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Solving Directly Third Order Ordinary Differential Equations Using One-Step Block Method with Generalized Three Hybrid Points and Fourth Derivative Presence

Mohammad Alkasassbeh and Zurniomar

Department of Mathematics, School of Quantitative Sciences, College of Arts and Sciences, Universiti Utara Malaysia, Malaysia

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Corresponding Author:
Mohammad Alkasassbeh
Department of Mathematics, School of Quantitative Sciences, College of Arts and Sciences, Universiti Utara Malaysia, Malaysia

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Abstract: This study proposes one-step block method with generalized three-hybrid points for solving initial value problems of third order ordinary differential equations directly using interpolation and collocation strategy. In deriving this method, the approximate power series function is interpolated at $\{x_n, x_{n+r}, x_{n+s}\}$ while its third and fourth derivatives are collocated at $\{x_n, x_{n+r}, x_{n+s}, x_{n+t}$ and $x_{n+1}\}$ in the given interval. Not only the developed method possesses good numerical method properties but it also out performs the existing methods when solving the same problems.

INTRODUCTION

This article considers the solution of general third order Initial Value Problems (IVPs) of the following form:

$$y''' = f(x, y, y', y''),\ \ y(a) = \alpha,$$
$$y'(a) = \beta,\ \ y''(a) = \gamma,\ \ x \in [a, b] \tag{1}$$

where, f is continuously differentiable. This kind of differential equation is frequently encountered in science, engineering, as well as other real life phenomena. Scholars such as[1-8] have developed several numerical methods to approximate the solution of (1).

Initially, Milne[9] in the early 1950's to increase the performance of the numerical method by reducing the execution time. Later, in order to improve the accuracy of the numerical methods[10] proposed predictor-correct or block scheme.

To overcome this drawback, hybrid methods were developed. According to Lambert[11], hybrid methods are not only capable of overcoming zero stability barrier occurred in block methods assumed by Gupta[12], they also have the ability of utilizing data at off-step points which contributes to better accuracy.

Furthermore, to enhance the numerical methods efficiency[13] developed a direct solution of second- order ordinary differential equation using a single-step hybrid block method of order five,

However, Ngwane and Jator[14] Proposed second derivative methods while[8] proposed a Simpson's type second derivative method for the solution of a first order stiff system of IVPs. Following those scholar's footsteps, a new fourth derivative method for solving third order ODEs directly using the approach of interpolation and collocation is derived.

DEVELOPMENT OF THE METHOD

The approximate solution considered for the third order IVPs given by:

$$p(x)=\sum_{j=0}^{2q+p-1} a_j\left(\frac{x-x_n}{h}\right)^j \tag{2}$$

where, p = 3 and q = 5 represent the number of interpolation and collocation points respectively. Differentiating (2) three and four times yield:

$$p'''(x)=\sum_{j=3}^{2q+p-1}\frac{j!a_j}{h^3(j-3)!}\left(\frac{x-x_n}{h}\right)^{j-3}=f(x,y,y',y'') \tag{3}$$

$$p^{(iv)}(x)=\sum_{j=4}^{2q+p-1}\frac{j!a_j}{h^4(j-4)!}\left(\frac{x-x_n}{h}\right)^{j-4}=g(x,y,y',y'',y''') \tag{4}$$

Interpolating (2) at $\{x_{n+j} = x_n + jh,\ j = 0, r, s\}$ and collocating (3) and (4) at all points $x_{n+j} = x_n + jh$, $j = \{0, r, s, t, 1\}$ where $\{r, s, t\}\in(0, 1)$, a system of equations in matrix form is produced:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0\\
1 & r & r^2 & r^3 & r^4 & r^5 & \cdots & r^{\hat{u}}\\
1 & s & s^2 & s^3 & s^4 & s^5 & \cdots & s^{\hat{u}}\\
0 & 0 & 0 & \frac{3!}{0!h^3} & 0 & 0 & \cdots & 0\\
0 & 0 & 0 & \frac{3!}{0!h^3} & \frac{4!r}{1!h^3} & \frac{5!r^2}{2!h^3} & \cdots & \frac{\hat{u}!r^{(\hat{u}-3)}}{(\hat{u}-3)!h^3}\\
0 & 0 & 0 & \frac{3!}{0!h^3} & \frac{4!s}{1!h^3} & \frac{5!s^2}{2!h^3} & \cdots & \frac{\hat{u}!s^{(\hat{u}-3)}}{(\hat{u}-3)!h^3}\\
0 & 0 & 0 & \frac{3!}{0!h^3} & \frac{4!t}{1!h^3} & \frac{5!t^2}{2!h^3} & \cdots & \frac{\hat{u}!t^{(\hat{u}-3)}}{(\hat{u}-3)!h^3}\\
0 & 0 & 0 & \frac{3!}{0!h^3} & \frac{4!}{1!h^3} & \frac{5!}{2!h^3} & \cdots & \frac{\hat{u}!}{(\hat{u}-3)!h^3}\\
0 & 0 & 0 & 0 & \frac{4!}{0!h^4} & 0 & \cdots & 0\\
0 & 0 & 0 & 0 & \frac{4!}{0!h^4} & \frac{5!r}{1!h^4} & \cdots & \frac{\hat{u}!r^{(\hat{u}-4)}}{(\hat{u}-4)!h^4}\\
0 & 0 & 0 & 0 & \frac{4!}{0!h^4} & \frac{5!s}{1!h^4} & \cdots & \frac{\hat{u}!s^{(\hat{u}-4)}}{(\hat{u}-4)!h^4}\\
0 & 0 & 0 & 0 & \frac{4!}{0!h^4} & \frac{5!t}{1!h^4} & \cdots & \frac{\hat{u}!t^{(\hat{u}-4)}}{(\hat{u}-4)!h^4}\\
0 & 0 & 0 & 0 & \frac{4!}{0!h^4} & \frac{5!}{1!h^4} & \cdots & \frac{\hat{u}!}{(\hat{u}-4)!h^4}
\end{pmatrix}
\begin{pmatrix} a_0\\ a_1\\ a_2\\ a_3\\ a_4\\ a_5\\ a_6\\ a_7\\ a_8\\ a_9\\ a_{10}\\ a_{11}\\ a_{12}\end{pmatrix}
=
\begin{pmatrix} y_n\\ y_{n+r}\\ y_{n+s}\\ f_n\\ f_{n+r}\\ f_{n+s}\\ f_{n+t}\\ f_{n+1}\\ g_n\\ g_{n+r}\\ g_{n+s}\\ g_{n+t}\\ g_{n+1}\end{pmatrix} \tag{5}$$

where, $\hat{u}$ = 2q + p-1. The resulting system (5) is then solved for the unknown coefficients $a'_i s, i = \{0, 1, 2,..., 12\}$ which then substituted into (1) to get:

$$p(x)=\sum_{i=0,r}\alpha_i y_{n+i}+h^3\sum_{i=0,r,s,1}\beta_i f_{n+i}+h^4\sum_{i=0,r,s,1}\gamma_i g_{n+i} \tag{6}$$

where, n = 0, 1, 2,..., N, $h = x_n-x_{n-1}$ is the constant step size for the partition π_N of the interval [a, b] which is given by $\pi_N = [a = x_0<x_1<...x_{N-1}<x_N = b]$, α_i, β_i, and γ_i are undetermined constants listed in Appendix I, $f_{n+1} = f(x+ ih)$ and:

$$g_{n+i}=\frac{df\left(x_{n+i},y_{n+i},y'_{n+i},y''_{n+i}\right)}{dx}$$

The first and second derivatives of (6) are:

$$p'(x)=\frac{1}{h}\sum_{i=0,r}\alpha'_i y_{n+i}+h^2\sum_{i=0,r,s,1}\beta'_i f_{n+i}+h^3\sum_{i=0,r,s,1}\gamma'_i g_{n+i} \tag{7}$$

$$p''(x)=\frac{1}{h^2}\sum_{i=0,r}\alpha''_i y_{n+i}+h\sum_{i=0,r,s,1}\beta''_i f_{n+i}+h^2\sum_{i=0,r,s,1}\gamma''_i g_{n+i} \tag{8}$$

Evaluating (6) at the non-interpolated points $\{x_{n+t}, x_{n+1}\}$. While, Eq. 7 and 8 are evaluated at all points $\{x_{n+i}\}$, i = {0, r, s, t, 1} to produce the following general equations in block form:

$$A^{[0]}Y_m^{[1]}=A^{[1]}Y_m^{[0]}+\sum_{i=0}^{1}B^{[i]}F_m^{[i]}+\sum_{i=0}^{1}D^{[i]}G_m^{[i]} \tag{9}$$

where, $A^{[0]}$ is an identity matrix of order 12 and:

$$A^{[1]}=\begin{bmatrix}
0&0&0&1&0&0&0&rh&0&0&0&\frac{r^2h^2}{2}\\
0&0&0&1&0&0&0&sh&0&0&0&\frac{s^2h^2}{2}\\
0&0&0&1&0&0&0&th&0&0&0&\frac{t^2h^2}{2}\\
0&0&0&1&0&0&0&h&0&0&0&\frac{h^2}{2}\\
0&0&0&0&0&0&0&1&0&0&0&rh\\
0&0&0&0&0&0&0&1&0&0&0&sh\\
0&0&0&0&0&0&0&1&0&0&0&th\\
0&0&0&0&0&0&0&1&0&0&0&h\\
0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&1
\end{bmatrix},$$

$$B^{[0]}=\begin{bmatrix}0&0&0&0&0&0&0&0&0&0&0&B^{[0]}_{112}\\0&0&0&0&0&0&0&0&0&0&0&B^{[0]}_{212}\\0&0&0&0&0&0&0&0&0&0&0&B^{[0]}_{312}\\0&0&0&0&0&0&0&0&0&0&0&B^{[0]}_{412}\\0&0&0&0&0&0&0&0&0&0&0&B^{[0]}_{512}\\0&0&0&0&0&0&0&0&0&0&0&B^{[0]}_{612}\\0&0&0&0&0&0&0&0&0&0&0&B^{[0]}_{712}\\0&0&0&0&0&0&0&0&0&0&0&B^{[0]}_{812}\\0&0&0&0&0&0&0&0&0&0&0&B^{[0]}_{912}\\0&0&0&0&0&0&0&0&0&0&0&B^{[0]}_{1012}\\0&0&0&0&0&0&0&0&0&0&0&B^{[0]}_{1112}\\0&0&0&0&0&0&0&0&0&0&0&B^{[0]}_{1212}\end{bmatrix},\ B^{[1]}=\begin{bmatrix}B^{[1]}_{11}&B^{[1]}_{12}&B^{[1]}_{13}&B^{[1]}_{14}\\B^{[1]}_{21}&B^{[1]}_{22}&B^{[1]}_{23}&B^{[1]}_{24}\\B^{[1]}_{31}&B^{[1]}_{32}&B^{[1]}_{33}&B^{[1]}_{34}\\B^{[1]}_{41}&B^{[1]}_{42}&B^{[1]}_{43}&B^{[1]}_{44}\\B^{[1]}_{51}&B^{[1]}_{52}&B^{[1]}_{53}&B^{[1]}_{54}\\B^{[1]}_{61}&B^{[1]}_{62}&B^{[1]}_{63}&B^{[1]}_{64}\\B^{[1]}_{71}&B^{[1]}_{72}&B^{[1]}_{73}&B^{[1]}_{74}\\B^{[1]}_{81}&B^{[1]}_{82}&B^{[1]}_{83}&B^{[1]}_{84}\\B^{[1]}_{91}&B^{[1]}_{92}&B^{[1]}_{93}&B^{[1]}_{94}\\B^{[1]}_{101}&B^{[1]}_{102}&B^{[1]}_{103}&B^{[1]}_{104}\\B^{[1]}_{111}&B^{[1]}_{112}&B^{[1]}_{113}&B^{[1]}_{114}\\B^{[1]}_{121}&B^{[1]}_{122}&B^{[1]}_{123}&B^{[1]}_{124}\end{bmatrix},$$

$$D^{[0]}=\begin{bmatrix}0&0&0&0&0&0&0&0&0&0&0&D^{[0]}_{112}\\0&0&0&0&0&0&0&0&0&0&0&D^{[0]}_{212}\\0&0&0&0&0&0&0&0&0&0&0&D^{[0]}_{312}\\0&0&0&0&0&0&0&0&0&0&0&D^{[0]}_{412}\\0&0&0&0&0&0&0&0&0&0&0&D^{[0]}_{512}\\0&0&0&0&0&0&0&0&0&0&0&D^{[0]}_{612}\\0&0&0&0&0&0&0&0&0&0&0&D^{[0]}_{712}\\0&0&0&0&0&0&0&0&0&0&0&D^{[0]}_{812}\\0&0&0&0&0&0&0&0&0&0&0&D^{[0]}_{912}\\0&0&0&0&0&0&0&0&0&0&0&D^{[0]}_{1012}\\0&0&0&0&0&0&0&0&0&0&0&D^{[0]}_{1112}\\0&0&0&0&0&0&0&0&0&0&0&D^{[0]}_{1212}\end{bmatrix},D^{[1]}=\begin{bmatrix}D^{[1]}_{11}&D^{[1]}_{12}&D^{[1]}_{13}&D^{[1]}_{14}\\D^{[1]}_{21}&D^{[1]}_{22}&D^{[1]}_{23}&D^{[1]}_{24}\\D^{[1]}_{31}&D^{[1]}_{32}&D^{[1]}_{33}&D^{[1]}_{34}\\D^{[1]}_{41}&D^{[1]}_{42}&D^{[1]}_{43}&D^{[1]}_{44}\\D^{[1]}_{51}&D^{[1]}_{52}&D^{[1]}_{53}&D^{[1]}_{54}\\D^{[1]}_{61}&D^{[1]}_{62}&D^{[1]}_{63}&D^{[1]}_{64}\\D^{[1]}_{71}&D^{[1]}_{72}&D^{[1]}_{73}&D^{[1]}_{74}\\D^{[1]}_{81}&D^{[1]}_{82}&D^{[1]}_{83}&D^{[1]}_{84}\\D^{[1]}_{91}&D^{[1]}_{92}&D^{[1]}_{93}&D^{[1]}_{94}\\D^{[1]}_{101}&D^{[1]}_{102}&D^{[1]}_{103}&D^{[1]}_{104}\\D^{[1]}_{111}&D^{[1]}_{112}&D^{[1]}_{113}&D^{[1]}_{114}\\D^{[1]}_{121}&D^{[1]}_{122}&D^{[1]}_{123}&D^{[1]}_{124}\end{bmatrix}$$

whose entries are listed in Appendix II while the vectors $Y^{[0]}_m, Y^{[1]}_m, F^{[0]}_m, Y^{[0]}_m, F^{[1]}_m, G^{[0]}_m, G^{[1]}_m$ are defined as follows:

$$Y^{[0]}_m=\begin{bmatrix}y_{n-t}\\y_{n-s}\\y_{n-r}\\y_n\\y'_{n-t}\\y'_{n-s}\\y'_{n-r}\\y'_n\\y''_{n-t}\\y''_{n-s}\\y''_{n-r}\\y''_n\end{bmatrix},\ Y^{[1]}_m=\begin{bmatrix}y_{n+r}\\y_{n+s}\\y_{n+t}\\y_{n+1}\\y'_{n+r}\\y'_{n+s}\\y'_{n+t}\\y'_{n+1}\\y''_{n+r}\\y''_{n+s}\\y''_{n+t}\\y''_{n+1}\end{bmatrix},\ F^{[0]}_m=\begin{bmatrix}f_{n-11}\\f_{n-10}\\f_{n-9}\\f_{n-8}\\f_{n-7}\\f_{n-6}\\f_{n-5}\\f_{n-4}\\f_{n-3}\\f_{n-2}\\f_{n-1}\\f_n\end{bmatrix},\ G^{[0]}_m=\begin{bmatrix}g_{n-11}\\g_{n-10}\\g_{n-9}\\g_{n-8}\\g_{n-7}\\g_{n-6}\\g_{n-5}\\g_{n-4}\\g_{n-3}\\g_{n-2}\\g_{n-1}\\g_n\end{bmatrix},F^{[1]}_m=\begin{bmatrix}f_{n+r}\\f_{n+s}\\f_{n+t}\\f_{n+1}\end{bmatrix},G^{[1]}_m=\begin{bmatrix}g_{n+r}\\g_{n+s}\\g_{n+t}\\g_{n+1}\end{bmatrix}$$

ANALYSIS OF THE METHOD

Zero stability (3.1)

Definition 3.1: The hybrid block method formula (9) is said to be zero stable if no root R_m of the first characteristic equation $\rho(R)$ has modulus greater than one, i.e., $|R_m|<1$ and if $R_m = 1$ then the multiplicity of R_m must not exceed three.

To demonstrate that the roots of the first characteristic equation satisfies the prior definition we assume that $\{r, s\}\in(0, 1)$ and hence $\rho(R) = \det[RA^{[0]}-A^{[1]}]$:

$$\rho(R)=\det\left[RA^{[0]}-A^{[1]}\right]=\begin{vmatrix} R & 0 & 0 & -1 & 0 & 0 & 0 & -rh & 0 & 0 & 0 & \frac{-r^2h^2}{2} \\ 0 & R & 0 & -1 & 0 & 0 & 0 & -sh & 0 & 0 & 0 & \frac{-s^2h^2}{2} \\ 0 & 0 & R & -1 & 0 & 0 & 0 & -th & 0 & 0 & 0 & \frac{-t^2h^2}{2} \\ 0 & 0 & 0 & R-1 & 0 & 0 & 0 & -h & 0 & 0 & 0 & \frac{-h^2}{2} \\ 0 & 0 & 0 & 0 & R & 0 & 0 & -1 & 0 & 0 & 0 & -rh \\ 0 & 0 & 0 & 0 & 0 & R & 0 & -1 & 0 & 0 & 0 & -sh \\ 0 & 0 & 0 & 0 & 0 & 0 & R & -1 & 0 & 0 & 0 & -th \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & R-1 & 0 & 0 & 0 & -h \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R-1 \end{vmatrix}$$

Which implies that:

$$R^9\left(R^3-1\right)=0\Rightarrow R_m=\begin{cases} 0, & \text{if } m=1(1)9 \\ 1, & \text{if } m=10(1)12 \end{cases}$$

As a result, the developed method is zero stable.

Order of the method (3.2): The linear operator Θ associated with the hybrid block methods Eq. 9 is defined as:

$$\Theta\{y(x),h\}= A^{[0]}Y_m^{[1]}-A^{[1]}Y_m^{[0]}-\sum_{i=0}^{1}B^{[i]}F_m^{[i]}-\sum_{i=0}^{1}D^{[i]}G_m^{[i]}$$

Expanding the above equation in Taylor series and combining like terms we wind up with:

$$\Theta\{y(x),h\}=C_0h^0y(x)+C_1h^1y'(x)+\cdots+C_{\rho+3}h^{\rho+3}y^{(\rho+3)}(x) \quad (10)$$

According to Fatunla[15] and Lambert[11] method (9) is said to be of order ρ if:

$$C_0=C_1=\cdots=C_{\rho+2}=0 \text{ and } C_{\rho+3}\neq 0$$

The term $C_{\rho+3}$ is called the error constant and the local truncation error is given by:

$$t_{n+k}=C_{\rho+3}h^{\rho+3}y^{(\rho+3)}(x)+O\left(h^{\rho+3}\right)$$

Comparing like terms of y^i and h^i in Eq. 10 produces the coefficients

$$C_0=C_1=\cdots=C_{12}=0$$

with vector of error constants:

$$C_{13}=\begin{bmatrix} C_{13}^1 & C_{13}^2 & C_{13}^3 & C_{13}^4 & C_{13}^5 & C_{13}^6 \\ C_{13}^7 & C_{13}^8 & C_{13}^9 & C_{13}^{10} & C_{13}^{11} & C_{13}^{12} \end{bmatrix}^T$$

Where:

$$\begin{aligned} C_{13}^1 = r^7(&28r\wedge 6-91r^5s-91r^5t-91r^5+78r^4s^2+ \\ &312r^4st+312r^4s+78r^4t^2+312r^4t+78r^4- \\ &286r^3s^2t-286r^3s^2-286r^3st^2-1144r^3st- \\ &286r^3s-286r^3t^2-286r^3t+286r^2s^2t^2+ \\ &1144r^2s^2t+286r^2s^2+1144r^2st^2+1144r^2st+ \\ &286r^2t^2-1287rs^2t^2-1287rs^2t-1287rst^2+ \\ &1716s^2t^2)/1307674368000 \end{aligned}$$

$$\begin{aligned} C_{13}^2 = s^7(&78r^2s^4-286r^2s^3t-286r^2s^3+286r^2s^2t^2+ \\ &1144r^2s^2t+286r^2s^2-1287r^2st^2-1287r^2st+ \\ &1716r^2t^2-91rs^5+312rs^4t+312rs^4- \\ &286rs^3t^2-1144rs^3t-286rs^3+1144rs^2t^2+ \\ &1144rs^2t-1287rst^2+28s^6-91s^5t-91s^5+ \\ &78s^4t^2+312s^4t+78s^4-286s^3t^2-286s^3t+ \\ &286s^2t^2)/1307674368000 \end{aligned}$$

$$\begin{aligned} C_{13}^3 = t^7(&286r^2s^2t^2-1287r^2s^2t+1716r^2s^2- \\ &286r^2st^3+1144r^2st^2-1287r^2st+ \\ &78r^2t^4-286r^2t^3+286r^2t^2-286rs^{2t^3}+ \\ &1144rs^{2t^2}-1287rs^{2t}+312rst^4-1144rst^3 \\ &+1144rst^2-91rt^5+312rt^4-286rt^3+ \\ &78s^{2t^4}-286s^{2t^3}+286s^{2t^2}-91st^5+286st^3 \\ &+28t^6-91t^5+78t^4)/1307674368000 \end{aligned}$$

$$C_{13}^{4} = (1716r^2s^2t^2 - 1287r^2s^2t + 286r^2s^2 - 1287r^2st^2 + 1144r^2st - 286r^2s + 286r^2t^2 - 286r^2t + 78r^2 - 1287rs^2t^2 + + 1144rs^2t - 286rs^2 + 1144rst^2 - 1144rst + 312rs - 286rt^2 + 312rt - 91r + 286s^{2t^2} - 286s^{2t} + 78s^2 - 286st^2 + 312st - 91s + 78t^2 - 91t + 28)/1307674368000$$

$$C_{13}^{5} = r^6(14r^6 - 42r^5s - 42r^5t - 42r^5 + 33r^4s^2 + 132r^4st + 132r^4s + 33r^4t^2 + 132r^4t + 33r^4 - 110r^3s^2t - 110r^3s^2 - 110r^3st^2 - 440r^3st - 110r^3s - 110r^3t^2 - 110r^3t + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 + 396r^2st^2 + 396r^2st + 99r^2t^2 - 396rs^2t^2 - 396rs^2t - 396rst^2 + 462s^2t^2)/100590336000$$

$$C_{13}^{6} = s^6(33r^2s^4 - 110r^2s^3t - 110r^2s^3 + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 - 396r^2st^2 - 396r^2st + 462r^2t^2 - 42rs^5 + 132rs^4t + 132rs^4 - 110rs^3t^2 - 440rs^3t - 110rs^3 + 396rs^2t^2 + 396rs^2t - 396rst^2 + 14s^6 - 42s^5t - 42s^5 + 33s^4t^2 + 132s^4t + 33s^4 - 110s^3t^2 - 110s^3t + 99s^2t^2)/100590336000$$

$$C_{13}^{7} = t^6\ (99r^2s^2t^2 - 396r^2s^2t + 462r^2s^2 - 110r^2st^3 + 396r^2st^2 - 396r^2st + 33r^2t^4 - 110r^2t^3 + 99r^2t^2 - 110rs^2t^3 + 396rs^2t^2 - 396rs^2t + 132rst^4 - 440rst^3 + 396rst^2 - 42rt^5 + 132rt^4 - 110rt^3 + 33s^2t^4 - 110s^2t^3 + 99s^2t^2 - 42st^5 + 132st^4 - 110st^3 + 14t^6 - 42t^5 + 33t^4)/100590336000$$

$$C_{13}^{8} = (462r^2s^2t^2 - 396r^2s^2t + 99r^2s^2 - 396r^2st^2 + 396r^2st - 110r^2s + 99r^2t^2 - 110r^2t + 33r^2 - 396rs^2t^2 + 396rs^2t - 110rs^2 + 396rst^2 - 440rst + 132rs - 110rt^2 + 132r - 42r + 99s^2t^2 - 110s^2t + 33s^2 - 110st^2 + 132st - 42s + 33t^2 - 42t + 14)/100590336000$$

$$C_{13}^{9} = r^5(28r^6 - 77r^5s - 77r^5t - 77r^5 + 55r^4s^2 + 220r^4st + 220r^4s + 55r^4t^2 + 220r^4t + 55r^4 - 165r^3s^2t - 165r^3s^2 - 165r^3st^2 - 660r^3st - 165r^3s - 165r^3t^2 - 165r^3t + 132r^2s^2t^2 + 528r^2s^2t + 132r^2s^2 + 528r^2st^2 + 528r^2st + 132r^2t^2 - 462rs^2t^2 - 462rs^2t - 462rst^2 + 462s^2t^2)/50295168000$$

$$C_{13}^{10} = s^5(55r^2s^4 - 165r^2s^3t - 165r^2s^3 + 132r^2s^2t^2 + 528r^2s^2t + 132r^2s^2 - 462r^2st^2 - 462r^2st + 462r^2t^2 - 77rs^5 + 220rs^4t + 220rs^4 - 165rs^3t^2 - 660rs^3t - 165rs^3 + 528rs^2t^2 + 528rs^2t - 462rst^2 + 28s^6 - 77s^5t - 77s^5 + 55s^4t^2 + 220s^4t + 55s^4 - 165s^3t^2 - 165s^3t + 132s^2t^2)/50295168000$$

$$C_{13}^{11} = t^5(132r^2s^2t^2 - 462r^2s^2t + 462r^2s^2 - 165r^2st^3 + 528r^2st^2 - 462r^2st + 55r^2t^4 - 165r^2t^3 + 132r^2t^2 - 165rs^2t^3 + 528rs^2t^2 - 462rs^2t + 220rst^4 - 660rst^3 + 528rst^2 - 77rt^5 + 220rt^4 - 165rt^3 + 55s^2t^4 - 165s^2t^3 + 132s^2t^2 - 77st^5 + 220st^4 - 165st^3 + 28t^6 - 77t^5 + 55t^4)/50295168000$$

$$C_{13}^{12} = (462r^2s^2t^2 - 462r^2s^2t + 132r^2s^2 - 462r^2st^2 + 528r^2st - 165r^2s + 132r^2t^2 - 165r^2t + 55r^2 - 462rs^2t^2 + 528rs^2t - 165rs^2 + 528rst^2 - 660rst + 220rs - 165rt^2 + 220rt - 77r + 132s^2t^2 - 165s^2 + 55s^2 - 165st^2 + 220st - 77s + 55t^2 - 77t + 28)/50295168000$$

which conclude that the order ρ of the developed method is 10.

