}$	74.1100
Left side of condition (d)	
$\left(\rho_r - \left\|H(s)(C(s)k_g)\right\|_{L1}\left\|r\right\|_{L_\infty} - \rho_{in}\right) / \left(L_{\rho r}\rho_r + B\right)$	136.3338
L1 condition (d), right side	

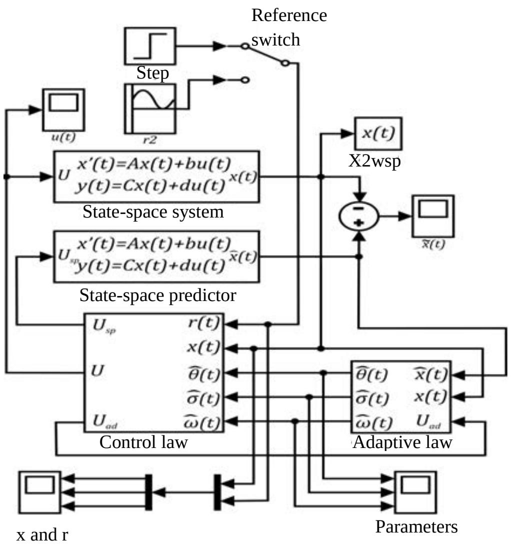

Fig. 2: Simulink main blocks for wing rock with strictly proper filter D(s) as described by Hovakymian and Cao[6] and Eq. 25. Results obtained are as shown in Fig. 3

where $c_0 \in R^n$ is a vector that renders $H_1(s)$ Bounded Input Bounded Output (BIBO) stable. Then, the system state vector x(t) tracks the reference system state vector $x_{ref}(t)$, and correspondingly the system control input tracks the reference control input $u_{ref}(t)$ and the error dynamics x(t) of L1 controlled system is brought arbitrarily small ($\leq \gamma_0$). The filter D(s) used for calculation of the norm of C(s) given by Hovakymian and Cao[6] is strictly proper:

$$D(s) = \frac{(s+500)(s+0.004)^2}{s(s+368)(s+0.00439)^2} \tag{25}$$

The filter gain k = 144 is used. If respective calculus are made for establishing if the L1 norm condition (d) is satisfied in the Wing Rock system according with respective norms and bounds, the results are shown in Table 2.

Which indeed is satisfied as right side must be greater than the left side. It is simulated with Simulink and MATLAB with main blocks as shown in Fig. 2 and 3.

Positive instability for the state vector $x(t) = [\varphi \ \dot{\varphi}]^T$ is obtained. To eliminate this instability and obtain a

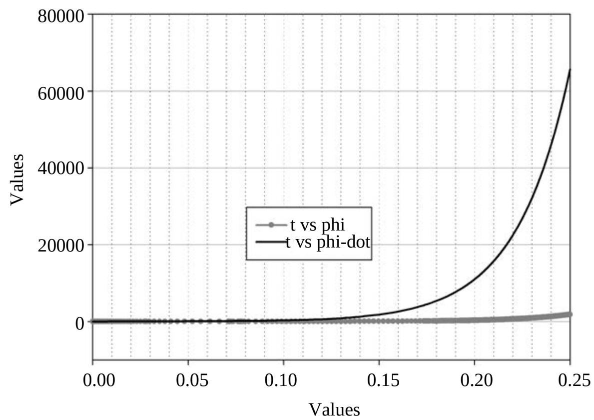

Fig. 3: Wing Rock State vector x(t) unstable response. $x(t) = [\varphi \ \dot{\varphi}]^T$ with L1 filter D(s) as Eq. 25

moderate tracking with an stationary error close to 30%, a modification of L1 filter was required, eliminating the pole at zero and increasing the filter gain k:

$$D_m(s) = \frac{(s+500)(s+0.004)^2}{(s+368)(s+0.00439)^2} \tag{26}$$

Gain k had to be modified to k = 144×150 = 21600. The result for the state vector evolution is shown in Fig. 4. Bode diagrams are shown for both Dm(s), D(s) L1 filters in Fig. 5a (Mag), Fig. 5b (Phase) and Fig. 6.

As depicted in Fig. 4a and 4b, there is tracking of reference command r(t) = 60 with a steady state error close to 30% with r(t) = 20 and k = 18720, a steady state error close to 16.6% is obtained.