Consistency (3.3)

Definition 3.2: A block method is said to be consistent if its order ρ is greater than one. Consistency property is achieved for the hybrid block method from the above analysis since the order ρ = 10>1.

Convergence (3.4)

Theorem 3.1[16]: Consistency and zero stability are sufficient conditions for a linear multi step method to be convergent. The hybrid block method (9) is convergent since it fulfills both the consistency and zero stability conditions.

NUMERICAL EXAMPLES

In this part, the efficiency and the performance of the general three hybrid one-step implicit hybrid block method (9) is investigated on four problems from the literature. The first, second and fourth problems employed the step size h = 1/10 while the third used the values h = {1/8, 1/16, 1/24, 1/32 and 1/40}. A various hybrid points are included in each example (Table 1).

Problem (1):

$$y''' + 4y' = x,\ y(0) = 0, y'(0) = 0, y''(0) = 1$$

Exact Solution[17]:

$$y = \frac{3}{16}\left(1 - \cos(2x)\right) + \frac{1}{8}x^2 \ \text{with}\ h = \frac{1}{10}$$

Problem (2):

$$y''' = e^x,\ \ y(0) = 3,\ \ y'(0) = 1,\ \ y''(0) = 5,\ \ 0 \le x \le 1$$

Exact solution[5] (Table 2):

Table 1: Comparison of the proposed method with Mohammed and Adeniyi[17]

X-value	Exact-solution	Computed-solution r = 1/4, s = 1/2, t = 3/4	Error in new method	Errorfor[17]
0.10	0.004987516654767195	0.004987516654767195	8.673617E(-19)	9.61000E(-10)
0.20	0.019801063624459044	0.019801063624459048	3.469447E(-18)	6.50000E(-09)
0.30	0.043999572204435337	0.043999572204435317	2.081668E(-17)	1.59700E(-08)
0.40	0.076867491997406501	0.076867491997406487	1.387779E(-17)	1.66400E(-08)
0.50	0.117443317649723790	0.117443317649723810	1.387779E(-17)	2.03000E(-08)
0.60	0.164557921035623690	0.164557921035623690	0.000000E(+00)	2.66000E(-08)
0.70	0.216881160706204810	0.216881160706204830	2.775558E(-17)	2.67000E(-08)
0.80	0.272974910431491580	0.272974910431491640	5.551115E(-17)	2.71000E(-08)
0.90	0.331350392754953760	0.331350392754953820	5.551115E(-17)	2.77000E(-08)
1.00	0.390527531852589150	0.390527531852589200	5.551115E(-17)	2.72000E(-08)

Table 2: Comparison of the proposed method with Kuboye and Omar[5]

X-value	Exact-solution	Computed-solution r = 1/7, s = 3/7, t = 5/7	Error in new method	Errorfor[5]
0.10	3.125170918075647700	3.125170918075647700	0.000000E(+00)	2.531308E(-14)
0.20	3.301402758160169700	3.301402758160169700	0.000000E(+00)	1.612044E(-13)
0.30	3.529858807576003300	3.529858807576003300	0.000000E(+00)	4.023448E(-13)
0.40	3.811824697641270600	3.811824697641270600	0.000000E(+00)	7.536194E(-13)
0.50	4.148721270700128200	4.148721270700129100	8.881784E(-16)	1.212364E(-12)
0.60	4.542118800390508900	4.542118800390509700	8.881784E(-16)	1.780798E(-12)
0.70	4.993752707470476600	4.993752707470477500	8.881784E(-16)	2.456702E(-12)
0.80	5.505540928492466800	5.505540928492468600	1.776357E(-15)	2.212097E(-11)
0.90	6.079603111156949100	6.079603111156950000	8.881784E(-16)	5.231993E(-11)
1.00	6.718281828459044600	6.718281828459045500	8.881784E(-16)	8.860113E(-11)

$$y = 2 + 2x^2 + e^x, \; h = \frac{1}{10}$$

Problem (3):

$$y''' = \frac{4}{(1+x)^3} - 2e^{-3y(x)}, \;\; y(0) = 0,$$

$$y'(0) = -1, \;\; y''(0) = 1, \;\; 0 \le x \le b$$

Exact solution[18] (Table 3):

$$y = \ln(1+x)$$

Its worth noting that the values {r, s and t} used for this problem are 1/4, 1/2, 3/4 respectively. On the other hand, for comparison reasons SJCM absolute errors elected corresponding to the values {α = β = 0}.

Problem (4); thin film flow problem: In this example, the derived method is implemented to solve the thin film flow of a liquid. Which is discussed by several scholars such as[1-3]. The discussion included the motion of the fluid on a plane surface in which the motion a long the plane is the same direction of the flow. In Momoniat[19], the numerical method is employed to solve special third order ODEs regarding the problem in thin film flow in the form (Table 4):

$$y''' = y^k, \; y(\rho) = \alpha, y(\rho) = \beta, y''(\rho) = \gamma$$

where, ρ = 0 and α = β = γ = 1.

Table 3: Comparison of the proposed method with Bhrawy and Abd-Elhameed[20]

Step size	Method employed	Absolute error at b = 1	Absolute error at b = 4
1/8	Proposed method	1.37E(-14)	1.07E(-13)
	SJCM method	3.38E(-7)	1.89E(-2)
1/16	Proposed method	2.22E(-16)	1.55E(-15)
	SJCM method	9.05E(-14)	8.95E(-8)
1/24	Proposed method	2.22E(-16)	1.55E(-15)
	SJCM method	3.33E(-16)	2.37E(-11)
1/32	Proposed method	2.22E(-16)	8.88E(-16)
	SJCM method	4.44E(-16)	8.88E(-15)
1/40	Proposed method	2.22E(-16)	4.44E(-16)
	SJCM method	4.44E(-16)	8.65E(-15)

Table IV: Comparison of the proposed method for the case

X-value	Exact-solution	Computed-solution for r = 1/7, s = 3/7, t = 5/7	Error in new method	Error for[2]
0.20	1.221211030	1.2212100045283234	1.02E(-6)	1.07E(-6)
0.40	1.488834893	1.4888347798657515	1.13E(-7)	4.13E(-7)
0.60	1.807361404	1.8073613976784422	6.32E(-9)	8.51E(-7)
0.80	2.179819234	2.1798192339156564	8.43E(-11)	1.71E(-6)
1.00	2.608275822	2.6082748675840373	9.54E(-7)	3.86E(-6)

CONCLUSION

A one-step block method with general three-hybrid points has been derived successfully. The derived method is employed to solve general third order IVP of ODEs directly. The numerical analysis of the method shows that the developed method is consistent and zeros table which conclude that its convergent. The computed results of the new method are then compared with the results of existing methods in terms of error by considering different values of {r, s and t}. The new derived method is out performed the other methods in terms of error as shown in the study.

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APPENDIXES

Appendix I:

$$\alpha_0 = (x_n - x + hr)(x_n - x + hs)/(h^2rs),$$

$$\alpha_r = -(x - x_n)(x_n - x + hs)/(h^2r(r-s)),$$

$$\alpha_s = (x - x_n)(x_n - x + hr)/(h^2s(r-s)),$$

$$\beta_0 = \frac{(x-x_n)^3}{6} - (x-x_n)^6(r^3s^3t^3 + 4r^3s^3t^2 + 4r^3s^3t + r^3s^3 + 4r^3s^2t^3 + 8r^3s^2t^2) + 4r^3s^2t + 4r^3st^3 + 4r^3st^2 + r^3t^3 + 4r^2s^3t^3 +$$
$$8r^2s^3t^2 + 4r^2s^3t + 8r^2s^2t^3 + 8r^2s^2t^2 + 4r^2st^3 + 4rs^3t^3 + 4rs^3t^2 + 4rs^2t^3 + s^3t^3))/60h3r^3s^3t^3) - ((x-x_n)^9(4r^3s^2t +$$
$$4r^3s^2 + 4r^3st^2 + 4rs^2t^3 + 36rs^2t^2 + 36rs^2t + 4rs^2 + 11rst^3 + 36rst^2 + 11rst + 4rt^3 + 4rt^2 + 4s^3t^2 + 4s^3t + 4s^2t^3 +$$
$$16s^2t^2 + 4s^2t + 4st^3 + 4st^2))/(504h6r^3s^3t^3) + ((x-x_n)^{(10)}(r^3st + r^3s + r^3t + 4r^2s^2t + 4r^2s^2 + 4r^2st^2 + 12r^2st +$$
$$4r^2s + 4r^2t^2 + 4r^2t + rs^3t + rs^3 + 4rs^2t^2 + 12rs^2t + 4rs^2 + rst^3 + 12rst^2 + 12rst + rs + rt^3 + 4rt^2 + rt + s^3t + 4s^2t^2 +$$
$$4s^2t + st^3 + 4st^2 + st))/(360h7r^3s^3t^3) - (h(x-x_n)^2(7r^{(11)}st + 7r^{(11)}s + 7r^{(11)}t - 17r^{(10)}s^2t - 17r^{(10)}s^2 - 24r^{(10)}st^2 -$$
$$52r^{(10)}st - 24r^{(10)}s - 24r^{(10)}t^2 - 24r^{(10)}t + 5r^9s^3t + 5r^9s^3 + 64r^9s^2t^2 + 100r^9s^2t + 64r^9s^2 + 22r^9st^3 + 128r^9st^2 + 128r^9st +$$
$$22r^9s + 22r^9t^3 + 88r^9t^2 + 22r^9t + 5r^8s^4t + 5r^8s^4 - 24r^8s^3t^2 - 32r^8s^3t - 24r^8s^3 - 66r^8s^2t^3 - 224r^8s^2t^2 - 224r^8s^2t -$$
$$66r^8s^2 - 110r^8st^3 - 264r^8st^2 - 110r^8st - 88r^8t^3 - 88r^8t^2 + 5r^7s^5t + 5r^7s^5 - 24r^7s^4t^2 32r^7s^4t - 24r^7s^4 + 33r^7s^3t^3 +$$
$$84r^7s^3t^2 + 84r^7s^3t + 33r^7s^3 + 198r^7s^2t^3 + 176r^7s^2t^2 + 198r^7s^2t + 220r^7st^3 + 220r^7st^2 + 99r^7t^3 + 5r^6s^6t + 5r^6s^6 - 24r^6s^5t^2 -$$
$$32r^6s^5t - +88r^6s^2t^3 + 88r^6s^2t^2 - 198r^6st^3 + 5r^5s^7t + 5r^5s^7 - 24r^5s^6t^2 - 32r^5s^6t - 24r^5s^6 + s^6t^3 + 84r^4s^6t^2 + 84r^4s^6t +$$
$$33r^4s^6 - 99r^4s^5t^3 + 44r^4s^5t^2 - 99r^4s^5t - 440r^4s^4t^3 - 440r^4s^4t^2 + 2970r^4s^3t^3 + 5r^3s^9t + 5r^3s^9 - 24r^3s^8t^2 - 32r^3s^8t -$$
$$24r^3s^8 + 33r^3s^7t^3 + 84r^3s^7t^2 + 84r^3s^7t + 33r^3s^7 - 99r^3s^6t^3 + 44r^3s^6t^2 - 99r^3s^6t - 440r^3s^5t^3 - 440r^3s^5t^2 + 2970r^3s^4t^3 -$$
$$17r^2s^{(10)}t - 17r^2s^{(10)} + 64r^2s^9t^2 + 100r^2s^9t\,64r^2s^9 - 66r^2s^8t^3 - 224r^2s^8t^2 - 224r^2s^8t - 66r^2s^8 + 198r^2s^7t^3 + 176r^2s^7t^2 +$$
$$198r^2s^7t + 88r^2s^6t^3 + 88r^2s^6t^2 - 726r^2s^5t^3 + 7rs^{(11)}t + 7rs^{(11)} - 24rs^{(10)}t^2 - 52rs^{(10)}t - 24rs^{(10)} + 22rs^9t^3 + 128rs^9t^2 +$$
$$128rs^9t + 22rs^9 - 110rs^8t^3 - 264rs^8t^2 - 110rs^8t + 220rs^7t^3 + 220rs^7t^2 - 198rs^6t^3 + 7s^{(11)}t - 24s^{(10)}t^2 - 24s^{(10)}t + 22s^9t^3 +$$
$$88s^9t^2 + 22s^9t - 88s^8t^3 - 88s^8t^2 + 99s^7t^3))/(27720r^3s^3t^3) + ((x-x_n)^{(12)}(rs + rt + st + rst))/(660h9r^3s^3t^3) - ((x-x_n)^5$$
$$(3r^2s^2t^2 + 4r^2s^2t + 3r^2s^2 + 4r^2st^2 + 4r^2st + 3r^2t^2 + 4rs^2t^2 + 4rs^2t + 4rst^2 + 3s^2t^2))/(60h2r^2s^2t^2) + ((x-x_n)^8(r^3s^3t + r^3s^3 +$$
$$4r^3s^2t^2 + 8r^3s^2t + 4r^3s^2 + r^3st^3 + 8r^3st^2 + 8r^3st + r^3s + r^3t^3 + 4r^3t^2 + r^3t + 4r^2s^3t^2 + 8r^2s^3t + 4r^2s^3 + 4r^2s^2t^3 + 24r^2s^2t^2 +$$
$$24r^2s^2t + 4r^2s^2 + 8r^2st^3 + 24r^2st^2 + 8r^2st + 4r^2t^3 + 4r^2t^2 + rs^3t^3 + 8rs^3t^2 + 8rs^3t + rs^3 + 8rs^2t^3 + 24rs^2t^2 + 8rs^2t + 8rst^3 +$$
$$8rst^2 + rt^3 + s^3t^3 + 4s^3t^2 + s^3t + 4s^2t^3 + 4s^2t^2 + st^3))/(168h5r^3s^3t^3) - ((x-x_n)^{(11)}(4r^2st + 4r^2s4r^2t + 4rs^2t + 4rs^2 +$$
$$4rst^2 + 15rst + 4rs + 4rt^2 + 4rt + 4s^2t + 4st^2 + 4st))/(990h8r^3s^3t^3) + (h2(x-x_n)(7r^{(10)}st + 7r^{(10)}s + 7r^{(10)}t - 17r^9s^2t -$$
$$17r^9s^2 - 24r^9st^2 - 52r^9st - 24r^9s - 24r^9t^2 - 24r^9t + 5r^8s^3t + 5r^8s^3 + 64r^8s^2t^2 + 100r^8s^2t + 64r^8s^2 + 22r^8st^3 + 128r^8st^2 +$$
$$128r^8st + 22r^8s + 22r^8t^3 + 88r^8t^2 + 22r^8t + 5r^7s^4t + 5r^7s^4 - 24r^7s^3t^2 - 32r^7s^3t - 24r^7s^3 - 66r^7s^2t^3 - 224r^7s^2t^2 - 224r^7s^2t -$$
$$66r^7s^2 - 110r^7st^3 - 264r^7st^2 - 110r^7st - 88r^7t^3 - 88r^7t^2 + 5r^6s^5t + 5r^6s^5 - 24r^6s^4t^2 - 32r^6s^4t - 24r^6s^4 + 33r^6s^3t^3 + 84r^6s^3t^2 +$$
$$84r^6s^3t + 33r^6s^3 + 198r^6s^2t^3 + 176r^6s^2t^2 + 198r^6s^2t + 220r^6st^3 + 220r^6st^2 + 99r^6t^3 + 5r^5s^6t + 5r^5s^6 - 24r^5s^5t^2 - 32r^5s^5t - 24r^5s^5 +$$
$$33r^5s^4t^3 + 84r^5s^4t^2 + 84r^5s^4t + 33r^5s^4 - 99r^5s^3t^3 + 44r^5s^3t^2 - 99r^5s^3t + 88r^5s^2t^3 + 88r^5s^2t^2 - 198r^5st^3 + 5r^4s^7t + 5r^4s^7 -$$
$$24r^4s^6t^2 - 32r^4s^6t - 24r^4s^6 + 33r^4s^5t^3 + 84r^4s^5t^2 + 84r^4s^5t + 33r^4s^5 - 99r^4s^4t^3 + 44r^4s^4t^2 - 99r^4s^4t - 440r^4s^3t^3 - 440r^4s^3t^2 -$$
$$726r^4s^2t^3 + 5r^3s^8t + 5r^3s^8 - 24r^3s^7t^2 - 32r^3s^7t - 24r^3s^7 + 33r^3s^6t^3 + 84r^3s^6t^2 + 84r^3s^6t + 33r^3s^6 - 99r^3s^5t^3 + 44r^3s^5t^2 - 99r^3s^5t -$$
$$440r^3s^4t^3 - 440r^3s^4t^2 + 2970r^3s^3t^3 - 17r^2s^9t - 17r^2s^9 + 64r^2s^8t^2 + 100r^2s^8t + 64r^2s^8 - 66r^2s^7t^3 - 224r^2s^7t^2 - 224r^2s^7t - 726r^2s^4t^3 +$$
$$7rs^{(10)}t + 7rs^{(10)} - 24rs^9t^2 - 52rs^9t - 24rs^9 + 22rs^8t^3 + 128rs^8t^2 + 128rs^8t + 22rs^8 - 110rs^7t^3 - 264rs^7t^2 - 110rs^7t + 220rs^6t^3 + 220r$$
$$s^6t^2 - 198rs^5t^3 + 7s^{(10)}t - 24s^9t^2 - 24s^9t + 22s^8t^3 + 8s^8t^2 + 22s^8t - 88s^7t^3 - 88s^7t^2 + 99s^6t^3))/(27720r^2s^2t^3) - ((x-x_n)^7(4r^3s^3t^2 +$$
$$7r^3s^3t + 4r^3s^3 + 4r^3s^2t^3 + 20r^3s^2t^2 + 20r^3s^2t + 4r^3s^2 + 7r^3st^3 + 20r^3st^2 + 7r^3st + 4r^3t^3 + 4r^3t^2 + 4r^2s^3t^3 + 20r^2s^3t^2 + 20r^2s^3t + 4r^2s^3 +$$
$$20r^2s^2t^3 + 48r^2s^2t^2 + 20r^2s^2t + 20r^2st^3 + 20r^2st^2 + 4r^2t^3 + 7rs^3t^3 + 20rs^3t^2 + 7rs^3t + 20rs^2t^3 + 20rs^2t^2 + 7rst^3 + 4s^3t^3 +$$
$$4s^3t^2 + 4s^2t^3))/(210h4r^3s^3t^3)$$

$$\begin{aligned}
\beta_r = {} & (((x-x_n)^{11}(9r^4+9r^3s+9r^3t+9r^3-12r^2s^2-19r^2st-19r^2s-12r^2t^2-19r^2t-12r^2+8rs^2t+8rs^2+8rst^2+21rst+8rs+8rt^2+8rt- \\
& 4s^2t-4st^2-4st))/(990h^8r^3(r-s)^3(r-t)^3(r-1)^3)+(h^2(x-x_n)(84r^{12}s-231r^{11}s^2-315r^{11}st-315r^{11}s+159r^{10}s^3+895r^{10}s^2t+ \\
& 895r^{10}s^2+390r^{10}st^2+1210r^{10}st+390r^{10}s+5r^9s^4-641r^9s^3t-641r^9s^3-1146r^9s^2t^2-3583r^9s^2t-1146r^9s^2-154r^9st^3-1536r^9st^2- \\
& 1536r^9st-154r^9s+5r^8s^5-25r^8s^4t-25r^8s^4+856r^8s^3t^2+2707r^8s^3t+856r^8s^3+462r^8s^2t^3+4754r^8s^2t^2+4754r^8s^2t+462r^8s^2+ \\
& 616r^8st^3+2002r^8st^2+616r^8st+5r^7s^6-25r^7s^5t-25r^7s^5+42r^7s^4t^2+133r^7s^4t+42r^7s^4-352r^7s^3t^3-3815r^7s^3t^2-3815r^7s^3t- \\
& 352r^7s^3-1958r^7s^2t^3-6567r^7s^2t^2-1958r^7s^2t-814r^7st^3-814r^7st^2+5r^6s^7-25r^6s^6t-25r^6s^6+42r^6s^5t^2+133r^6s^5t+42r^6s^5- \\
& 22r^6s^4t^3-240r^6s^4t^2-240r^6s^4t-22r^6s^4+1617r^6s^3t^3+5753r^6s^3t^2+1617r^6s^3t+2761r^6s^2t^3+2761r^6s^2t^2+330r^6st^3+5r^5s^8- \\
& 25r^5s^7t-25r^5s^7+42r^5s^6t^2+133r^5s^6t+42r^5s^6-22r^5s^5t^3-240r^5s^5t^2-240r^5s^5t-22r^5s^5+132r^5s^4t^3+473r^5s^4t^2+132r^5s^4t- \\
& 2519r^5s^3t^3-2519r^5s^3t^2-1155r^5s^2t^3+5r^4s^9-25r^4s^8t-25r^4s^8+42r^4s^7t^2+133r^4s^7t+42r^4s^7-22r^4s^6t^3-240r^4s^6t^2-240r^4s^6t- \\
& 22r^4s^6+132r^4s^5t^3+473r^4s^5t^2+132r^4s^5t-275r^4s^4t^3-275r^4s^4t^2+1089r^4s^3t^3-49r^3s^{10}+173r^3s^9t+173r^3s^9-156r^3s^8t^2- \\
& 659r^3s^8t-156r^3s^8-22r^3s^7t^3+651r^3s^7t^2+651r^3s^7t-22r^3s^7+132r^3s^6t^3-715r^3s^6t^2+132r^3s^6t-275r^3s^5t^3-275r^3s^5t^2+ \\
& 165r^3s^4t^3+21r^2s^{11}-35r^2s^{10}t-35r^2s^{10}-68r^2s^9t^2+r^2s^9t-68r^2s^9+132r^2s^8t^3+376r^2s^8t^2+376r^2s^8t+132r^2s^8-561r^2s^7t^3- \\
& 781r^2s^7t^2-561r^2s^7t+649r^2s^6t^3+649r^2s^6t^2+165r^2s^5t^3-14rs^{11}t-14rs^{11}+48rs^{10}t^2+71rs^{10}t+48rs^{10}-44rs^9t^3-130rs^9t^2- \\
& 130rs^9t-44rs^9+88rs^8t^3+88rs^8t+187rs^7t^3+187rs^7t^2-495rs^6t^3+7s^{11}t-24s^{10}t^2-24s^{10}t+22s^9t^3+88s^9t^2+22s^9t-88s^8t^3- \\
& 88s^8t^2+99s^7t^3))/(27720r^2(r-s)^3(r-t)^3(r-1)^3)+((x-x_n)^7(9r^4s^2t^2+36r^4s^2t+9r^4s^2+36r^4st^2+36r^4st+9r^4t^2-7r^3s^3t^2- \\
& 28r^3s^3t-7r^3s^3-7r^3s^2t^3-47r^3s^2t^2-47r^3s^2t-7r^3s^2-28r^3st^3-47r^3st^2-28r^3st-7r^3t^3-7r^3t^2+5r^2s^3t^3+13r^2s^3t^2+13r^2s^3t+ \\
& 5r^2s^3+13r^2s^2t^3+24r^2s^2t^2+13r^2s^2t+13r^2st^3+13r^2st^2+5r^2t^3+5rs^3t^3+4rs^3t^2+5rs^3t+4rs^2t^3+4rs^2t^2+5rst^3-4s^3t^3-4s^3t^2- \\
& 4s^2t^3))/(210h^4r^3(r-s)^3(r-t)^3(r-1)^3)-((x-x_n)^8(9r^4s^2t+9r^4s^2+9r^4st^2+36r^4st+9r^4s+9r^4t^2+9r^4t-7r^3s^3t-7r^3s^3- \\
& 10r^3s^2t^2-26r^3s^2t-10r^3s^2-7r^3st^3-26r^3st^2-26r^3st-7r^3s-7r^3t^3-10r^3t^2-7r^3t+2r^2s^3t^2-2r^2s^3t+2r^2s^3+2r^2s^2t^3+3r^2s^2t^2+ \\
& 3r^2s^2t+2r^2s^2-2r^2st^3+3r^2st^2-2r^2st+2r^2t^3+2r^2t^2+2rs^3t^3+7rs^3t^2+7rs^3t+2rs^3+7rs^2t^3+12rs^2t^2+7rs^2t+7rst^3+7rst^2+ \\
& 2rt^3-s^3t^3-4s^3t^2-s^3t-4s^2t^3-4s^2t^2-st^3))/(168h^5r^3(r-s)^3(r-t)^3(r-1)^3)-((x-x_n)^9(7r^3s^3-36r^4st-36r^4s-9r^4t^2-36r^4t- \\
& 9r^4-9r^4s^2+19r^3s^2t+19r^3s^2+19r^3st^2+20r^3st+19r^3s+7r^3t^3+19r^3t^2+19r^3t+7r^3+7r^2s^3t+7r^2s^3+4r^2s^2t^2+27r^2s^2t+4r^2s^2+ \\
& 7r^2st^3+27r^2st^2+27r^2st+7r^2s+7r^2t^3+4r^2t^2+7r^2t-8rs^3t^2-13rs^3t-8rs^3-8rs^2t^3-36rs^2t^2-36rs^2t-8rs^2-13rst^3-36rst^2- \\
& 13rst-8rt^3-8rt^2+4s^3t^2+4s^3t+36rs^2t^2-36rs^2t-8rs^2-13rst^3-36rst^2-13rst-8rt^3-8rt^2+4s^3t^2+4s^3t+4s^2t^3+16s^2t^2+ \\
& 4s^2t+4st^3+4st^2))/(504h^6r^3(r-s)^3(r-t)^3(r-1)^3)-(h(x-x_n)^2(84r^{13}-231r^{12}s-315r^{12}t-315r^{12}+159r^{11}s^2+895r^{11}st+ \\
& 895r^{11}s+390r^{11}t^2+1210r^{11}t+390r^{11}+5r^{10}s^3-641r^{10}s^2t-641r^{10}s^2-1146r^{10}st^2-3583r^{10}st-1146r^{10}s-154r^{10}t^3-1536r^{10}t^2 \\
& 1536r^{10}t-154r^{10}+5r^9s^4-25r^9s^3t-25r^9s^3+856r^9s^2t^2+2707r^9s^2t+856r^9s^2+462r^9st^3+4754r^9st^2+4754r^9st+462r^9s+ \\
& 616r^9t^3+2002r^9t^2+616r^9t+5r^8s^5-25r^8s^4t-25r^8s^4+42r^8s^3t^2+133r^8s^3t+42r^8s^3-352r^8s^2t^3-3815r^8s^2t^2-3815r^8s^2t- \\
& 352r^8s^2-1958r^8st^3-6567r^8st^2-1958r^8st-814r^8t^3-814r^8t^2+5r^7s^6-25r^7s^5t-25r^7s^5+42r^7s^4t^2+133r^7s^4t+42r^7s^4- \\
& 22r^7s^3t^3-240r^7s^3t^2-240r^7s^3t-22r^7s^3+1617r^7s^2t^3+5753r^7s^2t^2+1617r^7s^2t+2761r^7st^3+2761r^7st^2+330r^7t^3+5r^6s^7- \\
& 25r^6s^6t-25r^6s^6+42r^6s^5t^2+133r^6s^5t+42r^6s^5-22r^6s^4t^3-240r^6s^4t^2-240r^6s^4t-22r^6s^4+132r^6s^3t^3+473r^6s^3t^2+132r^6s^3t- \\
& 2519r^6s^2t^3-2519r^6s^2t^2-1155r^6st^3+5r^5s^8-25r^5s^7t-25r^5s^7+42r^5s^6t^2+133r^5s^6t+42r^5s^6-22r^5s^5t^3-240r^5s^5t^2-240r^5s^5t- \\
& 22r^5s^5+132r^5s^4t^3+473r^5s^4t^2+132r^5s^4t-275r^5s^3t^3-275r^5s^3t^2+1089r^5s^2t^3+5r^4s^9-25r^4s^8t-25r^4s^8+42r^4s^7t^2+133r^4s^7t+ \\
& 42r^4s^7-22r^4s^6t^3-240r^4s^6t^2-240r^4s^6t-22r^4s^6+132r^4s^5t^3+473r^4s^5t^2+132r^4s^5t-275r^4s^4t^3-275r^4s^4t^2+165r^4s^3t^3- \\
& 49r^3s^{10}+173r^3s^9t+173r^3s^9-156r^3s^8t^2-659r^3s^8t-156r^3s^8-22r^3s^7t^3+651r^3s^7t^2+651r^3s^7t-22r^3s^7+132r^3s^6t^3-715r^3s^6t^2+1 \\
& 32r^3s^6t-275r^3s^5t^3-275r^3s^5t^2+165r^3s^4t^3+21r^2s^{11}-35r^2s^{10}t-35r^2s^{10}-68r^2s^9t^2+r^2s^9t-68r^2s^9+132r^2s^8t^3+376r^2s^8t^2+ \\
& 376r^2s^8t+132r^2s^8-561r^2s^7t^3-781r^2s^7t^2-561r^2s^7t+649r^2s^6t^3+649r^2s^6t^2+165r^2s^5t^3-14rs^{11}t-14rs^{11}+48rs^{10}t^2+71rs^{10}t+ \\
& 48rs^{10}-44rs^9t^3-130rs^9t^2-130rs^9t-44rs^9+88rs^8t^3+88rs^8t+187rs^7t^3+187rs^7t^2-495rs^6t^3+7s^{11}t-24s^{10}t^2-24s^{10}t+22s^9t^3+ \\
& 88s^9t^2+22s^9t-88s^8t^3-88s^8t^2+99s^7t^3))/(27720r^3(r-s)^3(r-t)^3(r-1)^3)+((x-x_n)^{10}(3r^3s^2-9r^4t-9r^4-9r^4s-2r^3st-2r^3s+ \\
& 3r^3t^2-2r^3t+3r^3+3r^2s^3+10r^2s^2t+10r^2s^2+10r^2st^2+21r^2st+10r^2s+3r^2t^3+10r^2t^2+10r^2t+3r^2-2rs^3t-2rs^3-8rs^2t^2- \\
& 15rs^2t-8rs^2-2rst^3-15rst^2-15rst-2rs-2rt^3-8rt^2-2rt+s^3t+4s^2t^2+4s^2t+st^3+4st^2+st))/(360h^7r^3(r-s)^3(r-t)^3(r-1)^3)- \\
& ((x-x_n)^{12}(2rs+2rt-st-3r^2s-3r^2t-3r^2+4r^3+2rst))/(660h^9r^3(r-s)^3(r-t)^3(r-1)^3)+(s^2t^2(x-x_n)^5(5rs+5rt-3st-7r^2s- \\
& 7r^2t-7r^2+9r^3+5rst))/(60h^2r^2(r-s)^3(r-t)^3(r-1)^3)+(st(x-x_n)^6(7r^3s^2t-9r^4s-9r^4t-9r^4st+7r^3s^2+7r^3st^2+17r^3st+7r^3s+ \\
& 7r^3t^2+7r^3t-5r^2s^2t^2-7r^2s^2t-5r^2s^2-7r^2st^2-7r^2st5r^2t^2+rs^2t^2+rs^2t+rst^2+s^2t^2))/(60h^3r^3(r-s)^3(r-t)^3(r-1)^3)
\end{aligned}$$