In Fig. 5a, b the frequency response of the modified (proper) filtershows a frequency band between 10^{-2} to 10^{2} Hz, decaying close to 3 dB at higher frequencies. For the original filter shown in Fig. 6, at 10^{-2} Hz there is a gain decay close to 40 dB and continues to decay to 0 dB at 10^{2} Hz. Frequencies above 10^{2} Hz are then suppresed in the original filter compared to the modified filter.

Design and simulation was performed on two other examples for the literature: the wing rock with L1 filter of first order by Kharisov and Hovakimyan[8] and the Wing Rock as treated by Cao *et al.*[9].

The second example is taken from Kharisov and Hovakimyan[8] and consists of the Wing Rock problem but using the first order L1 filter proposed:

$$D(s) = 1/s \tag{27}$$

With filter gain k = 50000. Results are presented in Fig. 7 and 8. Again, simulation reproduces results

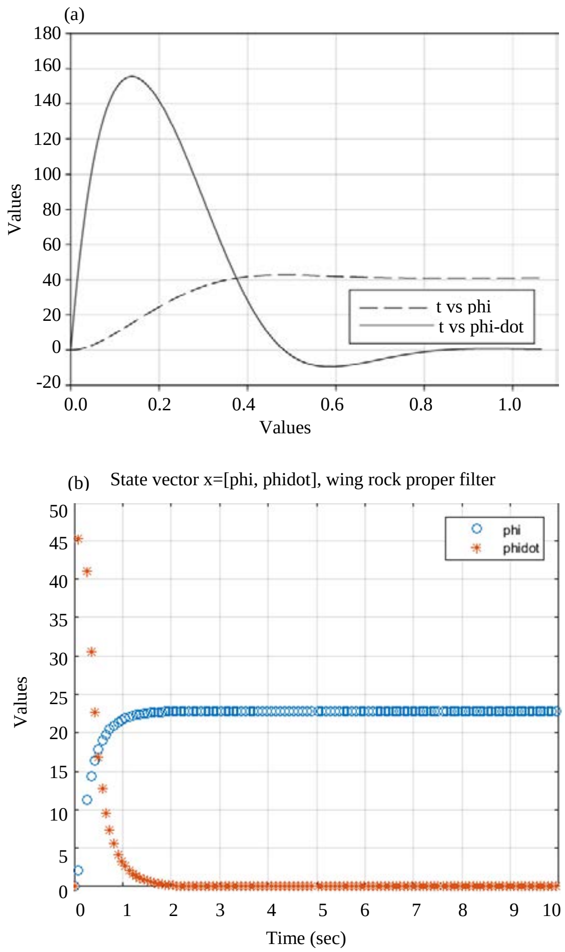

Fig. 4(a, b): (a) Wing Rock State vector x(t) stable response. $x(t) = [\varphi \;\; \dot{\varphi}]^T$ time in seconds with a reference command r(t) = 60 and (b) = Wing rock state vector x(t) stable response. $x(t) = [\varphi \;\; \dot{\varphi}]^T$, time in seconds, with a reference command r(t) = 20. Filter gain k = 144×130 = 18720

obtained by Kharisov and Hovakimyan[8]. Finally, the third example is taken from[9] and again considers the Wing Rock problem but with a different parameter estimation scheme in which the dynamics of the system are expressed as follows:

$$\dot{x} = A_m x(t) + b\left(u(t) + \theta^T(t) H(x(t))\right)$$

where:

$$x(0) = x_0,\; H(x(t)) = \begin{bmatrix} 1 & x_1 & x_2 & |x_1|x_2 & |x_2|x_2 & x_1^3 \end{bmatrix}^T$$
$$\text{and } \theta = \begin{bmatrix} d_0 & d_1 & d_2 & d_3 & d_4 & d_5 \end{bmatrix}$$