$$\beta_s = (((x-x_n)^8(2r^3s^2t^2 - 7r^3s^3 - 7r^3s^3t - 2r^3s^2t + 2r^3s^2 + 2r^3st^3 + 7r^3st^2 + 7r^3st + 2r^3s - r^3t^3 - 4r^3t^2 - r^3t + 9r^2s^4t +$$
$$9r^2s^4 - 10r^2s^3t^2 - 26r^2s^3t - 10r^2s^3 + 2r^2s^2t^3 + 3r^2s^2t^2 + 3r^2s^2t + 2r^2s^2 + 7r^2st^3 + 12r^2st^2 + 7r^2st - 4r^2t^3 - 4r^2t^2 +$$
$$9rs^4t^2 + 36rs^4t + 9rs^4 - 7rs^3t^3 - 26rs^3t^2 - 26rs^3t - 7rs^3 - 2rs^2t^3 + 3rs^2t^2 - 2rs^2t + 7rst^3 + 7rst^2 - rt^3 + 9s^4t^2 +$$
$$9s^4t - 7s^3t^3 - 10s^3t^2 - 7s^3t + 2s^2t^3 + 2s^2t^2 + 2st^3)) / (168h^5s^3(r-s)^3(s-t)^3(s-1)^3) - ((x-x_n)^{11}(8r^2st - 12r^2s^2 +$$
$$8r^2s - 4r^2t + 9rs^3 - 19rs^2t - 19rs^2 + 8rst^2 + 21rst + 8rs - 4rt^2 - 4rt + 9s^4 + 9s^3t + 9s^3 - 12s^2t^2 - 19s^2t - 12s^2 + 8st^2 +$$
$$8st)) / (990h^8s^3(r-s)^3(s-t)^3(s-1)^3) - ((x-x_n)^7(5r^3s^2t^3 - 28r^3s^3t - 7r^3s^3 - 7r^3s^3t^2 + 13r^3s^2t^2 + 13r^3s^2t + 5r^3s^2 +$$
$$5r^3st^3 + 4r^3st^2 + 5r^3st - 4r^3t^3 - 4r^3t^2 + 9r^2s^4t^2 + 36r^2s^4t + 9r^2s^4 - 7r^2s^3t^3 - 47r^2s^3t^2 - 47r^2s^3t - 7r^2s^3 + 13r^2s^2t^3 +$$
$$24r^2s^2t^2 + 13r^2s^2t + 4r^2st^3 + 4r^2st^2 - 4r^2t^3 + 36rs^4t^2 + 36rs^4t - 28rs^3t^3 - 47rs^3t^2 - 28rs^3t + 13rs^2t^3 + 13rs^2t^2 + 5rst^3 +$$
$$9s^4t^2 - 7s^3t^3 - 7s^3t^2 + 5s^2t^3)) / (210h^4s^3(r-s)^3(s-t)^3(s-1)^3) - (h^2(x-x_n)(21r^{11}s^2 - 14r^{11}st - 14r^{11}s + 7r^{11}t - 49r^{10}s^3 -$$
$$35r^{10}s^2t - 35r^{10}s^2 + 48r^{10}st^2 + 71r^{10}st + 48r^{10}s - 24r^{10}t^2 - 24r^{10}t + 5r^9s^4 + 173r^9s^3t + 173r^9s^3 - 68r^9s^2t^2 + r^9s^2t - 68r^9s^2 -$$
$$44r^9st^3 - 130r^9st^2 - 130r^9st - 44r^9s + 22r^9t^3 + 88r^9t^2 + 22r^9t + 5r^8s^5 - 25r^8s^4t - 25r^8s^4 - 156r^8s^3t^2 - 659r^8s^3t - 156r^8s^3 +$$
$$132r^8s^2t^3 + 376r^8s^2t^2 + 376r^8s^2t + 132r^8s^2 + 88r^8st^3 + 88r^8st - 88r^8t^3 - 88r^8t^2 + 5r^7s^6 - 25r^7s^5t - 25r^7s^5 + 42r^7s^4t^2 +$$
$$133r^7s^4t + 42r^7s^4 - 22r^7s^3t^3 + 651r^7s^3t^2 + 651r^7s^3t - 22r^7s^3 - 561r^7s^2t^3 - 781r^7s^2t^2 - 561r^7s^2t + 187r^7st^3 + 187r^7st^2 +$$
$$99r^7t^3 + 5r^6s^7 - 25r^6s^6t - 25r^6s^6 + 42r^6s^5t^2 + 133r^6s^5t + 42r^6s^5 - 22r^6s^4t^3 - 240r^6s^4t^2 - 240r^6s^4t - 22r^6s^4 + 132r^6s^3t^3 -$$
$$715r^6s^3t^2 + 132r^6s^3t + 649r^6s^2t^3 + 649r^6s^2t^2 - 495r^6st^3 + 5r^5s^8 - 25r^5s^7t - 25r^5s^7 + 42r^5s^6t^2 + 133r^5s^6t + 42r^5s^6 - 22r^5s^5t^3 -$$
$$240r^5s^5t^2 - 240r^5s^5t - 22r^5s^5 + 132r^5s^4t^3 + 473r^5s^4t^2 + 132r^5s^4t - 275r^5s^3t^3 - 275r^5s^3t^2 + 165r^5s^2t^3 + 5r^4s^9 - 25r^4s^8t -$$
$$25r^4s^8 + 42r^4s^7t^2 + 133r^4s^7t + 42r^4s^7 - 22r^4s^6t^3 - 240r^4s^6t^2 - 240r^4s^6t - 22r^4s^6 + 132r^4s^5t^3 + 473r^4s^5t^2 + 132r^4s^5t - 275r^4s^4t^3 -$$
$$275r^4s^4t^2 + 165r^4s^3t^3 + 159r^3s^{10} - 641r^3s^9t - 641r^3s^9 + 856r^3s^8t^2 + 2707r^3s^8t + 856r^3s^8 - 352r^3s^7t^3 - 3815r^3s^7t^2 - 3815r^3s^7t -$$
$$352r^3s^7 + 1617r^3s^6t^3 + 5753r^3s^6t^2 + 1617r^3s^6t - 2519r^3s^5t^3 - 2519r^3s^5t^2 + 1089r^3s^4t^3 - 231r^2s^{11} + 895r^2s^{10}t + 895r^2s^{10} -$$
$$1146r^2s^9t^2 - 3583r^2s^9t - 1146r^2s^9 + 462r^2s^8t^3 + 4754r^2s^8t^2 + 4754r^2s^8t + 462r^2s^8 - 1958r^2s^7t^3 - 6567r^2s^7t^2 - 1958r^2s^7t +$$
$$2761r^2s^6t^3 + 2761r^2s^6t^2 - 1155r^2s^5t^3 + 84rs^{12} - 315rs^{11}t - 315rs^{11} + 390rs^{10}t^2 + 1210rs^{10}t + 390rs^{10} - 154rs^9t^3 - 1536rs^9t^2 -$$
$$1536rs^9t - 154rs^9 + 616rs^8t^3 + 2002rs^8t^2 + 616rs^8t - 814rs^7t^3 - 814rs^7t^2 + 330rs^6t^3)) / (27720s^2(r-s)^3(s-t)^3(s-1)^3) + ((x -$$
$$x_n)^9(7r^3s^3 + 7r^3s^2t + 7r^3s^2 - 8r^3st^2 - 13r^3st - 8r^3s + 4r^3t^2 + 4r^3t - 9r^2s^4 + 19r^2s^3t + 19r^2s^3 + 4r^2s^2t^2 + 27r^2s^2t + 4r^2s^2 - 8r^2st^3 -$$
$$36r^2st^2 - 36r^2st - 8r^2s + 4r^2t^3 + 16r^2t^2 + 4r^2t - 36rs^4t - 36rs^4 + 19rs^3t^2 + 20rs^3t + 19rs^3 + 7rs^2t^3 + 27rs^2t^2 + 27rs^2t + 7rs^2 -$$
$$13rst^3 - 36rst^2 - 13rst + 4rt^3 + 4rt^2 - 9s^4t^2 - 36s^4t - 9s^4 + 7s^3t^3 + 19s^3t^2 + 19s^3t + 7s^3 + 7s^2t^3 + 4s^2t^2 + 7s^2t - 8st^3 - 8st^2)) /$$
$$(504h^6s^3(r-s)^3(s-t)^3(s-1)^3) + (h(x-x_n)^2(21r^{11}s^2 - 14r^{11}st - 14r^{11}s + 7r^{11}t - 49r^{10}s^3 - 35r^{10}s^2t - 35r^{10}s^2 + 48r^{10}st^2 + 71r^{10}st +$$
$$48r^{10}s - 24r^{10}t^2 - 24r^{10}t + 5r^9s^4 + 173r^9s^3t + 173r^9s^3 - 68r^9s^2t^2 + r^9s^2t - 68r^9s^2 - 44r^9st^3 - 130r^9st^2 - 130r^9st - 44r^9s + 22r^9t^3 +$$
$$88r^9t^2 + 22r^9t + 5r^8s^5 - 25r^8s^4t - 25r^8s^4 - 156r^8s^3t^2 - 659r^8s^3t - 156r^8s^3 + 132r^8s^2t^3 + 376r^8s^2t^2 + 376r^8s^2t + 132r^8s^2 + 88r^8st^3 +$$
$$88r^8st - 88r^8t^3 - 88r^8t^2 + 5r^7s^6 - 25r^7s^5t - 25r^7s^5 + 42r^7s^4t^2 + 133r^7s^4t + 42r^7s^4 - 22r^7s^3t^3 + 651r^7s^3t^2 + 651r^7s^3t - 22r^7s^3 -$$
$$561r^7s^2t^3 - 781r^7s^2t^2 - 561r^7s^2t + 187r^7st^3 + 187r^7st^2 + 99r^7t^3 + 5r^6s^7 - 25r^6s^6t - 25r^6s^6 + 42r^6s^5t^2 + 133r^6s^5t + 42r^6s^5 -$$
$$22r^6s^4t^3 - 240r^6s^4t^2 - 240r^6s^4t - 22r^6s^4 + 132r^6s^3t^3 - 715r^6s^3t^2 + 132r^6s^3t + 649r^6s^2t^3 + 649r^6s^2t^2 - 495r^6st^3 + 5r^5s^8 -$$
$$25r^5s^7t - 25r^5s^7 + 42r^5s^6t^2 + 133r^5s^6t + 42r^5s^6 - 22r^5s^5t^3 - 240r^5s^5t^2 - 240r^5s^5t - 22r^5s^5 + 132r^5s^4t^3 + 473r^5s^4t^2 +$$
$$132r^5s^4t - 275r^5s^3t^3 - 275r^5s^3t^2 + 165r^5s^2t^3 + 5r^4s^9 - 25r^4s^8t - 25r^4s^8 + 42r^4s^7t^2 + 133r^4s^7t + 42r^4s^7 - 22r^4s^6t^3 - 240r^4s^6t^2 -$$
$$240r^4s^6t - 22r^4s^6 + 132r^4s^5t^3 + 473r^4s^5t^2 + 132r^4s^5t - 275r^4s^4t^3 - 275r^4s^4t^2 + 165r^4s^3t^3 + 5r^3s^{10} - 25r^3s^9t - 25r^3s^9 + 42r^3s^8t^2 +$$
$$133r^3s^8t + 42r^3s^8 - 22r^3s^7t^3 - 240r^3s^7t^2 - 240r^3s^7t - 22r^3s^7 + 132r^3s^6t^3 + 473r^3s^6t^2 + 132r^3s^6t - 275r^3s^5t^3 - 275r^3s^5t^2 +$$
$$165r^3s^4t^3 + 159r^2s^{11} - 641r^2s^{10}t - 641r^2s^{10} + 856r^2s^9t^2 + 2707r^2s^9t + 856r^2s^9 - 352r^2s^8t^3 - 3815r^2s^8t^2 - 3815r^2s^8t - 352r^2s^8 +$$
$$1617r^2s^7t^3 + 5753r^2s^7t^2 + 1617r^2s^7t - 2519r^2s^6t^3 - 2519r^2s^6t^2 + 352r^2s^8 + 1617r^2s^7t^3 + 5753r^2s^7t^2 + 1617r^2s^7t - 2519r^2s^6t^3 -$$
$$2519r^2s^6t^2 + 1089r^2s^5t^3 - 231rs^{12} + 895rs^{11}t + 895rs^{11} - 1146rs^{10}t^2 - 3583rs^{10}t - 1146rs^{10} + 462rs^9t^3 + 4754rs^9t^2 + 4754rs^9t +$$
$$462rs^9 - 1958rs^8t^3 - 6567rs^8t^2 - 1958rs^8t + 2761rs^7t^3 + 2761rs^7t^2 - 1155rs^6t^3 + 84s^{13} - 315s^{12}t - 315s^{12} + 390s^{11}t^2 + 1210s^{11}t +$$
$$390s^{11} - 154s^{10}t^3 - 1536s^{10}t^2 - 1536s^{10}t - 154s^{10} + 616s^9t^3 + 2002s^9t^2 + 616s^9t - 814s^8t^3 - 814s^8t^2 + 330s^7t^3)) / (27720s^3$$
$$(r-s)^3(s-t)^3(s-1)^3) - ((x-x_n)^{10}(3r^3s^2 - 2r^3st - 2r^3s + r^3t + 3r^2s^3 + 10r^2s^2t + 10r^2s^2 - 8r^2st^2 - 15r^2st - 8r^2s + 4r^2t^2 +$$
$$4r^2t - 9rs^4 - 2rs^3t - 2rs^3 + 10rs^2t^2 + 21rs^2t + 10rs^2 - 2rst^3 - 15rst^2 - 15rst - 2rs + rt^3 + 4rt^2 + rt - 9s^4t - 9s^4 + 3s^3t^2 -$$
$$2s^3t + 3s^3 + 3s^2t^3 + 10s^2t^2 + 10s^2t + 3s^2 - 2st^3 - 8st^2 - 2st)) / (360h^7s^3(r-s)^3(s-t)^3(s-1)^3) + ((x-x_n)^{12}(2rs - rt +$$
$$2st - 3rs^2 - 3s^2t - 3s^2 + 4s^3 + 2rst)) / (660h^9s^3(r-s)^3(s-t)^3(s-1)^3) - (r^2t^2(x-x_n)^5(5rs - 3rt + 5st - 7rs^2 - 7s^2t - 7s^2 +$$
$$9s^3 + 5rst)) / (60h^2s^2(r-s)^3(s-t)^3(s-1)^3) - (rt(x-x_n)^6(7r^2s^3t + 7r^2s^3 - 5r^2s^2t^2 - 7r^2s^2t - 5r^2s^2 + r^2st^2 + r^2st + r^2t^2 -$$
$$9rs^4t - 9rs^4 + 7rs^3t^2 + 17rs^3t + 7rs^3 - 7rs^2t^2 - 7rs^2t + rst^2 - 9s^4t + 7s^3t^2 + 7s^3t - 5s^2t^2)) / (60h^3s^3(r-s)^3(s-t)^3(s-1)^3)$$

$$
\begin{aligned}
\beta_t = {} & ((x-x_n)^{11}(8r^2st-4r^2s-12r^2t^2+8r^2t+8rs^2t-4rs^2-19rst^2+21rst-4rs+9rt^3-19rt^2+8rt-12s^2t^2+8s^2t+9st^3-19st^2+8st+ \\
& 9t^4+9t^3-12t^2))/(990h^8t^3(r13r^2s^3t^2+4r^2s^3t-4r^2s^3+9r^2s^2t^4-47r^2s^2t^3+24r^2s^2t^2+4r^2s^2t+36r^2st^4-47r^2st^3+13r^2st^2+ \\
& 9r^2t^4-7r^2t^3-28rs^3t^3+13rs^3t^2+5rs^3t+36rs^2t^4-47rs^2t^3+13rs^2t^2+36rst^4-28rst^3-7s^3t^3+5s^3t^2+9s^2t^4-7s^2t^3))/(210h^4t^3 \\
& (r-t)^3(s-t)^3(t-1)^3)-((x-x_n)^8(2r^3s^3t-r^3s^3+2r^3s^2t^2+7r^3s^2t-4r^3s^2-7r^3st^3-2r^3st^2+7r^3st-r^3s-7r^3t^3+2r^3t^2+2r^3t+ \\
& 2r^2s^3t^2+7r^2s^3t-4r^2s^3-10r^2s^2t^3+3r^2s^2t^2+12r^2s^2t-4r^2s^2+9r^2st^4-26r^2st^3+3r^2st^2+7r^2st+9r^2t^4-10r^2t^3+2r^2t^2-7rs^3t^3- \\
& 2rs^3t^2+7rs^3t-rs^3+9rs^2t^4-26rs^2t^3+3rs^2t^2+7rs^2t+36rst^4-26rst^3-2rst^2+9rt^4-7rt^3-7s^3t^3+2s^3t^2+2s^3t+9s^2t^4-10s^2t^3+ \\
& 2s^2t^2+9st^4-7st^3))/(168h^5t^3(r-t)^3(s-t)^3(t-1)^3)-(h(x-x_n)^2(7r^{11}s-14r^{11}st+21r^{11}t^2-14r^{11}t+34r^{10}s^2t-17r^{10}s^2+50r^{10}st- \\
& 24r^{10}s-70r^{10}t^3-21r^{10}t^2+48r^{10}t-10r^9s^3t+5r^9s^3-116r^9s^2t^2-56r^9s^2t+64r^9s^2+138r^9st^3-91r^9st^2-58r^9st+22r^9s+54r^9t^4+ \\
& 208r^9t^3-116r^9t^2-44r^9t-10r^8s^4t+5r^8s^4+60r^8s^3t^2+10r^8s^3t-24r^8s^3+50r^8s^2t^3+415r^8s^2t^2-146r^8s^2t-66r^8s^2-144r^8st^4- \\
& 452r^8st^3+390r^8st^2+22r^8st-198r^8t^4-88r^8t^3+176r^8t^2-10r^7s^5t+5r^7s^5+60r^7s^4t^2+10r^7s^4t-24r^7s^4-104r^7s^3t^3-234r^7s^3t^2+ \\
& 129r^7s^3t+33r^7s^3+54r^7s^2t^4-177r^7s^2t^3\ 391r^7s^2t^2+297r^7s^2t+594r^7st^4+187r^7st^3\ 473r^7st^2+198r^7t^4\ 154r^7t^3\ 10r^6s^6t+5r^6s^6+. \\
& 60r^6s^5t^2+10r^6s^5t-24r^6s^5-104r^6s^4t^3-234r^6s^4t^2+129r^6s^4t+33r^6s^4+54r^6s^3t^4+516r^6s^3t^3+71r^6s^3t^2-297r^6s^3t-297r^6s^2t^4+ \\
& 253r^6s^2t^3-11r^6s^2t^2-693r^6st^4+539r^6st^3-10r^5s^7t+5r^5s^7+60r^5s^6t^2+10r^5s^6t-24r^5s^6-104r^5s^5t^3-234r^5s^5t^2+129r^5s^5t+ \\
& 33r^5s^5+54r^5s^4t^4+516r^5s^4t^3+71r^5s^4t^2-297r^5s^4t-297r^5s^3t^4-671r^5s^3t^3+649r^5s^3t^2+495r^5s^2t^4-385r^5s^2t^3-10r^4s^8t+ \\
& 5r^4s^8+60r^4s^7t^2+10r^4s^7t-24r^4s^7-104r^4s^6t^3-234r^4s^6t^2+129r^4s^6t+33r^4s^6+54r^4s^5t^4+516r^4s^5t^3+71r^4s^5t^2-297r^4s^5t- \\
& 297r^4s^4t^4-671r^4s^4t^3+649r^4s^4t^2+495r^4s^3t^4\ 385r^4s^3t^3-10r^3s^9t+5r^3s^9+60r^3s^8t^2+10r^3s^8t\ 24r^3s^8\ 104r^3s^7t^3\ 234r^3s^7t^2+ \\
& 129r^3s^7t+33r^3s^7+54r^3s^6t^4+516r^3s^6t^3+71r^3s^6t^2-297r^3s^6t-297r^3s^5t^4-671r^3s^5t^3+649r^3s^5t^2+495r^3s^4t^4-385r^3s^4t^3+ \\
& 34r^2s^{10}t-17r^2s^{10}-116r^2s^9t^2-56r^2s^9t+64r^2s^9+50r^2s^8t^3+415r^2s^8t^2-146r^2s^8t-66r^2s^8+54r^2s^7t^4-177r^2s^7t^3-391r^2s^7t^2+ \\
& 297r^2s^7t-297r^2s^6t^4+253r^2s^6t^3-11r^2s^6t^2+495r^2s^5t^4-385r^2s^5t^3-14rs^{11}t+7rs^{11}+50rs^{10}t-24rs^{10}+138rs^9t^3-91rs^9t^2- \\
& 58rs^9t+22rs^9-144rs^8t^4-452rs^8t^3+390rs^8t^2+22rs^8t+594rs^7t^4+187rs^7t^3-473rs^7t^2-693rs^6t^4+539rs^6t^3+21s^{11}t^2- \\
& 14s^{11}t-70s^{10}t^3-21s^{10}t^2+48s^{10}t+54s^9t^4+208s^9t^3-116s^9t^2-44s^9t-198s^8t^4-88s^8t^3+176s^8t^2+198s^7t^4-154s^7t^3))/ \\
& (27720t^3(r-t)^3(s-t)^3(t-1)^3)+(h^2(x-x_n)(7r^{11}s^2-14r^{11}s^2t+21r^{11}st^2-14r^{11}st+34r^{10}s^3t-17r^{10}s^3+50r^{10}s^2t-24r^{10}s^2- \\
& 70r^{10}st^3-21r^{10}st^2+48r^{10}st-10r^9s^4t+5r^9s^4-116r^9s^3t^2-56r^9s^3t+64r^9s^3+138r^9s^2t^3-91r^9s^2t^2-58r^9s^2t+22r^9s^2+54r^9st^4+ \\
& 208r^9st^3-116r^9st^2-44r^9st-10r^8s^5t+5r^8s^5+60r^8s^4t^2+10r^8s^4t-24r^8s^4+50r^8s^3t^3+415r^8s^3t^2-146r^8s^3t-66r^8s^3-144r^8s^2t^4- \\
& 452r^8s^2t^3+390r^8s^2t^2+22r^8s^2t-198r^8st^4-88r^8st^3+176r^8st^2-10r^7s^6t+5r^7s^6+60r^7s^5t^2+10r^7s^5t-24r^7s^5-104r^7s^4t^3- \\
& 234r^7s^4t^2+129r^7s^4t+33r^7s^4+54r^7s^3t^4\ 177r^7s^3t^3\ 391r^7s^3t^2+297r^7s^3t+594r^7s^2t^4+187r^7s^2t^3\ 473r^7s^2t^2+198r^7st^4\ 154r^7st^3 \\
& 10r^6s^7t+5r^6s^7+60r^6s^6t^2+10r^6s^6t-24r^6s^6-104r^6s^5t^3-234r^6s^5t^2+129r^6s^5t+33r^6s^5+54r^6s^4t^4+516r^6s^4t^3+71r^6s^4t^2- \\
& 297r^6s^4t-297r^6s^3t^4+253r^6s^3t^3-11r^6s^3t^2-693r^6s^2t^4+539r^6s^2t^3-10r^5s^8t+5r^5s^8+60r^5s^7t^2+10r^5s^7t-24r^5s^7-104r^5s^6t^3- \\
& 234r^5s^6t^2+129r^5s^6t+33r^5s^6+54r^5s^5t^4+516r^5s^5t^3+71r^5s^5t^2-297r^5s^5t-297r^5s^4t^4-671r^5s^4t^3+649r^5s^4t^2+495r^5s^3t^4- \\
& 385r^5s^3t^3-10r^4s^9t+5r^4s^9+60r^4s^8t^2+10r^4s^8t-24r^4s^8-104r^4s^7t^3-234r^4s^7t^2+129r^4s^7t+33r^4s^7+54r^4s^6t^4+516r^4s^6t^3+ \\
& 71r^4s^6t^2-297r^4s^6t-297r^4s^5t^4-671r^4s^5t^3+649r^4s^5t^2+495r^4s^4t^4-385r^4s^4t^3+34r^3s^{10}t-17r^3s^{10}-116r^3s^9t^2-56r^3s^9t+64r^3s^9+ \\
& 50r^3s^8t^3+415r^3s^8t^2-146r^3s^8t-66r^3s^8+54r^3s^7t^4-177r^3s^7t^3-391r^3s^7t^2+297r^3s^7t-297r^3s^6t^4+253r^3s^6t^3-11r^3s^6t^2+495r^3s^5t^4- \\
& 385r^3s^5t^3-14r^2s^{11}t+7r^2s^{11}+50r^2s^{10}t-24r^2s^{10}+138r^2s^9t^3-91r^2s^9t^2-58r^2s^9t+22r^2s^9-144r^2s^8t^4-452r^2s^8t^3+390r^2s^8t^2+ \\
& 22r^2s^8t+594r^2s^7t^4+187r^2s^7t^3-473r^2s^7t^2-693r^2s^6t^4+539r^2s^6t^3+21rs^{11}t^2-14rs^{11}t-70rs^{10}t^3-21rs^{10}t^2+48rs^{10}t+54rs^9t^4+ \\
& 208rs^9t^3-116rs^9t^2-44rs^9t-198rs^8t^4-88rs^8t^3+176rs^8t^2+198rs^7t^4-154rs^7t^3))/(27720t^3(r-t)^3(s-t)^3(t-1)^3)-((x-x_n)^9 \\
& (4r^3s^2-8r^3s^2t+7r^3st^2-13r^3st+4r^3s+7r^3t^3+7r^3t^2-8r^3t-8r^2s^3t+4r^2s^3+4r^2s^2t^2-36r^2s^2t+16r^2s^2+19r^2st^3+27r^2st^2-36r^2st+ \\
& 4r^2s-9r^2t^4+19r^2t^3+4r^2t^2-8r^2t+7rs^3t^2-13rs^3t+4rs^3+19rs^2t^3+27rs^2t^2-36rs^2t+4rs^2-36rst^4+20rst^3+27rst^2-13rst-36rt^4+ \\
& 19rt^3+7rt^2+7s^3t^3+7s^3t^2-8s^3t-9s^2t^4+19s^2t^3+4s^2t^2-8s^2t-36st^4+19st^3+7st^2-9t^4+7t^3))/(504h^6t^3(r-t)^3(s-t)^3(t-1)^3)+ \\
& ((x-x_n)^{12}(rs-2rt-2st+3rt^2+3st^2+3t^2-4t^3-2rst))/(660h^9t^3(r-t)^3(s-t)^3(t-1)^3)+((x-x_n)^{10}(r^3s-2r^3st+3r^3t^2-2r^3t- \\
& 8r^2s^2t+4r^2s^2+10r^2st^2-15r^2st+4r^2s+3r^2t^3+10r^2t^2-8r^2t-2rs^3t+rs^3+10rs^2t^2-15rs^2t+4rs^2-2rst^3+21rst^2-15rst+rs-9rt^4- \\
& 2rt^3+10rt^2-2rt+3s^3t^2-2s^3t+3s^2t^3+10s^2t^2-8s^2t-9st^4-2st^3+10st^2-2st-9t^4+3t^3+3t^2))/(360h^7t^3(r-t)^3(s-t)^3(t-1)^3)- \\
& (r^2s^2(x-x_n)^5(3rs-5rt-5st+7rt^2+7st^2+7t^2-9t^3-5rst))/(60h^2t^2(r-t)^3(s-t)^3(t-1)^3)+(rs(x-x_n)^6(r^2s^2t-5r^2s^2t^2+r^2s^2+ \\
& 7r^2st^3-7r^2st^2+r^2st+7r^2t^3-5r^2t^2+7rs^2t^3-7rs^2t^2+rs^2t-9rst^4+17rst^3-7rst^2-9rt^4+7rt^3+7s^2t^3-5s^2t^2-9st^4+7st^3))/ \\
& (60h^3t^3(r-t)^3(s-t)^3(t-1)^3)
\end{aligned}
$$

$$
\begin{aligned}
\beta_1 = {} & ((x-x_n)^8(2r^3s^3 - r^3s^3t - 4r^3s^2t^2 + 7r^3s^2t + 2r^3s^2 - r^3st^3 + 7r^3st^2 - 2r^3st - 7r^3s + 2r^3t^3 + 2r^3t^2 - 7r^3t - 4r^2s^3t^2 + 7r^2s^3t + 2r^2s^3 - \\
& 4r^2s^2t^3 + 12r^2s^2t^2 + 3r^2s^2t - 10r^2s^2 + 7r^2st^3 + 3r^2st^2 - 26r^2st + 9r^2s + 2r^2t^3 - 10r^2t^2 + 9r^2t - rs^3t^3 + 7rs^3t^2 - 2rs^3t - 7rs^3 + 7rs^2t^3 + \\
& 3rs^2t^2 - 26rs^2t + 9rs^2 - 2rst^3 - 26rst^2 + 36rst - 7rt^3 + 9rt^2 + 2s^3t^3 + 2s^3t^2 - 7s^3t + 2s^2t^3 - 10s^2t^2 + 9s^2t - 7st^3 + 9st^2))/(168h^5 \\
& (r-1)^3(s-1)^3(t-1)^3) - (h^2(x-x_n)(7r^{11}s^2t - 14r^{11}s^2 - 14r^{11}st + 21r^{11}s - 17r^{10}s^3t + 34r^{10}s^3 - 24r^{10}s^2t^2 + 50r^{10}s^2t + 48r^{10}st^2 - \\
& 21r^{10}st - 70r^{10}s + 5r^9s^4t - 10r^9s^4 + 64r^9s^3t^2 - 56r^9s^3t - 116r^9s^3 + 22r^9s^2t^3 - 58r^9s^2t^2 - 91r^9s^2t + 138r^9s^2 - 44r^9st^3 - 116r^9st^2 + \\
& 208r^9st + 54r^9s + 5r^8s^5t - 10r^8s^5 - 24r^8s^4t^2 + 10r^8s^4t + 60r^8s^4 - 66r^8s^3t^3 - 146r^8s^3t^2 + 415r^8s^3t + 50r^8s^3 + 22r^8s^2t^3 + 390r^8s^2t^2 - \\
& 452r^8s^2t - 144r^8s^2 + 176r^8st^3 - 88r^8st^2 - 198r^8st + 5r^7s^6t - 10r^7s^6 - 24r^7s^5t^2 + 10r^7s^5t + 60r^7s^5 + 33r^7s^4t^3 + 129r^7s^4t^2 - \\
& 234r^7s^4t - 104r^7s^4 + 297r^7s^3t^3 - 391r^7s^3t^2 - 177r^7s^3t + 54r^7s^3 - 473r^7s^2t^3 + 187r^7s^2t^2 + 594r^7s^2t - 154r^7st^3 + 198r^7st^2 + 5r^6s^7t -^{5} \\
& t - 10r^6s^7 - 24r^6s^6t^2 + 10r^6s^6t + 60r^6s^6 + 33r^6s^5t^3 + 129r^6s^5t^2 - 234r^6s104r^6s^5 - 297r^6s^4t^3 + 71r^6s^4t^2 + 516r^6s^4t + 54r^6s^4 - \\
& 11r^6s^3t^3 + 253r^6s^3t^2 - 297r^6s^3t + 539r^6s^2t^3 - 693r^6s^2t^2 + 5r^5s^8t - 10r^5s^8 - 24r^5s^7t^2 + 10r^5s^7t + 60r^5s^7 + 33r^5s^6t^3 + 129r^5s^6t^2 - \\
& 234r^5s^6t - 104r^5s^6 - 297r^5s^5t^3 + 71r^5s^5t^2 + 516r^5s^5t + 54r^5s^5 + 649r^5s^4t^3 - 671r^5s^4t^2 - 297r^5s^4t - 385r^5s^3t^3 + 495r^5s^3t^2 + \\
& 5r^4s^9t - 10r^4s^9 - 24r^4s^8t^2 + 10r^4s^8t + 60r^4s^8 + 33r^4s^7t^3 + 129r^4s^7t^2 - 234r^4s^7t - 104r^4s^7 - 297r^4s^6t^3 + 71r^4s^6t^2 + 516r^4s^6t + \\
& 54r^4s^6 + 649r^4s^5t^3 - 671r^4s^5t^2 - 297r^4s^5t - 385r^4s^4t^3 + 495r^4s^4t^2 - 17r^3s^{10}t + 34r^3s^{10} + 64r^3s^9t^2 - 56r^3s^9t - 116r^3s^9 - 66r^3s^8t^3 - \\
& 146r^3s^8t^2 + 415r^3s^8t + 50r^3s^8 + 297r^3s^7t^3 - 391r^3s^7t^2 - 177r^3s^7t + 54r^3s^7 - 11r^3s^6t^3 + 253r^3s^6t^2 - 297r^3s^6t - 385r^3s^5t^3 + \\
& 495r^3s^5t^2 + 7r^2s^{11}t - 14r^2s^{11} - 24r^2s^{10}t^2 + 50r^2s^{10}t + 22r^2s^9t^3 - 58r^2s^9t^2 - 91r^2s^9t + 138r^2s^9 + 22r^2s^8t^3 + 390r^2s^8t^2 - 452r^2s^8t - \\
& 144r^2s^8 - 473r^2s^7t^3 + 187r^2s^7t^2 + 594r^2s^7t + 539r^2s^6t^3 - 693r^2s^6t^2 - 14rs^{11}t + 21rs^{11} + 48rs^{10}t^2 - 21rs^{10}t - 70rs^{10} - 44rs^9t^3 - \\
& 116rs^9t^2 + 208rs^9t + 54rs^9 + 176rs^8t^3 - 88rs^8t^2 - 198rs^8t - 154rs^7t^3 + 198rs^7t^2))/(27720(r-1)^3(s-1)^3(t-1)^3) - ((x-x_n)^{12} \\
& (3r + 3s + 3t - 2rs - 2rt - 2st + rst - 4))/(660h^9(r-1)^3(s-1)^3(t-1)^3) - ((x-x_n)^{10}(r^3st - 2r^3s - 2r^3t + 3r^3 + 4r^2s^2t - 8r^2s^2 + \\
& 4r^2st^2 - 15r^2st + 10r^2s - 8r^2t^2 + 10r^2t + 3r^2 + rs^3t - 2rs^3 + 4rs^2t^2 - 15rs^2t + 10rs^2 + rst^3 - 15rst^2 + 21rst - 2rs - 2rt^3 + 10rt^2 - \\
& 2rt - 9r - 2s^3t + 3s^3 - 8s^2t^2 + 10s^2t + 3s^2 - 2st^3 + 10st^2 - 2st - 9s + 3t^3 + 3t^2 - 9t))/(360h^7(r-1)^3(s-1)^3(t-1)^3) + ((x-x_n)^9 \\
& (4r^3s^2t - 8r^3s^2 + 4r^3st^2 - 13r^3st + 7r^3s - 8r^3t^2 + 7r^3t + 7r^3 + 4r^2s^3t - 8r^2s^3 + 16r^2s^2t^2 - 36r^2s^2t + 4r^2s^2 + 4r^2st^3 - 36r^2st^2 + \\
& 27r^2st + 19r^2s - 8r^2t^3 + 4r^2t^2 + 19r^2t - 9r^2 + 4rs^3t^2 - 13rs^3t + 7rs^3 + 4rs^2t^3 - 36rs^2t^2 + 27rs^2t + 19rs^2 - 13rst^3 + 27rst^2 + \\
& 20rst - 36rs + 7rt^3 + 19rt^2 - 36rt - 8s^3t^2 + 7s^3t + 7s^3 - 8s^2t^3 + 4s^2t^2 + 19s^2t - 9s^2 + 7st^3 + 19st^2 - 36st + 7t^3 - 9t^2))/ \\
& (504h^6(r-1)^3(s-1)^3(t-1)^3) - ((x-x_n)^7(5r^3s^3t - 4r^3s^3t^2 + 5r^3s^3 - 4r^3s^2t^3 + 4r^3s^2t^2 + 13r^3s^2t - 7r^3s^2 + 5r^3st^3 + 13r^3st^2 - \\
& 28r^3st + 5r^3t^3 - 7r^3t^2 - 4r^2s^3t^3 + 4r^2s^3t^2 + 13r^2s^3t - 7r^2s^3 + 4r^2s^2t^3 + 24r^2s^2t^2 - 47r^2s^2t + 9r^2s^2 + 13r^2st^3 - 47r^2st^2 + \\
& 36r^2st - 7r^2t^3 + 9r^2t^2 + 5rs^3t^3 + 13rs^3t^2 - 28rs^3t + 13rs^2t^3 - 47rs^2t^2 + 36rs^2t - 28rst^3 + 36rst^2 + 5s^3t^3 - 7s^3t^2 - 7s^2t^3 + 9s^2t^2))/ \\
& (210h^4(r-1)^3(s-1)^3(t-1)^3) + (h(x-x_n)^2(7r^{11}st - 14r^{11}s - 14r^{11}t + 21r^{11} - 17r^{10}s^2t + 34r^{10}s^2 - 24r^{10}st^2 + 50r^{10}st + 48r^{10}t^2 - \\
& 21r^{10}t - 70r^{10} + 5r^9s^3t - 10r^9s^3 + 64r^9s^2t^2 - 56r^9s^2t - 116r^9s^2 + 22r^9st^3 - 58r^9st^2 - 91r^9st + 138r^9s - 44r^9t^3 - 116r^9t^2 + 208r^9t + \\
& 54r^9 + 5r^8s^4t - 10r^8s^4 - 24r^8s^3t^2 + 10r^8s^3t + 60r^8s^3 - 66r^8s^2t^3 - 146r^8s^2t^2 + 415r^8s^2t + 50r^8s^2 + 22r^8st^3 + 390r^8st^2 - 452r^8st - \\
& 144r^8s + 176r^8t^3 - 88r^8t^2 - 198r^8t + 5r^7s^5t - 10r^7s^5 - 24r^7s^4t^2 + 10r^7s^4t + 60r^7s^4 + 33r^7s^3t^3 + 129r^7s^3t^2 - 234r^7s^3t - 104r^7s^3 + \\
& 297r^7s^2t^3 - 391r^7s^2t^2 - 177r^7s^2t + 54r^7s^2 - 473r^7st^3 + 187r^7st^2 + 594r^7st - 154r^7t^3 + 198r^7t^2 + 5r^6s^6t - 10r^6s^6 - 24r^6s^5t^2 + \\
& 10r^6s^5t + 60r^6s^5 + 33r^6s^4t^3 + 129r^6s^4t^2 - 234r^6s^4t - 104r^6s^4 - 297r^6s^3t^3 + 71r^6s^3t^2 + 516r^6s^3t + 54r^6s^3 - 11r^6s^2t^3 + 253r^6s^2t^2 - \\
& 297r^6s^2t + 539r^6st^3 - 693r^6st^2 + 5r^5s^7t - 10r^5s^7 - 24r^5s^6t^2 + 10r^5s^6t + 60r^5s^6 + 33r^5s^5t^3 + 129r^5s^5t^2 - 234r^5s^5t - 104r^5s^5 - \\
& 297r^5s^4t^3 + 71r^5s^4t^2 + 516r^5s^4t + 54r^5s^4 + 649r^5s^3t^3 - 671r^5s^3t^2 - 297r^5s^3t - 385r^5s^2t^3 + 495r^5s^2t^2 + 5r^4s^8t - 10r^4s^8 - \\
& 24r^4s^7t^2 + 10r^4s^7t + 60r^4s^7 + 33r^4s^6t^3 + 129r^4s^6t^2 - 234r^4s^6t - 104r^4s^6 - 297r^4s^5t^3 + 71r^4s^5t^2 + 516r^4s^5t + 54r^4s^5 + 649r^4s^4t^3 - \\
& 671r^4s^4t^2 - 297r^4s^4t - 385r^4s^3t^3 + 495r^4s^3t^2 + 5r^3s^9t - 10r^3s^9 - 24r^3s^8t^2 + 10r^3s^8t + 60r^3s^8 + 33r^3s^7t^3 + 129r^3s^7t^2 - 234r^3s^7t - \\
& 104r^3s^7 - 297r^3s^6t^3 + 71r^3s^6t^2 + 516r^3s^6t + 54r^3s^6 + 649r^3s^5t^3 - 671r^3s^5t^2 - 297r^3s^5t - 385r^3s^4t^3 + 495r^3s^4t^2 - 17r^2s^{10}t + \\
& 34r^2s^{10} + 64r^2s^9t^2 - 56r^2s^9t - 116r^2s^9 - 66r^2s^8t^3 - 146r^2s^8t^2 + 415r^2s^8t + 50r^2s^8 + 297r^2s^7t^3 - 391r^2s^7t^2 - 177r^2s^7t + 54r^2s^7 - \\
& 11r^2s^6t^3 + 253r^2s^6t^2 - 297r^2s^6t - 385r^2s^5t^3 + 495r^2s^5t^2 + 7rs^{11}t - 14rs^{11} - 24rs^{10}t^2 + 50rs^{10}t + 22rs^9t^3 - 58rs^9t^2 - 91rs^9t + \\
& 138rs^9 + 22rs^8t^3 + 390rs^8t^2 - 452rs^8t - 144rs^8 - 473rs^7t^3 + 187rs^7t^2 + 594rs^7t + 539rs^6t^3 - 693rs^6t^2 - 14s^{11}t + 21s^{11} + \\
& 48s^{10}t^2 - 21s^{10}t - 70s^{10} - 44s^9t^3 - 116s^9t^2 + 208s^9t + 54s^9 + 176s^8t^3 - 88s^8t^2 - 198s^8t - 154s^7t^3 + 198s^7t^2))/(27720(r-1)^3 \\
& (s-1)^3(t-1)^3) - ((x-x_n)^{11}(8r^2s - 4r^2st + 8r^2t - 12r^2 - 4rs^2t + 8rs^2 - 4rst^2 + 21rst - 19rs + 8rt^2 - 19rt + 9r + 8s^2t - 12s^2 \\
& + 8st^2 - 19st + 9s - 12t^2 + 9t + 9))/(990h^8(r-1)^3(s-1)^3(t-1)^3) - (rst(x-x_n)^6(r^2s^2t^2 + r^2s^2t - 5r^2s^2 + r^2st^2 - 7r^2st + \\
& 7r^2s - 5r^2t^2 + 7r^2t + rs^2t^2 - 7rs^2t + 7rs^2 - 7rst^2 + 17rst - 9rs + 7rt^2 - 9rt - 5s^2t^2 + 7s^2t + 7st^2 - 9st))/(60h^3(r-1)^3(s-1)^3 \\
& (t-1)^3) + (r^2s^2t^2(x-x_n)^5(7r + 7s + 7t - 5rs - 5rt - 5st + 3rst - 9))/(60h^2(r-1)^3(s-1)^3(t-1)^3)
\end{aligned}
$$

$$\gamma_0 = (x-x_n)^4/24 + (x-x_n)^{12}/(1320h^8r^2s^2t^2) + ((x-x_n)^8(r^2s^2+4r^2st+4r^2s+r^2t^2+4r^2t+r^2+4rs^2t+4rs^2+4rst^2+16rst+4rs+$$
$$4rt^2+4rt+s^2t^2+4s^2t+s^2+4st^2+4st+t^2))/(336h^4r^2s^2t^2) - ((x-x_n)^9(r^2s+r^2t+r^2+rs^2+4rst+4rs+rt^2+4rt+r+s^2t+s^2+$$
$$st^2+4st+s+t^2+t))/(252h^5r^2s^2t^2) + (h^3(x-x_n)(7r^9-17r^8s-24r^8t-24r^8+5r^7s^2+64r^7st+64r^7s+22r^7t^2+88r^7t+22r^7+$$
$$5r^6s^3-24r^6s^2t-24r^6s^2-66r^6st^2-264r^6st-66r^6s-88r^6t^2-88r^6t+5r^5s^4-24r^5s^3t-24r^5s^3+33r^5s^2t^2+132r^5s^2t+33r^5s^2+$$
$$308r^5st^2+308r^5st+99r^5t^2+5r^4s^5-24r^4s^4t-24r^4s^4+33r^4s^3t^2+132r^4s^3t+33r^4s^3-220r^4s^2t^2-220r^4s^2t-429r^4st^2+5r^3s^6-$$
$$24r^3s^5t-24r^3s^5+33r^3s^4t^2+132r^3s^4t+33r^3s^4-220r^3s^3t^2-220r^3s^3t+495r^3s^2t^2+5r^2s^7-24r^2s^6t-24r^2s^6+33r^2s^5t^2+132r^2s^5t+$$
$$33r^2s^5-220r^2s^4t^2-220r^2s^4t+495r^2s^3t^2-17rs^8+64rs^7t+64rs^7-66rs^6t^2-264rs^6t-66rs^6+308rs^5t^2+308rs^5t-429rs^4t^2+$$
$$7s^9-24s^8t-24s^8+22s^7t^2+88s^7t+22s^7-88s^6t^2-88s^6t+99s^5t^2))/(55440rst^2) - ((x-x_n)^7(r^2s^2t+r^2s^2+r^2st^2+4r^2st+r^2s+$$
$$r^2t^2+r^2t+rs^2t^2+4rs^2t+rs^2+4rst^2+4rst+rt^2+s^2t^2+s^2t+st^2))/(105h^3r^2s^2t^2) - ((x-x_n)^{11}(r+s+t+1))/(495h^7r^2s^2t^2) -$$
$$((x-x_n)^5(rs+rt+st+rst))/(30hrst) + ((x-x_n)^6(r^2s^2t^2+4r^2s^2t+r^2s^2+4r^2st^2+4r^2st+r^2t^2+4rs^2t^2+4rs^2t+4rst^2+s^2t^2))/$$
$$(120h^2r^2s^2t^2) - (h^2(x-x_n)^2(7r^{10}-17r^9s-24r^9t-24r^9+5r^8s^2+64r^8st+64r^8s+22r^8t^2+88r^8t+22r^8+5r^7s^3-24r^7s^2t-24r^7s^2-$$
$$66r^7st^2-264r^7st-66r^7s-88r^7t^2-88r^7t+5r^6s^4-24r^6s^3t-24r^6s^3+33r^6s^2t^2+132r^6s^2t+33r^6s^2+308r^6st^2+308r^6st+99r^6t^2+$$
$$5r^5s^5-24r^5s^4t-24r^5s^4+33r^5s^3t^2+132r^5s^3t+33r^5s^3-220r^5s^2t^2-220r^5s^2t-429r^5st^2+5r^4s^6-24r^4s^5t-24r^4s^5+33r^4s^4t^2+$$
$$132r^4s^4t+33r^4s^4-220r^4s^3t^2-220r^4s^3t+495r^4s^2t^2+5r^3s^7-24r^3s^6t-24r^3s^6+33r^3s^5t^2+132r^3s^5t+33r^3s^5-220r^3s^4t^2-$$
$$220r^3s^4t+495r^3s^3t^2+5r^2s^8-24r^2s^7t-24r^2s^7+33r^2s^6t^2+132r^2s^6t+33r^2s^6-220r^2s^5t^2-220r^2s^5t+495r^2s^4t^2-17rs^9+64rs^8t+$$
$$64rs^8-66rs^7t^2-264rs^7t-66rs^7+308rs^6t^2+308rs^6t-429rs^5t^2+7s^{10}-24s^9t-24s^9+22s^8t^2+88s^8t+22s^8-88s^7t^2-88s^7t+$$
$$99s^6t^2))/(55440r^2s^2t^2) + ((x-x_n)^{10}(r^2+4rs+4rt+4r+s^2+4st+4s+t^2+4t+1))/(720h^6r^2s^2t^2)$$