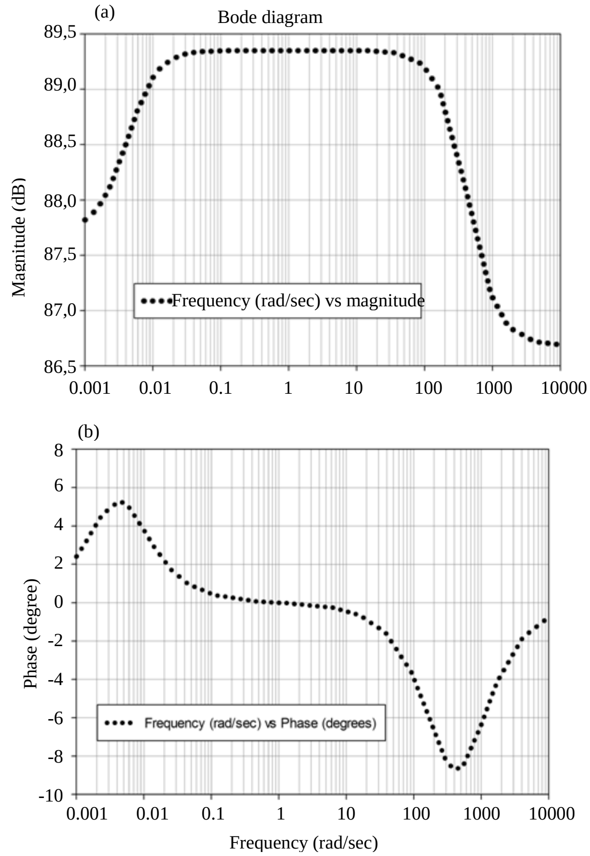

Fig. 5(a, b): (a) Bode plot (Mag) for L1 filter ($kD_m(s)$) modified with respect to Hovakymian and Cao, [6] for Wing Rock, following Eq. 26 and (b) Bode plot (Phase) for L1 filter (($kD_m(s)$) modified with respect to Hovakymian and Cao[6] for wing rock, following Eq. 26

θ is the 1 by 6 parameter vector to be estimated, which is different from the estimation done in the previous cases. The filter used is first order:

$$C(s) = 25/(s+25) \tag{28}$$

And the control input u(t) applies only the filtering to $\hat{\theta}^T H(x(t))$:

$$u(s) = k_g r(s) + C(s)\bar{r}(s)$$

where, $\bar{r}(s) = L\left(\theta^T H(x(t))\right)$ (L = Laplace Transform). Results obtained shown on Fig. 9, reproduce well what is described by Cao *et al.*[9].

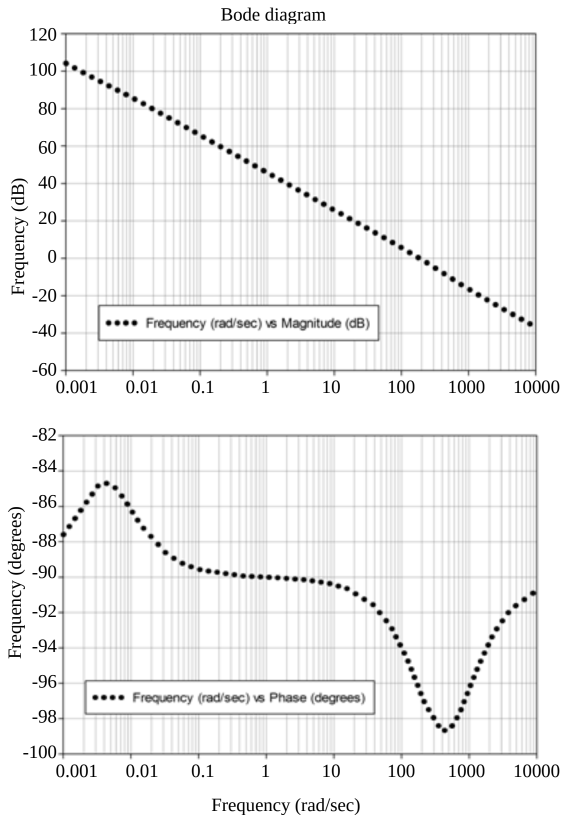

Fig. 6: Bode plot for L1 filter (kD(s)) following Eq. 25 and as in Hovakymian and Cao[6] for wing rock