$$\gamma_r = (x-x_n)^{12}/(1320h^8r^2(r-s)^2(r-t)^2(r-1)^2) - ((x-x_n)^9(r+2s+2t+4rs+4rt+8st+rs^2+rt^2+2st^2+2s^2t+2s^2+2t^2+4rst))/$$
$$(504h^5r^2(r-s)^2(r-t)^2(r-1)^2) + (h^2(x-x_n)^2(14r^{10}-28r^9s-42r^9t-42r^9+5r^8s^2+90r^8st+90r^8s+33r^8t^2+132r^8t+33r^8+$$
$$5r^7s^3-20r^7s^2t-20r^7s^2-77r^7st^2-308r^7st-77r^7s-110r^7t^2-110r^7t+5r^6s^4-20r^6s^3t-20r^6s^3+22r^6s^2t^2+88r^6s^2t+22r^6s^2+$$
$$286r^6st^2+286r^6st+99r^6t^2+5r^5s^5-20r^5s^4t-20r^5s^4+22r^5s^3t^2+88r^5s^3t+22r^5s^3-110r^5s^2t^2-110r^5s^2t-297r^5st^2+5r^4s^6-$$
$$20r^4s^5t-20r^4s^5+22r^4s^4t^2+88r^4s^4t+22r^4s^4-110r^4s^3t^2-110r^4s^3t+165r^4s^2t^2+5r^3s^7-20r^3s^6t-20r^3s^6+22r^3s^5t^2+88r^3s^5t+$$
$$22r^3s^5-110r^3s^4t^2-110r^3s^4t+165r^3s^3t^2+5r^2s^8-20r^2s^7t-20r^2s^7+22r^2s^6t^2+88r^2s^6t+22r^2s^6-110r^2s^5t^2-110r^2s^5t+$$
$$165r^2s^4t^2+5rs^9-20rs^8t-20rs^8+22rs^7t^2+88rs^7t+22rs^7-110rs^6t^2-110rs^6t+165rs^5t^2-7s^{10}+24s^9t+24s^9-22s^8t^2-88s^8t-$$
$$22s^8+88s^7t^2+88s^7t-99s^6t^2))/(55440r^2(r-s)^2(r-t)^2(r-1)^2) - (h^3(x-x_n)(14r^9s-28r^8s^2-42r^8st-42r^8s+5r^7s^3+90r^7s^2t+$$
$$90r^7s^2+33r^7st^2+132r^7st+33r^7s+5r^6s^4-20r^6s^3t-20r^6s^3-77r^6s^2t^2-308r^6s^2t-77r^6s^2-110r^6st^2-110r^6st+5r^5s^5-20r^5s^4t-$$
$$20r^5s^4+22r^5s^3t^2+88r^5s^3t+22r^5s^3+286r^5s^2t^2+286r^5s^2t+99r^5st^2+5r^4s^6-20r^4s^5t-20r^4s^5+22r^4s^4t^2+88r^4s^4t+22r^4s^4-$$
$$110r^4s^3t^2-110r^4s^3t-297r^4s^2t^2+5r^3s^7-20r^3s^6t-20r^3s^6+22r^3s^5t^2+88r^3s^5t+22r^3s^5-110r^3s^4t^2-110r^3s^4t+165r^3s^3t^2+5r^2s^8-$$
$$20r^2s^7t-20r^2s^7+22r^2s^6t^2+88r^2s^6t+22r^2s^6-110r^2s^5t^2-110r^2s^5t+165r^2s^4t^2+5rs^9-20rs^8t-20rs^8+22rs^7t^2+88rs^7t+22rs^7-$$
$$110rs^6t^2-110rs^6t+165rs^5t^2-7s^{10}+24s^9t+24s^9-22s^8t^2-88s^8t-22s^8+88s^7t^2+88s^7t-99s^6t^2))/(55440r(r-s)^2(r-t)^2$$
$$(r-1)^2) + ((x-x_n)^{10}(2r+4s+4t+2rs+2rt+4st+s^2+t^2+1))/(720h^6r^2(r-s)^2(r-t)^2(r-1)^2) + ((x-x_n)^8(s^2t^2+2rs+2rt+$$
$$4st+2rs^2+2rt^2+4st^2+4s^2t+s^2+t^2+2rst^2+2rs^2t+8rst))/(336h^4r^2(r-s)^2(r-t)^2(r-1)^2) - ((x-x_n)^7(2s^2t^2+rs^2+rt^2+2st^2+$$
$$2s^2t+4rst^2+4rs^2t+rs^2t^2+4rst))/(210h^3r^2(r-s)^2(r-t)^2(r-1)^2) - ((x-x_n)^{11}(r+2s+2t+2))/(990h^7r^2(r-s)^2(r-t)^2(r-1)^2) -$$
$$(s^2t^2(x-x_n)^5)/(60hr(r-s)^2(r-t)^2(r-1)^2) + (st(x-x_n)^6(2rs+2rt+st+2rst))/(120h^2r^2(r-s)^2(r-t)^2(r-1)^2)$$

$$\gamma_r = (x-x_n)^{12}/(1320h^8s^2(r-s)^2(s-t)^2(s-1)^2) - ((x-x_n)^9(2r+s+2t+4rs+8rt+4st+r^2s+2rt^2+2r^2t+st^2+2r^2+2t^2+4rst))/$$
$$(504h^5s^2(r-s)^2(s-t)^2(s-1)^2) + (h^2(x-x_n)^2(5r^9s-7r^{10}+24r^9t+24r^9+5r^8s^2-20r^8st-20r^8s-22r^8t^2-88r^8t-22r^8+5r^7s^3-$$
$$20r^7s^2t-20r^7s^2+22r^7st^2+88r^7st+22r^7s+88r^7t^2+88r^7t+5r^6s^4-20r^6s^3t-20r^6s^3+22r^6s^2t^2+88r^6s^2t+22r^6s^2-110r^6st^2-$$
$$110r^6st-99r^6t^2+5r^5s^5-20r^5s^4t-20r^5s^4+22r^5s^3t^2+88r^5s^3t+22r^5s^3-110r^5s^2t^2-110r^5s^2t+165r^5st^2+5r^4s^6-20r^4s^5t-$$
$$20r^4s^5+22r^4s^4t^2+88r^4s^4t+22r^4s^4-110r^4s^3t^2-110r^4s^3t+165r^4s^2t^2+5r^3s^7-20r^3s^6t-20r^3s^6+22r^3s^5t^2+88r^3s^5t+22r^3s^5-$$
$$110r^3s^4t^2-110r^3s^4t+165r^3s^3t^2+5r^2s^8-20r^2s^7t-20r^2s^7+22r^2s^6t^2+88r^2s^6t+22r^2s^6-110r^2s^5t^2-110r^2s^5t+165r^2s^4t^2-$$
$$28rs^9+90rs^8t+90rs^8-77rs^7t^2-308rs^7t-77rs^7+286rs^6t^2+286rs^6t-297rs^5t^2+14s^{10}-42s^9t-42s^9+33s^8t^2+132s^8t+$$
$$33s^8-110s^7t^2-110s^7t+99s^6t^2))/(55440s^2(r-s)^2(s-t)^2(s-1)^2) - (h^3(x-x_n)(5r^9s-7r^{10}+24r^9t+24r^9+5r^8s^2-20r^8st-$$
$$20r^8s-22r^8t^2-88r^8t-22r^8+5r^7s^3-20r^7s^2t-20r^7s^2+22r^7st^2+88r^7st+22r^7s+88r^7t^2+88r^7t+5r^6s^4-20r^6s^3t-20r^6s^3+$$
$$22r^6s^2t^2+88r^6s^2t+22r^6s^2-110r^6st^2-110r^6st-99r^6t^2+5r^5s^5-20r^5s^4t-20r^5s^4+22r^5s^3t^2+88r^5s^3t+22r^5s^3-110r^5s^2t^2-$$
$$110r^5s^2t+165r^5st^2+5r^4s^6-20r^4s^5t-20r^4s^5+22r^4s^4t^2+88r^4s^4t+22r^4s^4-110r^4s^3t^2-110r^4s^3t+165r^4s^2t^2+5r^3s^7-20r^3s^6t-$$

$$20r^3s^6+22r^3s^5t^2+88r^3s^5t+22r^3s^5-110r^3s^4t^2-110r^3s^4t+165r^3s^3t^2-28r^2s^8+90r^2s^7t+90r^2s^7-77r^2s^6t^2-308r^2s^6t-77r^2s^6+$$
$$286r^2s^5t^2+286r^2s^5t-297r^2s^4t^2+14rs^9-42rs^8t-42rs^8+33rs^7t^2+132rs^7t+33rs^7-110rs^6t^2-110rs^6t+99rs^5t^2))/(55440s$$
$$(r-s)^2(s-t)^2(s-1)^2)+((x-x_n)^{10}(4r+2s+4t+2rs+4rt+2st+r^2+t^2+1))/(720h^6s^2(r-s)^2(s-t)^2(s-1)^2)+((x-x_n)^8$$
$$(r^2t^2+2rs+4rt+2st+2r^2s+4rt^2+4r^2t+2st^2+r^2+t^2+2rst^2+2r^2st+8rst))/(336h^4s^2(r-s)^2(s-t)^2(s-1)^2)-((x-x_n)^7$$
$$(2r^2t^2+r^2s+2rt^2+2r^2t+st^2+4rst^2+4r^2st+r^2st^2+4rst))/(210h^3s^2(r-s)^2(s-t)^2(s-1)^2)-((x-x_n)^{11}(2r+s+2t+2))/$$
$$(990h^7s^2(r-s)^2(s-t)^2(s-1)^2)-(r^2t^2(x-x_n)^5)/(60hs(r-s)^2(s-t)^2(s-1)^2)+(rt(x-x_n)^6(2rs+rt+2st+2rst))/(120h^2s^2$$
$$(r-s)^2(s-t)^2(s-1)^2)$$

$$\gamma_t=(x-x_n)^{12}/(1320h^8t^2(r-t)^2(s-t)^2(t-1)^2)-((x-x_n)^9(2r+2s+t+8rs+4rt+4st+2rs^2+2r^2s+r^2t+s^2t+2r^2+2s^2+4rst))/$$
$$(504h^5t^2(r-t)^2(s-t)^2(t-1)^2)-(h^2(x-x_n)^2(33r^2s^6+33r^3s^5+33r^4s^4+33r^5s^3+33r^6s^2-24r^2s^7-24r^3s^6-24r^4s^5-24r^5s^4-$$
$$24r^6s^3-24r^7s^2+5r^2s^8+5r^3s^7+5r^4s^6+5r^5s^5+5r^6s^4+5r^7s^3+5r^8s^2-66rs^7-66r^7s+64rs^8+64r^8s-17rs^9-17r^9s-44r^7t+$$
$$44r^8t-12r^9t-44s^7t+44s^8t-12s^9t+22r^8-24r^9+7r^{10}+22s^8-24s^9+7s^{10}+154rs^6t+154r^6st-132rs^7t-132r^7st+32rs^8t+$$
$$32r^8st-110r^2s^5t-110r^3s^4t-110r^4s^3t-110r^5s^2t+66r^2s^6t+66r^3s^5t+66r^4s^4t+66r^5s^3t+66r^6s^2t-12r^2s^7t-12r^3s^6t-12r^4s^5t-$$
$$12r^5s^4t-12r^6s^3t-12r^7s^2t))/(55440t^2(r-t)^2(s-t)^2(t-1)^2)+((x-x_n)^{10}(4r+4s+2t+4rs+2rt+2st+r^2+s^2+1))/(720h^6t^2$$
$$(r-t)^2(s-t)^2(t-1)^2)+((x-x_n)^8(r^2s^2+4rs+2rt+2st+4rs^2+4r^2s+2r^2t+2s^2t+r^2+s^2+2rs^2t+2r^2st+8rst))/(336h^4t^2$$
$$(r-t)^2(s-t)^2(t-1)^2)-((x-x_n)^7(2r^2s^2+2rs^2+2r^2s+r^2t+s^2t+4rs^2t+4r^2st+r^2s^2t+4rst))/(210h^3t^2(r-t)^2(s-t)^2(t-1)^2)-$$
$$((x-x_n)^{11}(2r+2s+t+2))/(990h^7t^2(r-t)^2(s-t)^2(t-1)^2)-(r^2s^2(x-x_n)^5)/(60ht(r-t)^2(s-t)^2(t-1)^2)+(rs(x-x_n)^6(rs+$$
$$2rt+2st+2rst))/(120h^2t^2(r-t)^2(s-t)^2(t-1)^2)+(h^3rs(x-x_n)(33r^2s^5+33r^3s^4+33r^4s^3+33r^5s^2-24r^2s^6-24r^3s^5-24r^4s^4-$$
$$24r^5s^3-24r^6s^2+5r^2s^7+5r^3s^6+5r^4s^5+5r^5s^4+5r^6s^3+5r^7s^2-66rs^6-66r^6s+64rs^7+64r^7s-17rs^8-17r^8s-44r^6t+44r^7t-$$
$$12r^8t-44s^6t+44s^7t-12s^8t+22r^7-24r^8+7r^9+22s^7-24s^8+7s^9+154rs^5t+154r^5st-132rs^6t-132r^6st+32rs^7t+32r^7st-$$
$$110r^2s^4t-110r^3s^3t-110r^4s^2t+66r^2s^5t+66r^3s^4t+66r^4s^3t+66r^5s^2t-12r^2s^6t-12r^3s^5t-12r^4s^4t-12r^5s^3t-12r^6s^2t))/$$
$$(55440t^2(r-t)^2(s-t)^2(t-1)^2)$$

$$\gamma_1=(x-x_n)^{12}/(1320h^8(r-1)^2(s-1)^2(t-1)^2)-((x-x_n)^9(2r^2s+2r^2t+r^2+2rs^2+8rst+4rs+2rt^2+4rt+2s^2t+s^2+2st^2+$$
$$4st+t^2))/(504h^5(r-1)^2(s-1)^2(t-1)^2)+((x-x_n)^8(r^2s^2+4r^2st+2r^2s+r^2t^2+2r^2t+4rs^2t+2rs^2+4rst^2+8rst+2rt^2+$$
$$s^2t^2+2s^2t+2st^2))/(336h^4(r-1)^2(s-1)^2(t-1)^2)-(h^2(x-x_n)^2(7r^{10}-17r^9s-24r^9t-12r^9+5r^8s^2+64r^8st+32r^8s+$$
$$22r^8t^2+44r^8t+5r^7s^3-24r^7s^2t-12r^7s^2-66r^7st^2-132r^7st-44r^7t^2+5r^6s^4-24r^6s^3t-12r^6s^3+33r^6s^2t^2+66r^6s^2t+$$
$$154r^6st^2+5r^5s^5-24r^5s^4t-17r^9s-24r^9t-12r^9+5r^8s^2+64r^8st+32r^8s+22r^8t^2+44r^8t+5r^7s^3-24r^7s^2t-12r^7s^2-66r^7s$$
$$t^2-132r^7st-44r^7t^2+5r^6s^4-24r^6s^3t-12r^6s^3+33r^6s^2t^2+66r^6s^2t+154r^6st^2+5r^5s^5-24r^5s^4t-33r^2s^6t^2+66r^2s^6t-$$
$$110r^2s^5t^2-17rs^9+64rs^8t+32rs^8-66rs^7t^2-132rs^7t+154rs^6t^2+7s^{10}-24s^9t-12s^9+22s^8t^2+44s^8t-44s^7t^2))/$$
$$(55440(r-1)^2(s-1)^2(t-1)^2)-((x-x_n)^{11}(2r+2s+2t+1))/(990h^7(r-1)^2(s-1)^2(t-1)^2)-((x-x_n)^7(2r^2s^2t+r^2s^2+$$
$$2r^2st^2+4r^2st+r^2t^2+2rs^2t^2+4rs^2t+4rst^2+s^2t^2))/(210h^3(r-1)^2(s-1)^2(t-1)^2)+((x-x_n)^{10}(r^2+4rs+4rt+2r+$$
$$s^2+4st+2s+t^2+2t))/(720h^6(r-1)^2(s-1)^2(t-1)^2)+(h^3rs(x-x_n)(7r^9-17r^8s-24r^8t-12r^8+5r^7s^2+64r^7st+$$
$$32r^7s+22r^7t^2+44r^7t+5r^6s^3-24r^6s^2t-12r^6s^2-66r^6st^2-132r^6st-44r^6t^2+5r^5s^4-24r^5s^3t-12r^5s^3+33r^5s^2t^2+$$
$$66r^5s^2t+154r^5st^2+5r^4s^5-24r^4s^4t-12r^4s^4+33r^4s^3t^2+66r^4s^3t-110r^4s^2t^2+5r^3s^6-24r^3s^5t-12r^3s^5+33r^3s^4t^2+$$
$$66r^3s^4t-110r^3s^3t^2+5r^2s^7-24r^2s^6t-12r^2s^6+33r^2s^5t^2+66r^2s^5t-110r^2s^4t^2-17rs^8+64rs^7t+32rs^7-66rs^6t^2-$$
$$132rs^6t+154rs^5t^2+7s^9-24s^8t-12s^8+22s^7t^2+44s^7t-44s^6t^2))/(55440(r-1)^2(s-1)^2(t-1)^2)-(r^2s^2t^2(x-x_n)^5)/$$
$$(60h(r-1)^2(s-1)^2(t-1)^2)+(rst(x-x_n)^6(2rs+2rt+2st+rst))/(120h^2(r1)^2(s-1)^2(t-1)^2)-(r^2s^2t^2(x-x_n)^5)/$$
$$(60h(r-1)^2(s-1)^2(t-1)^2)+(rst(x-x_n)^6(2rs+2rt+2st+rst))/(120h^2(r-1)^2(s-1)^2(t-1)^2)$$

Appendix II:

$$B_{112}^{[0]}=\frac{-h^3r^3}{27720s^3t^3}(-7r^7st-7r^7s-7r^7t+24r^6s^2t+24r^6s^2+24r^6st^2+59r^6st+24r^6s+24r^6t^2+24r^6t-22r^5s^3t-22r^5s^3-88r^5s^2t^2-$$
$$152r^5s^2t-88r^5s^2-22r^5st^3-152r^5st^2-152r^5st-22r^5s-22r^5t^3-88r^5t^2-22r^5t+88r^4s^3t^2+132r^4s^3t+88r^4s^3+88r^4s^2t^3+$$
$$352r^4s^2t^2+352r^4s^2t+88r^4s^2+132r^4st^3+352r^4st^2+132r^4st+88r^4t^3+88r^4t^2-99r^3s^3t^3-308r^3s^3t^2-308r^3s^3t-99r^3s^3-$$
$$308r^3s^2t^3\ 440r^3s^2t^2-308r^3s^2t-308r^3st^3-308r^3st^2-99r^3t^3+297r^2s^3t^3+132r^2s^3t^2+297r^2s^3t+132r^2s^2t^3+132r^2s^2t^2+$$
$$297r^2st^3+528rs^3t^3+528rs^3t^2+528rs^2t^3-3696s^3t^3)$$

$$B_{212}^{[0]} = \frac{-h^3 s^3}{27720 r^3 t^3} x\,(-22r^3s^5t - 22r^3s^5 + 88r^3s^4t^2 + 132r^3s^4t + 88r^3s^4 - 99r^3s^3t^3 - 308r^3s^3t^2 - 308r^3s^3t - 99r^3s^3 + 297r^3s^2t^3 +$$
$$132r^3s^2t^2 + 297r^3s^2t + 528r^3st^3 + 528r^3st^2 - 3696r^3t^3 + 24r^2s^6t + 24r^2s^6 - 88r^2s^5t^2 - 152r^2s^5t - 88r^2s^5 + 88r^2s^4t^3 +$$
$$352r^2s^4t^2 + 352r^2s^4t + 88r^2s^4 - 308r^2s^3t^3 - 440r^2s^3t^2 - 308r^2s^3t + 132r^2s^2t^3 + 132r^2s^2t^2 + 528r^2st^3 - 7rs^7t - 7rs^7 +$$
$$24rs^6t^2 + 59rs^6t + 24rs^6 - 22rs^5t^3 - 152rs^5t^2 - 152rs^5t - 22rs^5 + 132rs^4t^3 + 352rs^4t^2 + 132rs^4t - 308rs^3t^3 -$$
$$308rs^3t^2 + 297rs^2t^3 - 7s^7t + 24s^6t^2 + 24s^6t - 22s^5t^3 - 88s^5t^2 - 22s^5t + 88s^4t^3 + 88s^4t^2 - 99s^3t^3)$$

$$B_{312}^{[0]} = \frac{-h^3 t^3}{27720 r^3 s^3}(-99r^3s^3t^3 + 297r^3s^3t^2 + 528r^3s^3t - 3696r^3s^3 + 88r^3s^2t^4 - 308r^3s^2t^3 + 132r^3s^2t^2 + 528r^3s^2t - 22r^3st^5 +$$
$$132r^3st^4 - 308r^3st^3 + 297r^3st^2 - 22r^3t^5 + 88r^3t^4 - 99r^3t^3 + 88r^2s^3t^4 - 308r^2s^3t^3 + 132r^2s^3t^2 + 528r^2s^3t - 88r^2s^2$$
$$t^5 + 352r^2s^2t^4 - 440r^2s^2t^3 + 132r^2s^2t^2 + 24r^2st^6 - 152r^2st^5 + 352r^2st^4 - 308r^2st^3 + 24r^2t^6 - 88r^2t^5 + 88r^2t^4 -$$
$$22rs^3t^5 + 132rs^3t^4 - 308rs^3t^3 + 297rs^3t^2 + 24rs^2t^6 - 152rs^2t^5 + 352rs^2t^4 - 308rs^2t^3 - 7rst^7 + 59rst^6 - 152rst^5 +$$
$$132rst^4 - 7rt^7 + 24rt^6 - 22rt^5 - 22s^3t^5 + 88s^3t^4 - 99s^3t^3 + 24s^2t^6 - 88s^2t^5 + 88s^2t^4 - 7st^7 + 24st^6 - 22st^5)$$

$$B_{412}^{[0]} = \frac{-h^3}{27720 r^3 s^3 t^3}(3696r^3s^3t^3 + 528r^3s^3t^2 + 297r^3s^3t\ 99r^3s^3 + 528r^3s^2t^3 + 132r^3s^2t^2\ 308r^3s^2t + 88r^2s^3 + 132r^2s^2t^3$$
$$440r^2s^2t^2 + 352r^2s^2t - 88r^2s^2 - 308r^2st^3 + 352r^2st^2 - 152r^2st + 24r^2s + 88r^2t^3 - 88r^2t^2 + 24r^2t + 297rs^3t^3 - 308rs^3$$
$$t^2 + 132rs^3t - 22rs^3 - 308rs^2t^3 + 352rs^2t^2 - 152rs^2t + 24rs^2 + 132rst^3 - 152rst^2 + 59rst - 7rs - 22rt^3 + 24rt^2 - 7rt -$$
$$99s^3t^3 + 88s^3t^2 - 22s^3t + 88s^2t^3 - 88s^2t^2 + 24s^2t - 22st^3 + 24st^2 - 7st)$$

$$B_{512}^{[0]} = \frac{-h^2 r^2}{27720 s^3 t^3}(-42r^7st - 42r^7s - 42r^7t + 132r^6s^2t + 132r^6s^2 + 132r^6st^2 + 321r^6st + 132r^6s + 132r^6t^2 + 132r^6t -$$
$$110r^5s^3t - 110r^5s^3 - 440r^5s^2t^2 - 748r^5s^2t - 440r^5s^2 - 110r^5st^3 - 748r^5st^2 - 748r^5st - 110r^5s - 110r^5t^3 -$$
$$440r^5t^2 - 110r^5t + 396r^4s^3t^2 + 583r^4s^3t + 396r^4s^3 + 396r^4s^2t^3 + 1540r^4s^2t^2 + 1540r^4s^2t + 396r^4s^2 + 583r^4st^3 +$$
$$1540r^4st^2 + 583r^4st + 396r^4t^3 + 396r^4t^2 - 396r^3s^3t^3 - 1188r^3s^3t^2 - 1188r^3s^3t - 396r^3s^3 - 1188r^3s^2t^3 -$$
$$1584r^3s^2t^2 - 1188r^3s^2t - 1188r^3st^3 - 1188r^3st^2 - 396r^3t^3 + 990r^2s^3t^3 + 264r^2s^3t^2 + 990r^2s^3t + 264r^2s^2t^3 +$$
$$264r^2s^2t^2 + 990r^2st^3 + 1848rs^3t^3 + 1848rs^3t^2 + 1848rs^2t^3 - 9702s^3t^3)$$

$$B_{612}^{[0]} = \frac{-h^2 s^2}{27720 r^3 t^3}(-110r^3s^5t - 110r^3s^5 + 396r^3s^4t^2 + 583r^3s^4t + 396r^3s^4 - 396r^3s^3t^3 - 1188r^3s^3t^2 - 1188r^3s^3t -$$
$$396r^3s^3 + 990r^3s^2t^3 + 264r^3s^2t^2 + 990r^3s^2t + 1848r^3st^3 + 1848r^3st^2 - 9702r^3t^3 132r^2s^6t + 132r^2s^6 - 440r^2s^5t^2 -$$
$$748r^2s^5t - 440r^2s^5 + 396r^2s^4t^3 + 1540r^2s^4t^2 + 1540r^2s^4t + 396r^2s^4 1188r^2s^3t^3 - 1584r^2s^3t^2 - 1188r^2s^3t +$$
$$264r^2s^2t^3 + 264r^2s^2t^2 + 1848r^2st^3 - 42rs^7t - 42rs^7 + 132rs^6t^2 + 321rs^6t + 132rs^6 - 110rs^5t^3 - 748rs^5t^2 -$$
$$748rs^5t - 110rs^5 + 583rs^4t^3 + 1540rs^4t^2 + 583rs^4t - 1188rs^3t^3 - 1188rs^3t^2 + 990rs^2t^3 - 42s^7t + 132s^6t^2 +$$
$$132s^6t - 110s^5t^3 - 440s^5t^2 - 110s^5t + 396s^4t^3 + 396s^4t^2 - 396s^3t^3)$$

$$B_{712}^{[0]} = \frac{-h^2 t^2}{27720 r^3 s^3}(-396r^3s^3t^3 + 990r^3s^3t^2 + 1848r^3s^3t - 9702r^3s^3 + 396r^3s^2t^4 - 1188r^3s^2t^3 + 264r^3s^2t^2 + 1848r^3s^2$$
$$t - 110r^3st^5 + 583r^3st^4 - 1188r^3st^3 + 990r^3st^2 - 110r^3t^5 + 396r^3t^4 - 396r^3t^3 + 396r^2s^3t^4 - 1188r^2s^3t^3 +$$
$$264r^2s^3t^2 + 1848r^2s^3t - 440r^2s^2t^5 + 1540r^2s^2t^4 - 1584r^2s^2t^3 + 264r^2s^2t^2 + 132r^2st^6 - 748r^2st^5 + 1540r^2$$
$$st^4\ 1188r^2st^3 + 132r^2t^6\ 440r^2t^5 + 396r^2t^4\ 110rs^3t^5 + 583rs^3t^4 - 1188rs^3t^3 + 990rs^3t^2 + 132rs^2t^6 - 748rs^2t^5 +$$
$$1540rs^2t^4 - 1188rs^2t^3 - 42rst^7 + 321rst^6 - 748rst^5 + 583rst^4 - 42rt^7 + 132rt^6 - 110rt^5 - 110s^3t^5 + 396s^3t^4 -$$
$$396s^3t^3 + 132s^2t^6 - 440s^2t^5 + 396s^2t^4 - 42st^7 + 132st^6 - 110st^5)$$