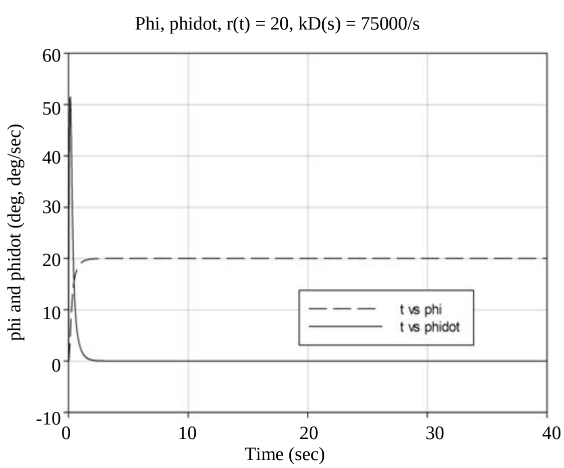

Fig. 7: Wing rock state vector x(t) step command (r(t) = 20) stable response. $x(t) = [\varphi \ \dot{\varphi}]^T$ with first order L1 filter[8] and wing rock simulation with filters Eq. 25 and 26

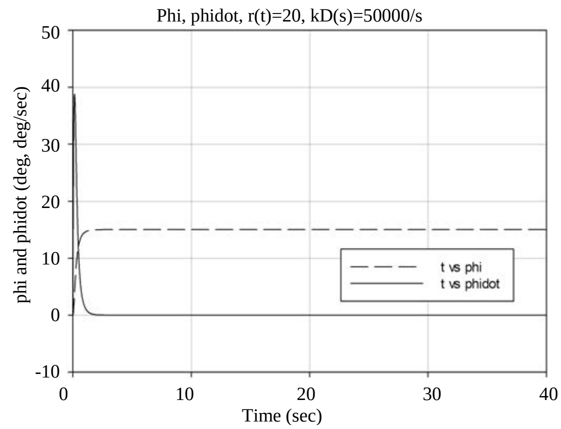

Fig. 8: Wing Rock state vector x(t) step command (r(t) = 15) stable response. $x(t) = [\varphi \ \dot{\varphi}]^T$ with first order L1 filter as proposed by Kharisov and Hovakimyan[8] and simulink software as used in Wing Rock simulation with filters Eq. 25 and 26

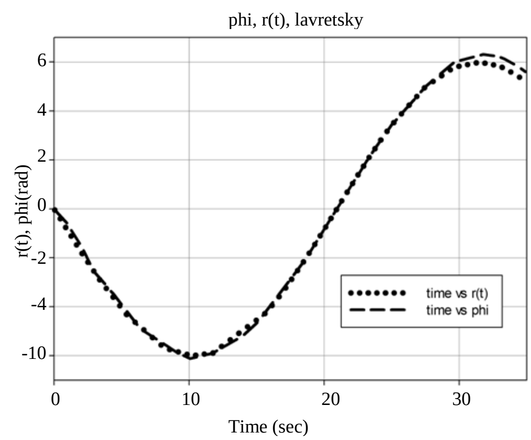

Fig. 9: Wing rock state vector x(t) with sinusoidal command (r(t) = -6s in (0.15t) stable response[8]. $x(t) = [\varphi \ \dot{\varphi}]^T$ with first order L1 filter and different parameter estimation as compared to parameters estimated in Hovakymian and Cao[6] and Kharisov and Hovakimyan[7] and simulink software with changes compared to Wing Rock simulation with filters Eq. 25 and 26

CONCLUSION

Although, L1 assumptions, bounds and conditions hold in Wing Rock, no stable response is obtainable with parameters and L1 filter proposed and used in Hovakymian and Cao[6]. A change in L1 filter contrary to L1 norm necessary and sufficient condition (d) as it is not strictly proper, gives stable step response but with stationary error. Nevertheless, L1 controller shows good stability and performance results, according to simulations performed in this article which reproduce results obtained by Kharisov and Hovakimyan[8]. Good reproduction by Kharisov and Hovakimyan[8] obtained of wing rock treatment, requiring changes in estimation scheme, reassure the validity of the simulation software developed and provides confidence about the exception to L1 adaptive control theory found and exposed in this article.

In many other cases such as Rohr´s example and other aerospace applications[5] with matched uncertainties, L1 adaptive controllers perform well. As L1 filter designed is considered an open question in systems such as un-modeled actuator dynamics[6] where the unknown transfer function F(s) of the actuator is non relative degree one and minimum phase and in other systems with unmatched nonlinearities, it is required a general procedure for L1 filter designed, or changes to L1 Adaptive algorithm.

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