$$B_{812}^{[0]} = \frac{-h^2}{27720 r^3 s^3 t^3}(-9702r^3s^3t^3 + 1848r^3s^3t^2 + 990r^3s^3t - 396r^3s^3 + 1848r^3s^2t^3 + 264r^3s^2t^2 - 1188r^3s^2t + 396r^3s^2 +$$
$$990r^3st^3 - 1188r^3st^2 + 583r^3st - 110r^3s - 396r^3t^3 + 396r^3t^2 - 110r^3t + 1848r^2s^3t^3 + 264r^2s^3t^2 - 1188r^2s^3t +$$
$$396r^2s^3 + 264r^2s^2t^3 - 1584r^2s^2t^2 + 1540r^2s^2t - 440r^2s^2 - 1188r^2st^3 + 1540r^2st^2 - 748r^2st + 132r^2s + 396r^2t^3 -$$
$$440r^2t^2 + 132r^2t + 990rs^3t^3 - 1188rs^3t^2 + 583rs^3t - 110rs^3 - 1188rs^2t^3 + 1540rs^2t^2 - 748rs^2t + 132rs^2 +$$
$$583rst^3 - 748rst^2 + 321rst - 42rs - 110rt^3 + 132rt^2 - 42rt - 396s^3t^3 + 396s^3t^2 - 110s^3t + 396s^2t^3 - 440s^2t^2 +$$
$$132s^2t - 110st^3 + 132st^2 - 42st)$$

$$B_{912}^{[0]} = \frac{hr}{1260s^3t^3}(7r^7st + 7r^7s + 7r^7t - 20r^6s^2t - 20r^6s^2 - 20r^6st^2 - 48r^6st - 20r^6s - 20r^6t^2 - 20r^6t + 15r^5s^3t + 15r^5s^3 + 60r^5s^2t^2 + 100r^5s^2t + 60r^5s^2 + 15r^5st^3 + 100r^5st^2 + 100r^5st + 15r^5s + 15r^5t^3 + 60r^5t^2 + 15r^5t - 48r^4s^3t^2 - 69r^4s^3t - 48r^4s^3 - 48r^4s^2t^3 - 180r^4s^2t^2 - 180r^4s^2t - 48r^4s^2 - 69r^4st^3 - 180r^4st^2 - 69r^4st - 48r^4t^3 - 48r^4t^2 + 42r^3s^3t^3 + 120r^3s^3t^2 + 120r^3s^3t + 42r^3s^3 + 120r^3s^2t^3 + 144r^3s^2t^2 + 120r^3s^2t + 120r^3st^3 + 120r^3st^2 + 42r^3t^3 - 84r^2s^3t^3 - 84r^2s^3t - 84r^2st^3 - 168rs^3t^3 - 168rs^3t^2 - 168rs^2t^3 + 630s^3t^3)$$

$$B_{1012}^{[0]} = \frac{hs}{1260r^3t^3}(15r^3s^5t + 15r^3s^5 - 48r^3s^4t^2 - 69r^3s^4t - 48r^3s^4 + 42r^3s^3t^3 + 120r^3s^3t^2 + 120r^3s^3t + 42r^3s^3 - 84r^3s^2t^3 - 84r^3s^2t - 168r^3st^3 - 168r^3st^2 + 630r^3t^3 - 20r^2s^6t - 20r^2s^6 + 60r^2s^5t^2 + 100r^2s^5t + 60r^2s^5 - 48r^2s^4t^3 - 180r^2s^4t^2 - 180r^2s^4t - 48r^2s^4 + 120r^2s^3t^3 + 144r^2s^3t^2 + 120r^2s^3t - 168r^2st^3 + 7rs^7t + 7rs^7 - 20rs^6t^2 - 48rs^6t - 20rs^6 + 15rs^5t^3 + 100rs^5t^2 + 100rs^5t + 15rs^5 - 69rs^4t^3 - 180rs^4t^2 - 69rs^4t + 120rs^3t^3 + 120rs^3t^2 - 84rs^2t^3 + 7s^7t - 20s^6t^2 - 20s^6t + 15s^5t^3 + 60s^5t^2 + 15s^5t - 48s^4t^3 - 48s^4t^2 + 42s^3t^3)$$

$$B_{1112}^{[0]} = \frac{ht}{1260r^3s^3}(42r^3s^3t^3 - 84r^3s^3t^2 - 168r^3s^3t + 630r^3s^3 - 48r^3s^2t^4 + 120r^3s^2t^3 - 168r^3s^2t + 15r^3st^5 - 69r^3st^4 + 120r^3st^3 - 84r^3st^2 + 15r^3t^5 - 48r^3t^4 + 42r^3t^3 - 48r^2s^3t^4 + 120r^2s^3t^3 - 168r^2s^3t + 60r^2s^2t^5 - 180r^2s^2t^4 + 144r^2s^2t^3 - 20r^2st^6 + 100r^2st^5 - 180r^2st^4 + 120r^2st^3 - 20r^2t^6 + 60r^2t^5 - 48r^2t^4 + 15rs^3t^5 - 69rs^3t^4 + 120rs^3t^3 - 84rs^3t^2 - 20rs^2t^6 + 100rs^2t^5 - 180rs^2t^4 + 120rs^2t^3 + 7rst^7 - 48rst^6 + 100rst^5 - 69rst^4 + 7rt^7 - 20rt^6 + 15rt^5 + 15s^3t^5 - 48s^3t^4 + 42s^3t^3 - 20s^2t^6 + 60s^2t^5 - 48s^2t^4 + 7st^7 - 20st^6 + 15st^5)$$

$$B_{1212}^{[0]} = \frac{h}{1260r^3s^3t^3}(630r^3s^3t^3 - 168r^3s^3t^2 - 84r^3s^3t + 42r^3s^3 - 168r^3s^2t^3 + 120r^3s^2t - 48r^3s^2 - 84r^3st^3 + 120r^3st^2 - 69r^3st + 15r^3s + 42r^3t^3 - 48r^3t^2 + 15r^3t - 168r^2s^3t^3 + 120r^2s^3t - 48r^2s^3 + 144r^2s^2t^2 - 180r^2s^2t + 60r^2s^2 + 120r^2st^3 - 180r^2st^2 + 100r^2st - 20r^2s - 48r^2t^3 + 60r^2t^2 - 20r^2t84rs^3t^3 + 120rs^3t^2 - 69rs^3t + 15rs^3 + 120rs^2t^3 - 180rs^2t^2 + 100rs^2t - 20rs^2 - 69rst^3 + 100rst^2 - 48rst + 7rs + 15rt^3 - 20rt^2 + 7rt + 42s^3t^3 - 48s^3t^2 + 15s^3t - 48s^2t^3 + 60s^2t^2 - 20s^2t + 15st^3 - 20st^2 + 7st)$$

$$B_{11}^{[1]} = \frac{h^3r^3}{27720(r-s)^3(r-t)^3(r-1)^3}(84r^9 - 315r^8s - 315r^8t - 315r^8 + 390r^7s^2 + 1210r^7st + 1210r^7s + 390r^7t^2 + 1210r^7t + 390r^7 - 154r^6s^3 - 1536r^6s^2t - 1536r^6s^2 - 1536r^6st^2 - 4793r^6st - 1536r^6s - 154r^6t^3 - 1536r^6t^2 - 1536r^6t - 154r^6 + 616r^5s^3t + 616r^5s^3 + 2002r^5s^2t^2 + 6290r^5s^2t + 2002r^5s^2 + 616r^5st^3 + 6290r^5st^2 + 6290r^5st + 616r^5s + 616r^5t^3 + 2002r^5t^2 + 616r^5t - 814r^4s^3t^2 - 2574r^4s^3t - 814r^4s^3 - 814r^4s^2t^3 - 8569r^4s^2t^2 - 8569r^4s^2t - 814r^4s^2 - 2574r^4st^3 - 8569r^4st^2 - 2574r^4st - 814r^4t^3 - 814r^4t^2 + 330r^3s^3t^3 + 3575r^3s^3t^2 + 3575r^3s^3t + 330r^3s^3 + 3575r^3s^2t^3 + 12320r^3s^2t^2 + 3575r^3s^2t + 3575r^3st^3 + 3575r^3st^2 + 330r^3t^3 - 1485r^2s^3t^3 - 5280r^2s^3t^2 - 1485r^2s^3t - 5280r^2s^2t^3 - 5280r^2s^2t^2 - 1485r^2st^3 + 2244rs^3t^3 + 2244rs^3t^2 + 2244rs^2t^3 - 924s^3t^3)$$

$$B_{21}^{[1]} = \frac{h^3s^7}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(54r^4s^4 - 198r^4s^3t - 198r^4s^3 + 198r^4s^2t^2 + 792r^4s^2t + 198r^4s^2 - 891r^4st^2 - 891r^4st + 1188r^4t^2\ 70r^3s^5 + 208r^3s^4t + 208r^3s^4 - 88r^3s^3t^2 - 660r^3s^3t - 88r^3s^3 - 154r^3s^2t^3 + 275r^3s^2t^2 + 275r^3s^2t - 154r^3s^2 + 693r^3st^3 + 66r^3st^2 + 693r^3st - 924r^3t^3 - 924r^3t^2 + 21r^2s^6 - 21r^2s^5t - 21r^2s^5 - 116r^2s^4t^2 - 70r^2s^4t - 116r^2s^4 + 176r^2s^3t^3 + 506r^2s^3t^2 + 506r^2s^3t + 176r^2s^3 - 649r^2s^2t^3 - 781r^2s^2t^2 - 649r^2s^2t + 462r^2st^3 + 462r^2st^2 + 660r^2t^3 - 14rs^6t - 14rs^6 + 48rs^5t^2 + 64rs^5t + 48rs^5 - 44rs^4t^3 - 106rs^4t^2 - 106rs^4t - 44rs^4 + 66rs^3t^3 - 88rs^3t^2 - 88s^3t^2 + 99s^2t^3)$$

$$B_{21}^{[1]} = \frac{h^3 s^7}{27720 r^3 (r-s)^3 (r-t)^3 (r-1)^3}(54r^4s^4 - 198r^4s^3t - 198r^4s^3 + 198r^4s^2t^2 + 792r^4s^2t + 198r^4s^2 - 891r^4st^2 -$$
$$891r^4st + 1188r^4t^2 - 70r^3s^5 + 208r^3s^4t + 208r^3s^4 - 88r^3s^3t^2 - 660r^3s^3t - 88r^3s^3 - 154r^3s^2t^3 + 275r^3s^2t^2 +$$
$$275r^3s^2t - 154r^3s^2 + 693r^3st^3 + 66r^3st^2 + 693r^3st - 924r^3t^3 - 924r^3t^2 + 21r^2s^6 - 21r^2s^5t - 21r^2s^5 - 116r^2s^4$$
$$t^2 - 70r^2s^4t - 116r^2s^4 + 176r^2s^3t^3 + 506r^2s^3t^2 + 506r^2s^3t + 176r^2s^3 - 649r^2s^2t^3 - 781r^2s^2t^2 - 649r^2s^2t +$$
$$462r^2st^3 + 462r^2st^2 + 660r^2t^3 - 14rs^6t - 14rs^6 + 48rs^5t^2 + 64rs^5t + 48rs^5 - 44rs^4t^3 - 106rs^4t^2 - 106rs^4$$
$$t - 44rs^4 + 66rs^3t^3 - 88rs^3t^2 + 66rs^3t + 275rs^2t^3 + 275rs^2t^2 - 594rst^3 + 7s^6t - 24s^5t^2 - 24s^5t + 22s^4t^3 +$$
$$88s^4t^2 + 22s^4t - 88s^3t^3 - 88s^3t^2 + 99s^2t^3)$$

$$B_{31}^{[1]} = \frac{h^3 t^7}{27720 r^3 (r-s)^3 (r-t)^3 (r-1)^3}(198r^4s^2t^2 - 891r^4s^2t + 1188r^4s^2 198r^4st^3 + 792r^4st^2 - 891r^4st + 54r^4t^4 -$$
$$198r^4t^3 + 198r^4t^2 - 154r^3s^3t^2 + 693r^3s^3t - 924r^3s^3 - 88r^3s^2t^3 + 275r^3s^2t^2 + 66r^3s^2t - 924r^3s^2 + 208r^3s$$
$$t^4 - 660r^3st^3 + 275r^3st^2 + 693r^3st - 70r^3t^5 + 208r^3t^4 - 88r^3t^3 - 154r^3t^2 + 176r^2s^3t^3 - 649r^2s^3t^2 +$$
$$462r^2s^3t + 660r^2s^3 - 116r^2s^2t^4 + 506r^2s^2t^3 - 781r^2s^2t^2 + 462r^2s^2t - 21r^2st^5 - 70r^2st^4 + 506r^2st^3 -$$
$$649r^2st^2 + 21r^2t^6 - 21r^2t^5 - 116r^2t^4 + 176r^2t^3 - 44rs^3t^4 + 66rs^3t^3 + 275rs^3t^2 - 594rs^3t + 48rs^2t^5 -$$
$$106rs^2t^4 - 88rs^2t^3 + 275rs^2t^2 - 14rst^6 + 64rst^5 - 106rst^4 + 66rst^3 - 14rt^6 + 48rt^5 - 44rt^4 + 22s^3t^4 -$$
$$88s^3t^3 + 99s^3t^2 - 24s^2t^5 + 88s^2t^4 - 88s^2t^3 + 7st^6 - 24st^5 + 22st^4)$$

$$B_{41}^{[1]} = \frac{-h^3}{27720 r^3 (r-s)^3 (r-t)^3 (r-1)^3}(-1188r^4s^2t^2 + 891r^4s^2t - 198r^4s^2 + 891r^4st^2 - 792r^4st + 198r^4s - 198r^4t^2 +$$
$$198r^4t - 54r^4 + 924r^3s^3t^2 - 693r^3s^3t + 154r^3s^3 + 924r^3s^2t^3 - 66r^3s^2t^2 - 275r^3s^2t + 88r^3s^2 - 693r^3st^3 -$$
$$275r^3st^2 + 660r^3st - 208r^3s + 154r^3t^3 + 88r^3t^2 - 208r^3t + 70r^3 660r^2s^3t^3 462r^2s^3t^2 + 649r^2s^3t - 176r^2s^3$$
$$462r^2s^2t^3 + 781r^2s^2t^2 - 506r^2s^2t + 116r^2s^2 + 649r^2st^3 - 506r^2st^2 + 70r^2st + 21r^2s - 176r^2t^3 + 116r^2t^2 +$$
$$21r^2t - 21r^2 + 594rs^3t^3 - 275rs^3t^2 - 66rs^3t + 44rs^3 - 275rs^2t^3 + 88rs^2t^2 + 106rs^2t - 48rs^2 - 66rst^3 + 106rst^2 -$$
$$64rst + 14rs + 44rt^3 - 48rt^2 + 14rt - 99s^3t^3 + 88s^3t^2 - 22s^3t + 88s^2t^3 - 88s^2t^2 + 24s^2t - 22st^3 + 24st^2 - 7st)$$

$$B_{51}^{[1]} = \frac{h^2 r^2}{27720 (r-s)^3 (r-t)^3 (r-1)^3}(756r^9 - 2646r^8s - 2646r^8t - 2646r^8 + 3069r^7s^2 + 9420r^7st + 9420r^7s + 3069r^7$$
$$t^2 + 9420r^7t + 3069r^7 - 1155r^6s^3 - 11121r^6s^2t - 11121r^6s^2 - 11121r^6st^2 - 34278r^6st - 11121r^6s - 1155r^6t^3 -$$
$$11121r^6t^2 - 11121r^6t - 1155r^6 + 4235r^5s^3t + 4235r^5s^3 + 13376r^5s^2t^2 + 41437r^5s^2t + 13376r^5s^2 + 4235r^5st^3 +$$
$$41437r^5st^2 + 41437r^5st + 4235r^5s + 4235r^5t^3 + 13376r^5t^2 + 4235r^5t 5148r^4s^3t^2 - 16027r^4s^3t - 5148r^4s^3$$
$$5148r^4s^2t^3 - 51436r^4s^2t^2 - 51436r^4s^2t - 5148r^4s^2 - 16027r^4st^3 - 51436r^4st^2 - 16027r^4st - 5148r^4t^3 -$$
$$5148r^4t^2 + 1980r^3s^3t^3 + 20196r^3s^3t^2 + 20196r^3s^3t + 1980r^3s^3 + 20196r^3s^2t^3 + 66330r^3s^2t^2 + 20196r^3s^2t +$$
$$20196r^3st^3 + 20196r^3st^2 + 1980r^3t^3 - 7920r^2s^3t^3 - 26598r^2s^3t^2 - 7920r^2s^3t - 26598r^2s^2t^3 - 26598r^2s^2$$
$$t^2 - 7920r^2st^3 + 10626rs^3t^3 + 10626rs^3t^2 + 10626rs^2t^3 - 4158s^3t^3)$$

$$B_{61}^{[1]} = \frac{h^2 s^6}{27720 r^3 (r-s)^3 (r-t)^3 (r-1)^3}(297r^4s^4 - 990r^4s^3t - 990r^4s^3 + 891r^4s^2t^2 + 3564r^4s^2t + 891r^4s^2 - 3564r^4$$
$$st^2 - 3564r^4st + 4158r^4t^2 - 399r^3s^5 + 1067r^3s^4t + 1067r^3s^4 - 363r^3s^3t^2 - 2992r^3s^3t - 363r^3s^3 - 693r^3s^2$$
$$t^3 + 891r^3s^2t^2 + 891r^3s^2t 693r^3s^2 + 2772r^3st^3 + 726r^3st^2 + 2772r^3st 3234r^3t^3 3234r^3t^2 + 126r^2s^6 - 105r^2s^5t$$
$$105r^2s^5 - 616r^2s^4t^2 - 407r^2s^4t - 616r^2s^4 + 825r^2s^3t^3 + 2387r^2s^3t^2 + 2387r^2s^3t + 825r^2s^3 - 2673r^2s^2t^3 -$$
$$3168r^2s^2t^2 - 2673r^2s^2t + 1518r^2st^3 + 1518r^2st^2 + 2310r^2t^3 - 84rs^6t - 84rs^6 + 264rs^5t^2 + 345rs^5t + 264rs^5 -$$
$$220rs^4t^3 - 506rs^4t^2 - 506rs^4t - 220rs^4 + 275rs^3t^3 - 484rs^3t^2 + 275rs^3t + 1188rs^2t^3 + 1188rs^2t^2 - 2178rst^3 +$$
$$42s^6t - 132s^5t^2 - 132s^5t + 110s^4t^3 + 440s^4t^2 + 110s^4t - 396s^3t^3 - 396s^3t^2 + 396s^2t^3)$$

$$B_{71}^{[1]}=\frac{h^2t^6}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(891r^4s^2t^2-3564r^4s^2t+4158r^4s^2-990r^4st^3+3564r^4st^2-3564r^4st+297r^4$$
$$t^4-990r^4t^3+891r^4t^2-693r^3s^3t^2+2772r^3s^3t-3234r^3s^3-363r^3s^2t^3+891r^3s^2t^2+726r^3s^2t-3234r^3s^2+$$
$$1067r^3st^4-2992r^3st^3+891r^3st^2+2772r^3st-399r^3t^5+1067r^3t^4-363r^3t^3-693r^3t^2+825r^2s^3t^3-2673r^2s^3$$
$$t^2+1518r^2s^3t+2310r^2s^3-616r^2s^2t^4+2387r^2s^2t^3-3168r^2s^2t^2+1518r^2s^2t-105r^2st^5-407r^2st^4+2387r^2$$
$$st^3-2673r^2st^2+126r^2t^6-105r^2t^5-616r^2t^4+825r^2t^3-220rs^3t^4+275rs^3t^3+1188rs^3t^2-2178rs^3t+264rs^2$$
$$t^5-506rs^2t^4-484rs^2t^3+1188rs^2t^2-84rst^6+345rst^5-506rst^4+275rst^3-84rt^6+264rt^5-220rt^4+110s^3$$
$$t^4-396s^3t^3+396s^3t^2-132s^2t^5+440s^2t^4-396s^2t^3+42st^6-132st^5+110st^4)$$

$$B_{81}^{[1]}=\frac{-h^2}{27720r^3(r-s)^3(r-t)^3(r-1)^3}(-4158r^4s^2t^2+3564r^4s^2t-891r^4s^2+3564r^4st^2-3564r^4st+990r^4s-891r^4$$
$$t^2+990r^4t-297r^4+3234r^3s^3t^2-2772r^3s^3t+693r^3s^3+3234r^3s^2t^3-726r^3s^2t^2-891r^3s^2t+363r^3s^2-$$
$$2772r^3st^3-891r^3st^2+2992r^3st-1067r^3s+693r^3t^3+363r^3t^2-1067r^3t+399r^3-2310r^2s^3t^3-1518r^2s^3$$
$$t^2+2673r^2s^3t-825r^2s^3-1518r^2s^2t^3+3168r^2s^2t^2\ 2387r^2s^2t+616r^2s^2+2673r^2st^3-2387r^2st^2+$$
$$407r^2st+105r^2s-825r^2t^3+616r^2t^2+105r^2t-126r^2+2178rs^3t^3-1188rs^3t^2-275rs^3t+220rs^3-$$
$$1188rs^2t^3+484rs^2t^2+506rs^2t-264rs^2-275rst^3+506rst^2-345rst+84rs+220rt^3-264rt^2+84rt-$$
$$396s^3t^3+396s^3t^2-110s^3t+396s^2t^3-440s^2t^2+132s^2t-110st^3+132st^2-42st)$$

$$B_{91}^{[1]}=\frac{hr}{1260(r-s)^3(r-t)^3(r-1)^3}(252r^9-819r^8s-819r^8t-819r^8+885r^7s^2+2686r^7st+2686r^7s+885r^7$$
$$t^2+2686r^7t+885r^7-315r^6s^3-2930r^6s^2t-2930r^6s^2-2930r^6st^2-8913r^6st-2930r^6s-315r^6t^3$$
$$2930r^6t^2\,2930r^6t-315r^6+1050r^5s^3t+1050r^5s^3+3228r^5s^2t^2+9847r^5s^2t+3228r^5s^2+1050r^5st^3+$$
$$9847r^5st^2+9847r^5st+1050r^5s+1050r^5t^3+3228r^5t^2+1050r^5t-1164r^4s^3t^2-3561r^4s^3t-1164r^4-$$
$$s^3-1164r^4s^2t^3\,11034r^4s^2t^2-11034r^4s^2t-1164r^4s^2-3561r^4st^3-11034r^4st^2-12618r^3s^2t^2+$$
$$4026r^3s^2t+4026r^3st^3+4026r^3st^2+420r^3t^3-1470r^2s^3t^3-4662r^2s^3t^2-1470r^2s^3t-4662r^2s^2t^3-$$
$$4662r^2s^2t^2-1470r^2st^3+1722rs^3t^3+1722rs^3t^2+1722rs^2t^3-630s^3t^3)$$

$$B_{101}^{[1]}=\frac{hs^5}{1260r^3(r-s)^3(r-t)^3(r-1)^3}(45r^4s^4-135r^4s^3t-135r^4s^3+108r^4s^2t^2+432r^4s^2t+108r^4s^2-378r^4st^2-$$
$$378r^4st+378r^4t^2-63r^3s^5+150r^3s^4t+150r^3s^4-39r^3s^3t^2-366r^3s^3t-39r^3s^3-84r^3s^2t^3+66r^3s^2t^2+$$
$$66r^3s^2t-84r^3s^2+294r^3st^3+126r^3st^2+294r^3st-294r^3t^3-294r^3t^2+21r^2s^6-14r^2s^5t-14r^2s^5-90r^2s^4t^2$$
$$65r^2s^4t-90r^2s^4+105r^2s^3t^3+306r^2s^3t^2+306r^2s^3t+105r^2s^3-294r^2s^2t^3-342r^2s^2t^2-294r^2s^2t+$$
$$126r^2st^3+126r^2st^2+210r^2t^3-14rs^6t-14rs^6+40rs^5t^2+51rs^5t+40rs^5-30rs^4t^3-65rs^4t^2-65rs^4$$
$$t-30rs^4+30rs^3t^3-72rs^3t^2+30rs^3t+138rs^2t^3+138rs^2t^2-210rst^3+7s^6t-20s^5t^2-20s^5t+15s^4t^3+$$
$$60s^4t^2+15s^4t-48s^3t^3-48s^3t^2+42s^2t^3)$$

$$B_{111}^{[1]}=\frac{ht^5}{1260r^3(r-s)^3(r-t)^3(r-1)^3}(108r^4s^2t^2-378r^4s^2t+378r^4s^2-135r^4st^3+432r^4st^2-378r^4st+$$
$$45r^4t^4-135r^4t^3+108r^4t^2-84r^3s^3t^2+294r^3s^3t-294r^3s^3-39r^3s^2t^3+66r^3s^2t^2+126r^3s^2t-$$
$$294r^3s^2+150r^3st^4\ 366r^3st^3+66r^3st^2+294r^3st\ 63r^3t^5+150r^3t^4-39r^3t^3-84r^3t^2+105r^2s^3t^3$$
$$294r^2s^3t^2+126r^2s^3t+210r^2s^3-90r^2s^2t^4+306r^2s^2t^3-342r^2s^2t^2+126r^2s^2t-14r^2st^5-65r^2s$$
$$t^4+306r^2st^3-294r^2st^2+21r^2t^6-14r^2t^5-90r^2t^4+105r^2t^3-30rs^3t^4+30rs^3t^3+138rs^3t^2-$$
$$210rs^3t+40rs^2t^5-65rs^2t^4-72rs^2t^3+138rs^2t^2-14rst^6+51rst^5-65rst^4+30rst^3-14rt^6+$$
$$40rt^5-30rt^4+15s^3t^4-48s^3t^3+42s^3t^2-20s^2t^5+60s^2t^4-48s^2t^3+7st^6-20st^5+15st^4)$$

$$B_{121}^{[1]}=\frac{-h}{1260r^3(r-s)^3(r-t)^3(r-1)^3}(-378r^4s^2t^2+378r^4s^2t-108r^4s^2+378r^4st^2-432r^4st+135r^4s-$$
$$108r^4t^2+135r^4t-45r^4+294r^3s^3t^2-294r^3s^3t+84r^3s^3+294r^3s^2t^3-126r^3s^2t^2-66r^3s^2t+$$
$$39r^3s^2-294r^3st^3-66r^3st^2+366r^3st-150r^3s+84r^3t^3+39r^3t^2-150r^3t+63r^3-210r^2s^3t^3-$$
$$126r^2s^3t^2+294r^2s^3t-105r^2s^3-126r^2s^2t^3+342r^2s^2t^2-306r^2s^2t+90r^2s^2+294r^2st^3-306r^2st^2+$$
$$65r^2st+14r^2s-105r^2t^3+90r^2t^2+14r^2t-21r^2+210rs^3t^3-138rs^3t^2-30rs^3t+30rs^3-138rs^2t^3+$$
$$72rs^2t^2+65rs^2t-40rs^2-30rst^3+65rst^2-51rst+14rs+30rt^3-40rt^2+14rt-42s^3t^3+48s^3t^2-$$
$$15s^3t+48s^2t^3-60s^2t^2+20s^2t-15st^3+20st^2-7st)$$

$$B_{12}^{[1]}=\frac{h^3r^7}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(-21r^6s^2+14r^6st+14r^6s-7r^6t+70r^5s^3+21r^5s^2t+21r^5s^2-48r^5st^2-$$
$$64r^5st-48r^5s+24r^5t^2+24r^5t-54r^4s^4-208r^4s^3t-208r^4s^3+116r^4s^2t^2+70r^4s^2t+116r^4s^2+$$
$$44r^4st^3+106r^4st^2+106r^4st+44r^4s-22r^4t^3-88r^4t^2-22r^4t+198r^3s^4t+198r^3s^4+88r^3s^3t^2+$$
$$660r^3s^3t+88r^3s^3-176r^3s^2t^3-506r^3s^2t^2-506r^3s^2t-176r^3s^2-66r^3st^3+88r^3st^2-66r^3st+$$
$$88r^3t^3+88r^3t^2-198r^2s^4t^2-792r^2s^4t-198r^2s^4+154r^2s^3t^3-275r^2s^3t^2-275r^2s^3t+154r^2s^3+$$
$$649r^2s^2t^3+781r^2s^2t^2+649r^2s^2t-275r^2st^3-275r^2st^2-99r^2t^3+891rs^4t^2+891rs^4t-693rs^3t^3-$$
$$66rs^3t^2-693rs^3t-462rs^2t^3-462rs^2t^2+594rst^3-1188s^4t^2+924s^3t^3+924s^3t^2-660s^2t^3)$$

$$B_{22}^{[1]}=\frac{h^3s^3}{27720(r-s)^3(s-t)^3(s-1)^3}(154r^3s^6-616r^3s^5t-616r^3s^5+814r^3s^4t^2+2574r^3s^4t+814r^3s^4-$$
$$330r^3s^3t^3-3575r^3s^3t^2-3575r^3s^3t-330r^3s^3+1485r^3s^2t^3+5280r^3s^2t^2+1485r^3s^2t-2244r^3st^3-$$
$$2244r^3st^2+924r^3t^3-390r^2s^7+1536r^2s^6t+1536r^2s^6-2002r^2s^5t^2-6290r^2s^5t-2002r^2s^5+$$
$$814r^2s^4t^3+8569r^2s^4t^2+8569r^2s^4t+814r^2s^4-3575r^2s^3t^3-12320r^2s^3t^2-3575r^2s^3t+$$
$$5280r^2s^2t^3+5280r^2s^2t^2-2244r^2st^3+315rs^8-1210rs^7t-1210rs^7+1536rs^6t^2+4793rs^6t+$$
$$1536rs^6-616rs^5t^3-6290rs^5t^2-6290rs^5t-616rs^5+2574rs^4t^3+8569rs^4t^2+2574rs^4t-$$
$$3575rs^3t^3-3575rs^3t^2+1485rs^2t^3-84s^9+315s^8t+315s^8-390s^7t^2-1210s^7t-390s^7+$$
$$154s^6t^3+1536s^6t^2+1536s^6t+154s^6-616s^5t^3-2002s^5t^2-616s^5t+814s^4t^3+814s^4t^2-330s^3t^3)$$

$$B_{32}^{[1]}=\frac{h^3t^7}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(154r^3s^3t^2-693r^3s^3t+924r^3s^3176r^3s^2t^3+649r^3s^2t^2-462r^3s^2t-$$
$$660r^3s^2+44r^3st^4-66r^3st^3-275r^3st^2+594r^3st-22r^3t^4+88r^3t^3-99r^3t^2-198r^2s^4t^2+891r^2s^4t-$$
$$1188r^2s^4+88r^2s^3t^3-275r^2s^3t^2-66r^2s^3t+924r^2s^3+116r^2s^2t^4\ 506r^2s^2t^3+781r^2s^2t^2\ 462r^2s^2t$$
$$48r^2st^5+106r^2st^4+88r^2st^3-275r^2st^2+24r^2t^5-88r^2t^4+88r^2t^3+198rs^4t^3-792rs^4t^2+891rs^4t-$$
$$208rs^3t^4+660rs^3t^3-275rs^3t^2-693rs^3t+21rs^2t^5+70rs^2t^4-506rs^2t^3+649rs^2t^2+14rst^6-64rst^5+$$
$$106rst^4-66rst^3-7rt^6+24rt^5-22rt^4-54s^4t^4+198s^4t^3-198s^4t^2+70s^3t^5-208s^3t^4+88s^3t^3+$$
$$154s^3t^2-21s^2t^6+21s^2t^5+116s^2t^4-176s^2t^3+14st^6-48st^5+44st^4)$$

$$B_{42}^{[1]}=\frac{h^3}{27720s^3(r-s)^3(s-t)^3(s-1)^3}(924r^3s^3t^2-693r^3s^3t+154r^3s^3-660r^3s^2t^3-462r^3s^2t^2+649r^3s^2t-$$
$$176r^3s^2+594r^3st^3-275r^3st^2-66r^3st+44r^3s-99r^3t^3+88r^3t^2-22r^3t-1188r^2s^4t^2+891r^2s^4t-$$
$$198r^2s^4+924r^2s^3t^3-66r^2s^3t^2-275r^2s^3t+88r^2s^3\ 462r^2s^2t^3+781r^2s^2t^2\ 506r^2s^2t+116r^2s^2$$
$$275r^2st^3+88r^2st^2+106r^2st-48r^2s+88r^2t^3-88r^2t^2+24r^2t+891rs^4t^2-792rs^4t+198rs^4-$$
$$693rs^3t^3-275rs^3t^2+660rs^3t-208rs^3+649rs^2t^3-506rs^2t^2+70rs^2t+21rs^2-66rst^3+106rst^2-$$
$$64rst+14rs-22rt^3+24rt^2-7rt-198s^4t^2+198s^4t-54s^4+154s^3t^3+88s^3t^2-208s^3t+70s^3-$$
$$176s^2t^3+116s^2t^2+21s^2t-21s^2+44st^3-48st^2+14st)$$

$$B_{52}^{[1]} = \frac{h^2 r^6}{27720 s^3 (r-s)^3 (s-t)^3 (s-1)^3} (-126r^6s^2 + 84r^6st + 84r^6s - 42r^6t + 399r^5s^3 + 105r^5s^2t + 105r^5s^2 - 264r^5st^2 - 345r^5st - 264r^5s + 132r^5t^2 + 132r^5t - 297r^4s^4 - 1067r^4s^3t - 1067r^4s^3 + 616r^4s^2t^2 + 407r^4s^2t + 616r^4s^2 + 220r^4st^3 + 506r^4st^2 + 506r^4st + 220r^4s - 110r^4t^3\ 440r^4t^2 - 110r^4t + 990r^3s^4t + 990r^3s^4 + 363r^3s^3t^2 + 2992r^3s^3t + 363r^3s^3 - 825r^3s^2t^3 - 2387r^3s^2t^2 - 2387r^3s^2t - 825r^3s^2 - 275r^3st^3 + 484r^3st^2 - 275r^3st + 396r^3t^3 + 396r^3t^2 - 891r^2s^4t^2 - 3564r^2s^4t - 891r^2s^4 + 693r^2s^3t^3 - 891r^2s^3t^2 - 891r^2s^3t + 693r^2s^3 + 2673r^2s^2t^3 + 3168r^2s^2t^2 + 2673r^2s^2t - 1188r^2st^3 - 1188r^2st^2 - 396r^2t^3 + 3564rs^4t^2 + 3564rs^4t - 2772rs^3t^3 - 726rs^3t^2 - 2772rs^3t - 1518rs^2t^3 - 1518rs^2t^2 + 2178rst^3 - 4158s^4t^2 + 3234s^3t^3 + 3234s^3t^2 - 2310s^2t^3)$$

$$B_{62}^{[1]} = \frac{h^2 s^2}{27720 (r-s)^3 (s-t)^3 (s-1)^3} (1155r^3s^6 - 4235r^3s^5t - 4235r^3s^5 + 5148r^3s^4t^2 + 16027r^3s^4t + 5148r^3s^4 - 1980r^3s^3t^3 - 20196r^3s^3t^2 - 20196r^3s^3t - 1980r^3s^3 + 7920r^3s^2t^3 + 26598r^3s^2t^2 + 7920r^3s^2t - 10626r^3st^3 - 10626r^3st^2 + 4158r^3t^3 - 3069r^2s^7 + 11121r^2s^6t + 11121r^2s^6 - 13376r^2s^5t^2 - 41437r^2s^5t - 13376r^2s^5 + 5148r^2s^4t^3 + 51436r^2s^4t^2 + 51436r^2s^4t + 5148r^2s^4 - 20196r^2s^3t^3 - 66330r^2s^3t^2 - 20196r^2s^3t + 26598r^2s^2t^3 + 26598r^2s^2t^2 - 10626r^2st^3 + 2646rs^8 - 9420rs^7t - 9420rs^7 + 11121rs^6t^2 + 34278rs^6t + 11121rs^6 - 4235rs^5t^3 - 41437rs^5t^2 - 41437rs^5t - 4235rs^5 + 16027rs^4t^3 + 51436rs^4t^2 + 16027rs^4t - 20196rs^3t^3 - 20196rs^3t^2 + 7920rs^2t^3 - 756s^9 + 2646s^8t + 2646s^8 - 3069s^7t^2 - 9420s^7t - 3069s^7 + 1155s^6t^3 + 11121s^6t^2 + 11121s^6t + 1155s^6 - 4235s^5t^3 - 13376s^5t^2 - 4235s^5t + 5148s^4t^3 + 5148s^4t^2 - 1980s^3t^3)$$

$$B_{72}^{[1]} = \frac{h^2 t^6}{27720 s^3 (r-s)^3 (s-t)^3 (s-1)^3} (693r^3s^3t^2 - 2772r^3s^3t + 3234r^3s^3 - 825r^3s^2t^3 + 2673r^3s^2t^2 - 1518r^3s^2t - 2310r^3s^2 + 220r^3st^4 - 275r^3st^3 - 1188r^3st^2 + 2178r^3st - 110r^3t^4 + 396r^3t^3 - 396r^3t^2 - 891r^2s^4t^2 + 3564r^2s^4t - 4158r^2s^4 + 363r^2s^3t^3 - 891r^2s^3t^2 - 726r^2s^3t + 3234r^2s^3 + 616r^2s^2t^4 - 2387r^2s^2t^3 + 3168r^2s^2t^2 - 1518r^2s^2t - 264r^2st^5 + 506r^2st^4 + 484r^2st^3 - 1188r^2st^2 + 132r^2t^5 - 440r^2t^4 + 396r^2t^3 + 990rs^4t^3 - 3564rs^4t^2 + 3564rs^4t - 1067rs^3t^4 + 2992rs^3t^3 - 891rs^3t^2 - 2772rs^3t + 105rs^2t^5 + 407rs^2t^4 - 2387rs^2t^3 + 2673rs^2t^2 + 84rst^6 - 345rst^5 + 506rst^4 - 275rst^3 - 42rt^6 + 132rt^5 - 110rt^4 - 297s^4t^4 + 990s^4t^3 - 891s^4t^2 + 399s^3t^5 - 1067s^3t^4 + 363s^3t^3 + 693s^3t^2 - 126s^2t^6 + 105s^2t^5 + 616s^2t^4 - 825s^2t^3 + 84st^6 - 264st^5 + 220st^4)$$

$$B_{82}^{[1]} = \frac{h^2}{27720 s^3 (r-s)^3 (s-t)^3 (s-1)^3} (3234r^3s^3t^2 - 2772r^3s^3t + 693r^3s^3 - 2310r^3s^2t^3 - 1518r^3s^2t^2 + 2673r^3s^2t - 825r^3s^2 + 2178r^3st^3 - 1188r^3st^2 - 275r^3st + 220r^3s - 396r^3t^3 + 396r^3t^2 - 110r^3t - 4158r^2s^4t^2 + 3564r^2s^4t - 891r^2s^4 + 3234r^2s^3t^3 - 726r^2s^3t^2 - 891r^2s^3t + 363r^2s^3\ 518r^2s^2t^3 + 3168r^2s^2t^2 - 2387r^2s^2t + 616r^2s^2 - 1188r^2st^3 + 484r^2st^2 + 506r^2st - 264r^2s + 396r^2t^3 - 440r^2t^2 + 132r^2t + 3564rs^4t^2 - 3564rs^4t + 990rs^4 - 2772rs^3t^3 - 891rs^3t^2 + 2992rs^3t - 1067rs^3 + 2673rs^2t^3 - 2387rs^2t^2 + 407rs^2t + 105rs^2 - 275rst^3 + 506rst^2 - 345rst + 84rs - 110rt^3 + 132rt^2 - 42rt - 891s^4t^2 + 990s^4t - 297s^4 + 693s^3t^3 + 363s^3t^2 - 1067s^3t + 399s^3 - 825s^2t^3 + 616s^2t^2 + 105s^2t - 126s^2 + 220st^3 - 264st^2 + 84st)$$

$$B_{92}^{[1]} = \frac{h r^5}{1260 s^3 (r-s)^3 (s-t)^3 (s-1)^3} (-21r^6s^2 + 14r^6st + 14r^6s - 7r^6t + 63r^5s^3 + 14r^5s^2t + 14r^5s^2 - 40r^5st^2 - 51r^5st - 40r^5s + 20r^5t^2 + 20r^5t - 45r^4s^4 - 150r^4s^3t - 150r^4s^3 + 90r^4s^2t^2 + 65r^4s^2t + 90r^4s^2 + 30r^4st^3 + 65r^4st^2 + 65r^4st + 30r^4s - 15r^4t^3 - 60r^4t^2 - 15r^4t + 135r^3s^4t + 135r^3s^4 + 39r^3s^3t^2 +$$

$$366r^3s^3t+39r^3s^3-105r^3s^2t^3-306r^3s^2t^2-306r^3s^2t-105r^3s^2-30r^3st^3+72r^3st^2-30r^3st+48r^3 t^3+48r^3t^2-108r^2s^4t^2-432r^2s^4t-108r^2s^4+84r^2s^3t^3-66r^2s^3t^2-66r^2s^3t+84r^2s^3+294r^2s^2t^3+342r^2s^2t^2+294r^2s^2t-138r^2st^3-138r^2st^2-42r^2t^3+378rs^4t^2+378rs^4t-294rs^3t^3-126rs^3t^2-294rs^3t-126rs^2t^3-126rs^2t^2+210rst^3-378s^4t^2+294s^3t^3+294s^3t^2-210s^2t^3)$$

$$B_{102}^{[1]}=\frac{hs}{1260(r-s)^3(s-t)^3(s-1)^3}(315r^3s^6-1050r^3s^5t-1050r^3s^5+1164r^3s^4t^2+3561r^3s^4t+1164r^3s^4-420r^3s^3t^3-4026r^3s^3t^2-4026r^3s^3t-420r^3s^3+1470r^3s^2t^3+4662r^3s^2t^2+1470r^3s^2t-1722r^3st^3-1722r^3st^2+630r^3t^3-885r^2s^7+2930r^2s^6t+2930r^2s^6-3228r^2s^5t^2-9847r^2s^5t-3228r^2s^5+1164r^2s^4t^3+11034r^2s^4t^2+11034r^2s^4t+1164r^2s^4-4026r^2s^3t^3-12618r^2s^3t^2-4026r^2s^3t+4662r^2s^2t^3+4662r^2s^2t^2-1722r^2st^3+819rs^8-2686rs^7t-2686rs^7+2930rs^6t^2+8913rs^6t+2930rs^6-1050rs^5t^3-9847rs^5t^2-9847rs^5t-1050rs^5+3561rs^4t^3+11034rs^4t^2+3561rs^4t-4026rs^3t^3-4026rs^3t^2+1470rs^2t^3-252s^9+819s^8t+819s^8-885s^7t^2-2686s^7t-885s^7+315s^6t^3+2930s^6t^2+2930s^6t+315s^6-1050s^5t^3-3228s^5t^2-1050s^5t+1164s^4t^3+1164s^4t^2-420s^3t^3)$$

$$B_{112}^{[1]}=\frac{ht^5}{1260s^3(r-s)^3(s-t)^3(s-1)^3}(84r^3s^3t^2-294r^3s^3t+294r^3s^3-105r^3s^2t^3+294r^3s^2t^2-126r^3s^2t-210r^3s^2+30r^3st^4-30r^3st^3-138r^3st^2+210r^3st-15r^3t^4+48r^3t^3-42r^3t^2-108r^2s^4t^2+378r^2s^4t-378r^2s^4+39r^2s^3t^3-66r^2s^3t^2-126r^2s^3t+294r^2s^3+90r^2s^2t^4-306r^2s^2t^3+342r^2s^2t^2-126r^2s^2t-40r^2st^5+65r^2st^4+72r^2st^3-138r^2st^2+20r^2t^5-60r^2t^4+48r^2t^3+135rs^4t^3-432rs^4t^2+378rs^4t-150rs^3t^4+366rs^3t^3-66rs^3t^2-294rs^3t+14rs^2t^5+65rs^2t^4-306rs^2t^3+294rs^2t^2+14rst^6-51rst^5+65rst^4-30rst^3-7rt^6+20rt^5-15rt^4-45s^4t^4+135s^4t^3-108s^4t^2+63s^3t^5-150s^3t^4+39s^3t^3+84s^3t^2-21s^2t^6+14s^2t^5+90s^2t^4-105s^2t^3+14st^6-40st^5+30st^4)$$

$$B_{122}^{[1]}=\frac{h}{1260s^3(r-s)^3(s-t)^3(s-1)^3}(294r^3s^3t^2-294r^3s^3t+84r^3s^3-210r^3s^2t^3-126r^3s^2t^2+294r^3s^2t-105r^3s^2+210r^3st^3-138r^3st^2-30r^3st+30r^3s-42r^3t^3+48r^3t^2-15r^3t378r^2s^4t^2+378r^2s^4t-108r^2s^4+294r^2s^3t^3-126r^2s^3t^2-66r^2s^3t+39r^2s^3-126r^2s^2t^3+342r^2s^2t^2-306r^2s^2t+90r^2s^2-138r^2st^3+72r^2st^2+65r^2st-40r^2s+48r^2t^3-60r^2t^2+20r^2t+378rs^4t^2-432rs^4t+135rs^4-294rs^3t^3-66rs^3t^2+366rs^3t-150rs^3+294rs^2t^3-306rs^2t^2+65rs^2t+14rs^2-30rst^3+65rst^2-51rst+14rs-15rt^3+20rt^2-7rt-108s^4t^2+135s^4t-45s^4+84s^3t^3+39s^3t^2-150s^3t+63s^3-105s^2t^3+90s^2t^2+14s^2t-21s^2+30st^3-40st^2+14st)$$

$$B_{13}^{[1]}=\frac{h^3r^7}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(-14r^6st+7r^6s+21r^6t^2-14r^6t+48r^5s^2t-24r^5s^2-21r^5st^2+64r^5st-24r^5s-70r^5t^3-21r^5t^2+48r^5t-44r^4s^3t+22r^4s^3-116r^4s^2t^2-106r^4s^2t+88r^4s^2+208r^4st^3-70r^4st^2-106r^4st+22r^4s+54r^4t^4+208r^4t^3-116r^4t^2-44r^4t+176r^3s^3t^2+66r^3s^3t-88r^3s^3-88r^3s^2t^3+506r^3s^2t^2-88r^3s^2t-88r^3s^2-198r^3st^4-660r^3st^3+506r^3st^2+66r^3st-198r^3t^4-88r^3t^3+176r^3t^2-154r^2s^3t^3-649r^2s^3t^2+275r^2s^3t+99r^2s^3+198r^2s^2t^4+275r^2s^2t^3-781r^2s^2t^2+275r^2s^2t+792r^2st^4+275r^2st^3-649r^2st^2+198r^2t^4-154r^2t^3+693rs^3t^3+462rs^3t^2-594rs^3t-891rs^2t^4+66rs^2t^3+462rs^2t^2-891rst^4+693rst^3-924s^3t^3+660s^3t^2+1188s^2t^4-924s^2t^3)$$

$$B_{23}^{[1]}=\frac{-h^3s^7}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(44r^3s^4t-22r^3s^4-176r^3s^3t^2-66r^3s^3t+88r^3s^3+154r^3s^2t^3+649r^3s^2t^2-275r^3s^2t-99r^3s^2-693r^3st^3-462r^3st^2+594r^3st+924r^3t^3-660r^3t^2-48r^2s^5t+24r^2s^5+116r^2s^4t^2+106r^2s^4t-88r^2s^4+88r^2s^3t^3-506r^2s^3t^2+88r^2s^3t+88r^2s^3-198r^2s^2t^4-$$

$$275r^2s^2t^3+781r^2s^2t^2-275r^2s^2t+891r^2st^4-66r^2st^3-462r^2st^2-1188r^2t^4+924r^2t^3+14rs^6t-7rs^6+21rs^5t^2-64rs^5t+24rs^5-208rs^4t^3+70rs^4t^2+106rs^4t-22rs^4+198rs^3t^4+660rs^3t^3-506rs^3t^2-66rs^3t-792rs^2t^4-275rs^2t^3+649rs^2t^2+891rst^4-693rst^3-21s^6t^2+14s^6t+70s^5t^3+21s^5t^2-48s^5t-54s^4t^4-208s^4t^3+116s^4t^2+44s^4t+198s^3t^4+88s^3t^3-176s^3t^2-198s^2t^4+154s^2t^3)$$

$$B_{33}^{[1]}=\frac{-h^3t^3}{27720(r-t)^3(s-t)^3(t-1)^3}(-330r^3s^3t^3+1485r^3s^3t^2-2244r^3s^3t+924r^3s^3+814r^3s^2t^4-3575r^3s^2t^3+5280r^3s^2t^2-2244r^3s^2t-616r^3st^5+2574r^3st^4-3575r^3st^3+1485r^3st^2+154r^3t^6-616r^3t^5+814r^3t^4-330r^3t^3+814r^2s^3t^4-3575r^2s^3t^3+5280r^2s^3t^2-2244r^2s^3t-2002r^2s^2t^5+8569r^2s^2t^4-12320r^2s^2t^3+5280r^2s^2t^2+1536r^2st^6-6290r^2st^5+8569r^2st^4-3575r^2st^3-390r^2t^7+1536r^2t^6-2002r^2t^5+814r^2t^4-616rs^3t^5+2574rs^3t^4-3575rs^3t^3+1485rs^3t^2+1536rs^2t^6-6290rs^2t^5+8569rs^2t^4-3575rs^2t^3-1210rst^7+4793rst^6-6290rst^5+2574rst^4+315rt^8-1210rt^7+1536rt^6-616rt^5+154s^3t^6-616s^3t^5+814s^3t^4-330s^3t^3-390s^2t^7+1536s^2t^6-2002s^2t^5+814s^2t^4+315st^8-1210st^7+1536st^6-616st^5-84t^9+315t^8-390t^7+154t^6)$$

$$B_{43}^{[1]}=\frac{h^3}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(660r^3s^3t^2-594r^3s^3t+99r^3s^3-924r^3s^2t^3+462r^3s^2t^2+275r^3s^2t-88r^3s^2+693r^3st^3-649r^3st^2+66r^3st+22r^3s-154r^3t^3+176r^3t^2-44r^3t-924r^2s^3t^3+462r^2s^3t^2+275r^2s^3t-88r^2s^3+1188r^2s^2t^4+66r^2s^2t^3-781r^2s^2t^2-88r^2s^2t+88r^2s^2-891r^2st^4+275r^2st^3+506r^2st^2-106r^2st-24r^2s+198r^2t^4-88r^2t^3-116r^2t^2+48r^2t+693rs^3t^3-649rs^3t^2+66rs^3t+22rs^3-891rs^2t^4+275rs^2t^3+506rs^2t^2-106rs^2t-24rs^2+792rst^4-660rst^3-70rst^2+64rst+7rs-198rt^4+208rt^3-21rt^2-14rt-154s^3t^3+176s^3t^2-44s^3t+198s^2t^4-88s^2t^3-116s^2t^2+48s^2t-198st^4+208st^3-21st^2-14st+54t^4-70t^3+21t^2)$$

$$B_{53}^{[1]}=\frac{h^2r^6}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(-84r^6st+42r^6s+126r^6t^2-84r^6t+264r^5s^2t-132r^5s^2-105r^5st^2+345r^5st-132r^5s-399r^5t^3-105r^5t^2+264r^5t-220r^4s^3t+110r^4s^3-616r^4s^2t^2-506r^4s^2t+440r^4s^2+1067r^4st^3-407r^4st^2-506r^4st+110r^4s+297r^4t^4+1067r^4t^3-616r^4t^2-220r^4t+825r^3s^3t^2+275r^3s^3t-396r^3s^3-363r^3s^2t^3+2387r^3s^2t^2-484r^3s^2t-396r^3s^2-990r^3st^4-2992r^3st^3+2387r^3st^2+275r^3st-990r^3t^4-363r^3t^3+825r^3t^2-693r^2s^3t^3-2673r^2s^3t^2+1188r^2s^3t+396r^2s^3+891r^2s^2t^4+891r^2s^2t^3-3168r^2s^2t^2+1188r^2s^2t+3564r^2st^4+891r^2st^3-2673r^2st^2+891r^2t^4-693r^2t^3+2772rs^3t^3+1518rs^3t^2-2178rs^3t-3564rs^2t^4+726rs^2t^3+1518rs^2t^2-3564rst^4+2772rst^3-3234s^3t^3+2310s^3t^2+4158s^2t^4-3234s^2t^3)$$

$$B_{63}^{[1]}=\frac{-h^2s^6}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(220r^3s^4t-110r^3s^4-825r^3s^3t^2-275r^3s^3t+396r^3s^3+693r^3s^2t^3+2673r^3s^2t^2-1188r^3s^2t-396r^3s^2-2772r^3st^3-1518r^3st^2+2178r^3st+3234r^3t^3-2310r^3t^2-264r^2s^5t+132r^2s^5+616r^2s^4t^2+506r^2s^4t-440r^2s^4+363r^2s^3t^3-2387r^2s^3t^2+484r^2s^3t+396r^2s^3-891r^2s^2t^4-891r^2s^2t^3+3168r^2s^2t^2-1188r^2s^2t+3564r^2st^4-726r^2st^3-1518r^2st^2-4158r^2t^4+3234r^2t^3+84rs^6t-42rs^6+105rs^5t^2-345rs^5t+132rs^5-1067rs^4t^3+407rs^4t^2+506rs^4t-110rs^4+990rs^3t^4+2992rs^3t^3-2387rs^3t^2-275rs^3t-3564rs^2t^4-891rs^2t^3+2673rs^2t^2+3564rst^4-2772rst^3-126s^6t^2+84s^6t+399s^5t^3+105s^5t^2-264s^5t-297s^4t^4-1067s^4t^3+616s^4t^2+220s^4t+990s^3t^4+363s^3t^3-825s^3t^2-891s^2t^4+693s^2t^3)$$

$$B_{73}^{[1]}=\frac{-h^2t^2}{27720(r-t)^3(s-t)^3(t-1)^3}(-1980r^3s^3t^3+7920r^3s^3t^2-10626r^3s^3t+4158r^3s^3+5148r^3s^2t^4-20196r^3s^2t^3+26598r^3s^2t^2-10626r^3s^2t-4235r^3st^5+16027r^3st^4-20196r^3st^3+7920r^3st^2+1155r^3t^6-4235r^3t^5+5148r^3t^4-1980r^3t^3+5148r^2s^3t^4-20196r^2s^3t^3+26598r^2s^3t^2-10626r^2s^3t-13376r^2s^2t^5+51436r^2s^2t^4-66330r^2s^2t^3+26598r^2s^2t^2+11121r^2st^6-41437r^2st^5+51436r^2st^4-20196r^2st^3-3069r^2t^7+11121r^2t^6-13376r^2t^5+5148r^2t^4-4235rs^3t^5+16027rs^3t^4-20196rs^3t^3+7920rs^3t^2+11121rs^2t^6-41437rs^2t^5+51436rs^2t^4-20196rs^2t^3-9420rst^7+34278rst^6-41437rst^5+16027rst^4+2646rt^8-9420rt^7+11121rt^6-4235rt^5+1155s^3t^6-4235s^3t^5+5148s^3t^4-1980s^3t^3-3069s^2t^7+11121s^2t^6-13376s^2t^5+5148s^2t^4+2646st^8-9420st^7+11121st^6-4235st^5-756t^9+2646t^8-3069t^7+1155t^6)$$

$$B_{83}^{[1]}=\frac{h^2}{27720t^3(r-t)^3(s-t)^3(t-1)^3}(2310r^3s^3t^2-2178r^3s^3t+396r^3s^3-3234r^3s^2t^3+1518r^3s^2t^2+1188r^3s^2t-396r^3s^2+2772r^3st^3-2673r^3st^2+275r^3st+110r^3s-693r^3t^3+825r^3t^2-220r^3t-3234r^2s^3t^3+1518r^2s^3t^2+1188r^2s^3t-396r^2s^3+4158r^2s^2t^4+726r^2s^2t^3-3168r^2s^2t^2-484r^2s^2t+440r^2s^2-3564r^2st^4+891r^2st^3+2387r^2st^2-506r^2st-132r^2s+891r^2t^4-363r^2t^3-616r^2t^2+264r^2t+2772rs^3t^3-2673rs^3t^2+275rs^3t+110rs^3-3564rs^2t^4+891rs^2t^3+2387rs^2t^2-506rs^2t-132rs^2+3564rst^4-2992rst^3-407rst^2+345rst+42rs-990rt^4+1067rt^3-105rt^2-84rt-693s^3t^3+825s^3t^2-220s^3t+891s^2t^4-363s^2t^3-616s^2t^2+264s^2t-990st^4+1067st^3-105st^2-84st+297t^4-399t^3+126t^2)$$

$$B_{93}^{[1]}=\frac{hr^5}{1260t^3(r-t)^3(s-t)^3(t-1)^3}(-14r^6st+7r^6s+21r^6t^2-14r^6t+40r^5s^2t-20r^5s^2-14r^5st^2+51r^5st-20r^5s-63r^5t^3-14r^5t^2+40r^5t-30r^4s^3t+15r^4s^3-90r^4s^2t^2-65r^4s^2t+60r^4s^2+150r^4st^3-65r^4st^2-65r^4st+15r^4s+45r^4t^4+150r^4t^3-90r^4t^2-30r^4t+105r^3s^3t^2+30r^3s^3t-48r^3s^3-39r^3s^2t^3+306r^3s^2t^2-72r^3s^2t-48r^3s^2-135r^3st^4-366r^3st^3+306r^3st^2+30r^3st-135r^3t^4-39r^3t^3+105r^3t^2-84r^2s^3t^3-294r^2s^3t^2+138r^2s^3t+42r^2s^3+108r^2s^2t^4+66r^2s^2t^3-342r^2s^2t^2+138r^2s^2t+432r^2st^4+66r^2st^3-294r^2st^2+108r^2t^4-84r^2t^3+294rs^3t^3+126rs^3t^2-210rs^3t-378rs^2t^4+126rs^2t^3+126rs^2t^2-378rst^4+294rst^3-294s^3t^3+210s^3t^2+378s^2t^4-294s^2t^3)$$

$$B_{103}^{[1]}=\frac{-hs^5}{1260t^3(r-t)^3(s-t)^3(t-1)^3}(30r^3s^4t-15r^3s^4-105r^3s^3t^2-30r^3s^3t+48r^3s^3+84r^3s^2t^3+294r^3s^2t^2-138r^3s^2t-42r^3s^2-294r^3st^3-126r^3st^2+210r^3st+294r^3t^3-210r^3t^2-40r^2s^5t+20r^2s^5+90r^2s^4t^2+65r^2s^4t-60r^2s^4+39r^2s^3t^3-306r^2s^3t^2+72r^2s^3t+48r^2s^3-108r^2s^2t^4-66r^2s^2t^3+342r^2s^2t^2-138r^2s^2t+378r^2st^4-126r^2st^3-126r^2st^2-378r^2t^4+294r^2t^3+14rs^6t-7rs^6+14rs^5t^2-51rs^5t+20rs^5-150rs^4t^3+65rs^4t^2+65rs^4t-15rs^4+135rs^3t^4+366rs^3t^3-306rs^3t^2-30rs^3t-432rs^2t^4-66rs^2t^3+294rs^2t^2+378rst^4-294rst^3-21s^6t^2+14s^6t+63s^5t^3+14s^5t^2-40s^5t-45s^4t^4-150s^4t^3+90s^4t^2+30s^4t+135s^3t^4+39s^3t^3-105s^3t^2-108s^2t^4+84s^2t^3)$$

$$B_{113}^{[1]}=\frac{-ht}{1260(r-t)^3(s-t)^3(t-1)^3}(-420r^3s^3t^3+1470r^3s^3t^2-1722r^3s^3t+630r^3s^3+1164r^3s^2t^4-4026r^3s^2t^3+4662r^3s^2t^2-1722r^3s^2t-1050r^3st^5+3561r^3st^4-4026r^3st^3+1470r^3st^2+315r^3t^6-1050r^3t^5+1164r^3t^4-420r^3t^3+1164r^2s^3t^4-4026r^2s^3t^3+4662r^2s^3t^2-1722r^2s^3t-3228r^2s^2t^5+11034r^2s^2t^4-12618r^2s^2t^3+4662r^2s^2t^2+2930r^2st^6-9847r^2st^5+11034r^2st^4-4026r^2st^3-885r^2t^7+2930r^2t^6-3228r^2t^5+1164r^2t^4-1050rs^3t^5+3561rs^3t^4-4026rs^3t^3+1470rs^3t^2+2930rs^2t^6-9847rs^2t^5+$$

$$11034rs^2t^4-4026rs^2t^3-2686rst^7+8913rst^6-9847rst^5+3561rst^4+819rt^8-2686rt^7+2930rt^6-$$
$$1050rt^5+315s^3t^6-1050s^3t^5+1164s^3t^4-420s^3t^3-885s^2t^7+2930s^2t^6-3228s^2t^5+1164s^2t^4+$$
$$819st^8-2686st^7+2930st^6-1050st^5-252t^9+819t^8-885t^7+315t^6)$$

$$B_{123}^{[1]}=\frac{h}{1260t^3(r-t)^3(s-t)^3(t-1)^3}(210r^3s^3t^2-210r^3s^3t+42r^3s^3-294r^3s^2t^3+126r^3s^2t^2+138r^3s^2t-$$
$$48r^3s^2+294r^3st^3-294r^3st^2+30r^3st+15r^3s-84r^3t^3+105r^3t^2-30r^3t-294r^2s^3t^3+126r^2s^3t^2+$$
$$138r^2s^3t-48r^2s^3+378r^2s^2t^4+126r^2s^2t^3-342r^2s^2t^2-72r^2s^2t+60r^2s^2-378r^2st^4+66r^2st^3+$$
$$306r^2st^2-65r^2st-20r^2s+108r^2t^4-39r^2t^3-90r^2t^2+40r^2t+294rs^3t^3-294rs^3t^2+30rs^3t+$$
$$15rs^3-378rs^2t^4+66rs^2t^3+306rs^2t^2-65rs^2t-20rs^2+432rst^4-366rst^3-65rst^2+51rst+7rs-$$
$$135rt^4+150rt^3-14rt^2-14rt-84s^3t^3+105s^3t^2-30s^3t+108s^2t^4-39s^2t^3-90s^2t^2+40s^2t-$$
$$135st^4+150st^3-14st^2-14st+45t^4-63t^3+21t^2)$$

$$B_{14}^{[1]}=\frac{-h^3r^7}{27720(r-1)^3(s-1)^3(t-1)^3}(7r^6st-14r^6s-14r^6t+21r^6-24r^5s^2t+48r^5s^2-24r^5st^2+64r^5st-21r^5s+$$
$$48r^5t^2-21r^5t-70r^5+22r^4s^3t-44r^4s^3+88r^4s^2t^2-106r^4s^2t-116r^4s^2+22r^4st^3-106r^4st^2-$$
$$70r^4st+208r^4s-44r^4t^3-116r^4t^2+208r^4t+54r^4-88r^3s^3t^2+66r^3s^3t+176r^3s^3-88r^3s^2t^3-$$
$$88r^3s^2t^2+506r^3s^2t-88r^3s^2+66r^3st^3+506r^3st^2-660r^3st-198r^3s+176r^3t^3-88r^3t^2-198r^3t+$$
$$99r^2s^3t^3+275r^2s^3t^2-649r^2s^3t-154r^2s^3+275r^2s^2t^3-781r^2s^2t^2+275r^2s^2t+198r^2s^2-649r^2st^3+$$
$$275r^2st^2+792r^2st-154r^2t^3+198r^2t^2-594rs^3t^3+462rs^3t^2+693rs^3t+462rs^2t^3+66rs^2t^2-$$
$$891rs^2t+693rst^3-891rst^2+660s^3t^3-924s^3t^2-924s^2t^3+1188s^2t^2)$$

$$B_{24}^{[1]}=\frac{-h^3s^7}{27720(r-1)^3(s-1)^3(t-1)^3}(22r^3s^4t-44r^3s^4-88r^3s^3t^2+66r^3s^3t+176r^3s^3+99r^3s^2t^3+275r^3s^2t^2-$$
$$649r^3s^2t-154r^3s^2-594r^3st^3+462r^3st^2+693r^3st+660r^3t^3-924r^3t^2-24r^2s^5t+48r^2s^5+88r^2s^4$$
$$t^2-106r^2s^4t-116r^2s^4-88r^2s^3t^3-88r^2s^3t^2+506r^2s^3t-88r^2s^3+275r^2s^2t^3-781r^2s^2t^2+275r^2s^2$$
$$t+198r^2s^2+462r^2st^3+66r^2st^2-891r^2st-924r^2t^3+1188r^2t^2+7rs^6t-14rs^6-24rs^5t^2+64rs^5t-$$
$$21rs^5+22rs^4t^3-106rs^4t^2-70rs^4t+208rs^4+66rs^3t^3+506rs^3t^2-660rs^3t-198rs^3-649rs^2t^3+$$
$$275rs^2t^2+792rs^2t+693rst^3-891rst^2-14s^6t+21s^6+48s^5t^2-21s^5t-70s^5-44s^4t^3-116s^4t^2+$$
$$208s^4t+54s^4+176s^3t^3-88s^3t^2-198s^3t-154s^2t^3+198s^2t^2)$$

$$B_{34}^{[1]}=\frac{-h^3t^7}{27720(r-1)^3(s-1)^3(t-1)^3}(99r^3s^3t^2-594r^3s^3t+660r^3s^3-88r^3s^2t^3+275r^3s^2t^2+462r^3s^2t-924r^3s^2+$$
$$22r^3st^4+66r^3st^3-649r^3st^2+693r^3st-44r^3t^4+176r^3t^3-154r^3t^2-88r^2s^3t^3+275r^2s^3t^2+462r^2s^3t-$$
$$924r^2s^3+88r^2s^2t^4-88r^2s^2t^3-781r^2s^2t^2+66r^2s^2t+1188r^2s^2-24r^2st^5-106r^2st^4+506r^2st^3+$$
$$275r^2st^2-891r^2st+48r^2t^5-116r^2t^4-88r^2t^3+198r^2t^2+22rs^3t^4+66rs^3t^3-649rs^3t^2+693rs^3t-$$
$$24rs^2t^5-106rs^2t^4+506rs^2t^3+275rs^2t^2-891rs^2t+7rst^6+64rst^5-70rst^4-660rst^3+792rst^2-$$
$$14rt^6-21rt^5+208rt^4-198rt^3-44s^3t^4+176s^3t^3-154s^3t^2+48s^2t^5-116s^2t^4-88s^2t^3+198s^2t^2-$$
$$14st^6-21st^5+208st^4-198st^3+21t^6-70t^5+54t^4)$$

$$B_{44}^{[1]}=\frac{h^3}{27720(r-1)^3(s-1)^3(t-1)^3}(924r^3s^3t^3-2244r^3s^3t^2+1485r^3s^3t-330r^3s^3-2244r^3s^2t^3+5280r^3s^2t^2-$$
$$3575r^3s^2t+814r^3s^2+1485r^3st^3-3575r^3st^2+2574r^3st-616r^3s-330r^3t^3+814r^3t^2-616r^3t+154r^3-$$
$$2244r^2s^3t^3+5280r^2s^3t^2-3575r^2s^3t+814r^2s^3+5280r^2s^2t^3\text{-}12320r^2s^2t^2+8569r^2s^2t\text{-}2002r^2s^2\text{-}$$
$$3575r^2st^3+8569r^2st^2-6290r^2st+1536r^2s+814r^2t^3-2002r^2t^2+1536r^2t-390r^2+1485rs^3t^3-$$
$$3575rs^3t^2+2574rs^3t-616rs^3-3575rs^2t^3+8569rs^2t^2-6290rs^2t+1536rs^2+2574rst^3-6290rst^2+$$
$$4793rst-1210rs-616rt^3+1536rt^2-1210rt+315r-330s^3t^3+814s^3t^2-616s^3t+154s^3+814s^2t^3-$$
$$2002s^2t^2+1536s^2t-390s^2-616st^3+1536st^2-1210st+315s+154t^3-390t^2+315t-84)$$

$$
\begin{aligned}
B_{54}^{[1]} = \frac{-h^2r^6}{27720(r-1)^3(s-1)^3(t-1)^3}(&42r^6st - 84r^6s - 84r^6t + 126r^6 - 132r^5s^2t + 264r^5s^2 - 132r^5st^2 + 345r^5st - \\
&105r^5s + 264r^5t^2 - 105r^5t - 399r^5 + 110r^4s^3t - 220r^4s^3 + 440r^4s^2t^2 - 506r^4s^2t - 616r^4s^2 + 110r^4st^3 - \\
&506r^4st^2 - 407r^4st + 1067r^4s - 220r^4t^3 - 616r^4t^2 + 1067r^4t + 297r^4 - 396r^3s^3t^2 + 275r^3s^3t + 825r^3s^3 - \\
&396r^3s^2t^3 - 484r^3s^2t^2 + 2387r^3s^2t - 363r^3s^2 + 275r^3st^3 + 2387r^3st^2 - 2992r^3st - 990r^3s + 825r^3t^3 - \\
&363r^3t^2 - 990r^3t + 396r^2s^3t^3 + 1188r^2s^3t^2 - 2673r^2s^3t - 693r^2s^3 + 1188r^2s^2t^3 - 3168r^2s^2t^2 + 891r^2s^2 \\
&t + 891r^2s^2 - 2673r^2st^3 + 891r^2st^2 + 3564r^2st - 693r^2t^3 + 891r^2t^2 - 2178rs^3t^3 + 1518rs^3t^2 + 2772rs^3t + \\
&1518rs^2t^3 + 726rs^2t^2 - 3564rs^2t + 2772rst^3 - 3564rst^2 + 2310s^3t^3 - 3234s^3t^2 - 3234s^2t^3 + 4158s^2t^2)
\end{aligned}
$$

$$
\begin{aligned}
B_{64}^{[1]} = \frac{-h^2s^6}{27720(r-1)^3(s-1)^3(t-1)^3}(&110r^3s^4t - 220r^3s^4 - 396r^3s^3t^2 + 275r^3s^3t + 825r^3s^3 + 396r^3s^2t^3 + 1188r^3s^2 \\
&t^2 - 2673r^3s^2t - 693r^3s^2 - 2178r^3st^3 + 1518r^3st^2 + 2772r^3st + 2310r^3t^3 - 3234r^3t^2 - 132r^2s^5t + 264r^2s^5 + \\
&440r^2s^4t^2 - 506r^2s^4t - 616r^2s^4 - 396r^2s^3t^3 - 484r^2s^3t^2 + 2387r^2s^3t - 363r^2s^3 + 1188r^2s^2t^3 - 3168r^2s^2 \\
&t^2 + 891r^2s^2t + 891r^2s^2 + 1518r^2st^3 + 726r^2st^2 - 3564r^2st - 3234r^2t^3 + 4158r^2t^2 + 42rs^6t - 84rs^6 - 132rs^5 \\
&t^2 + 345rs^5t - 105rs^5 + 110rs^4t^3 - 506rs^4t^2 - 407rs^4t + 1067rs^4 + 275rs^3t^3 + 2387rs^3t^2 - 2992rs^3t - 990rs^3 - \\
&2673rs^2t^3 + 891rs^2t^2 + 3564rs^2t + 2772rst^3 - 3564rst^2 - 84s^6t + 126s^6 + 264s^5t^2 - 105s^5t - 399s^5 - 220s^4t^3 - \\
&616s^4t^2 + 1067s^4t + 297s^4 + 825s^3t^3 - 363s^3t^2 - 990s^3t - 693s^2t^3 + 891s^2t^2)
\end{aligned}
$$

$$
\begin{aligned}
B_{74}^{[1]} = \frac{-h^2t^6}{27720(r-1)^3(s-1)^3(t-1)^3}(&396r^3s^3t^2 - 2178r^3s^3t + 2310r^3s^3 - 396r^3s^2t^3 + 1188r^3s^2t^2 + 1518r^3s^2t - \\
&3234r^3s^2 + 110r^3st^4 + 275r^3st^3 - 2673r^3st^2 + 2772r^3st - 220r^3t^4 + 825r^3t^3 - 693r^3t^2 - 396r^2s^3t^3 + \\
&1188r^2s^3t^2 + 1518r^2s^3t - 3234r^2s^3 + 440r^2s^2t^4 - 484r^2s^2t^3 - 3168r^2s^2t^2 + 726r^2s^2t + 4158r^2s^2 - 132r^2st^5 - \\
&506r^2st^4 + 2387r^2st^3 + 891r^2st^2 - 3564r^2st + 264r^2t^5 - 616r^2t^4 - 363r^2t^3 + 891r^2t^2 + 110rs^3t^4 + \\
&275rs^3t^3 - 2673rs^3t^2 + 2772rs^3t - 132rs^2t^5 - 506rs^2t^4 + 2387rs^2t^3 + 891rs^2t^2 - 3564rs^2t + 42rst^6 + \\
&345rst^5 - 407rst^4 - 2992rst^3 + 3564rst^2 - 84rt^6 - 105rt^5 + 1067rt^4 - 990rt^3 - 220s^3t^4 + 825s^3t^3 - 693s^3 \\
&t^2 + 264s^2t^5 - 616s^2t^4 - 363s^2t^3 + 891s^2t^2 - 84st^6 - 105st^5 + 1067st^4 - 990st^3 + 126t^6 - 399t^5 + 297t^4)
\end{aligned}
$$

$$
\begin{aligned}
B_{84}^{[1]} = \frac{h^2}{27720(r-1)^3(s-1)^3(t-1)^3}(&4158r^3s^3t^3 - 10626r^3s^3t^2 + 7920r^3s^3t - 1980r^3s^3 - 10626r^3s^2t^3 + 26598r^3s^2 \\
&t^2 - 20196r^3s^2t + 5148r^3s^2 + 7920r^3st^3 - 20196r^3st^2 + 16027r^3st - 4235r^3s - 1980r^3t^3 + 5148r^3t^2 - 4235r^3 \\
&t + 1155r^3 - 10626r^2s^3t^3 + 26598r^2s^3t^2 - 20196r^2s^3t + 5148r^2s^3 + 26598r^2s^2t^3 - 66330r^2s^2t^2 + 51436r^2s^2 \\
&t - 13376r^2s^2 - 20196r^2st^3 + 51436r^2st^2 - 41437r^2st + 11121r^2s + 5148r^2t^3 - 13376r^2t^2 + 11121r^2t - 3069r^2 + \\
&7920rs^3t^3 - 20196rs^3t^2 + 16027rs^3t - 4235rs^3 - 20196rs^2t^3 + 51436rs^2t^2 - 41437rs^2t + 11121rs^2 + 16027rst^3 - \\
&41437rst^2 + 34278rst - 9420rs - 4235rt^3 + 11121rt^2 - 9420rt + 2646r - 1980s^3t^3 + 5148s^3t^2 - 4235s^3t + 1155s^3 + \\
&5148s^2t^3 - 13376s^2t^2 + 11121s^2t - 3069s^2 - 4235st^3 + 11121st^2 - 9420st + 2646s + 1155t^3 - 3069t^2 + 2646t - 756)
\end{aligned}
$$

$$
\begin{aligned}
B_{104}^{[1]} = \frac{-hs^5}{1260(r-1)^3(s-1)^3(t-1)^3}(&15r^3s^4t - 30r^3s^4 - 48r^3s^3t^2 + 30r^3s^3t + 105r^3s^3 + 42r^3s^2t^3 + 138r^3s^2t^2 - 294r^3s^2t - \\
&84r^3s^2 - 210r^3st^3 + 126r^3st^2 + 294r^3st + 210r^3t^3 - 294r^3t^2 - 20r^2s^5t + 40r^2s^5 + 60r^2s^4t^2 - 65r^2s^4t - 90r^2s^4 - \\
&48r^2s^3t^3 - 72r^2s^3t^2 + 306r^2s^3t - 39r^2s^3 + 138r^2s^2t^3 - 342r^2s^2t^2 + 66r^2s^2t + 108r^2s^2 + 126r^2st^3 + 126r^2st^2 - \\
&378r^2st - 294r^2t^3 + 378r^2t^2 + 7rs^6t - 14rs^6 - 20rs^5t^2 + 51rs^5t - 14rs^5 + 15rs^4t^3 - 65rs^4t^2 - 65rs^4t + 150rs^4 + \\
&30rs^3t^3 + 306rs^3t^2 - 366rs^3t - 135rs^3 - 294rs^2t^3 + 66rs^2t^2 + 432rs^2t + 294rst^3 - 378rst^2 - 14s^6t + 21s^6 + \\
&40s^5t^2 - 14s^5t - 63s^5 - 30s^4t^3 - 90s^4t^2 + 150s^4t + 45s^4 + 105s^3t^3 - 39s^3t^2 - 135s^3t - 84s^2t^3 + 108s^2t^2)
\end{aligned}
$$

$$B_{114}^{[1]} = \frac{-ht^5}{1260(r-1)^3(s-1)^3(t-1)^3}(42r^3s^3t^2 - 210r^3s^3t + 210r^3s^3 - 48r^3s^2t^3 + 138r^3s^2t^2 + 126r^3s^2t - 294r^3s^2 + 15r^3st^4 +$$
$$30r^3st^3 - 294r^3st^2 + 294r^3st - 30r^3t^4 + 105r^3t^3 - 84r^3t^2 - 48r^2s^3t^3 + 138r^2s^3t^2 + 126r^2s^3t - 294r^2s^3 + 60r^2s^2t^4 -$$
$$72r^2s^2t^3 - 342r^2s^2t^2 + 126r^2s^2t + 378r^2s^2 - 20r^2st^5 - 65r^2st^4 + 306r^2st^3 + 66r^2st^2 - 378r^2st + 40r^2t^5 - 90r^2t^4 - 39r^2t^3 +$$
$$108r^2t^2 + 15rs^3t^4 + 30rs^3t^3 - 294rs^3t^2 + 294rs^3t - 20rs^2t^5 - 65rs^2t^4 + 306rs^2t^3 + 66rs^2t^2 - 378rs^2t + 7rst^6 +$$
$$51rst^5 - 65rst^4 - 366rst^3 + 432rst^2 - 14rt^6 - 14rt^5 + 150rt^4 - 135rt^3 - 30s^3t^4 + 105s^3t^3 - 84s^3t^2 + 40s^2t^5 - 90s^2t^4 -$$
$$39s^2t^3 + 108s^2t^2 - 14st^6 - 14st^5 + 150st^4 - 135st^3 + 21t^6 - 63t^5 + 45t^4)$$

$$B_{124}^{[1]} = \frac{h}{1260(r-1)^3(s-1)^3(t-1)^3}(630r^3s^3t^3 - 1722r^3s^3t^2 + 1470r^3s^3t - 420r^3s^3 - 1722r^3s^2t^3 + 4662r^3s^2t^2 - 4026r^3s^2t +$$
$$1164r^3s^2 + 1470r^3st^3 - 4026r^3st^2 + 3561r^3st - 1050r^3s - 420r^3t^3 + 1164r^3t^2 - 1050r^3t + 315r^3 - 1722r^2s^3t^3 +$$
$$4662r^2s^3t^2 - 4026r^2s^3t + 1164r^2s^3 + 4662r^2s^2t^3 - 12618r^2s^2t^2 + 11034r^2s^2t - 3228r^2s^2 - 4026r^2st^3 +$$
$$11034r^2st^2 - 9847r^2st + 2930r^2s + 1164r^2t^3 - 3228r^2t^2 + 2930r^2t - 885r^2 + 1470rs^3t^3 - 4026rs^3t^2 + 3561rs^3t -$$
$$1050rs^3 - 4026rs^2t^3 + 11034rs^2t^2 - 9847rs^2t + 2930rs^2 + 3561rst^3 - 9847rst^2 + 8913rst - 2686rs - 1050rt^3 +$$
$$2930rt^2 - 2686rt + 819r - 420s^3t^3 + 1164s^3t^2 - 1050s^3t + 315s^3 + 1164s^2t^3 - 3228s^2t^2 + 2930s^2t - 885s^2 -$$
$$1050st^3 + 2930st^2 - 2686st + 819s + 315t^3 - 885t^2 + 819t - 252)$$

$$D_{112}^{[0]} = \frac{h^4r^4}{55440s^2t^2}(7r^6 - 24r^5s - 24r^5t - 24r^5 + 22r^4s^2 + 88r^4st + 88r^4s + 22r^4t^2 + 88r^4t + 22r^4 - 88r^3s^2t - 88r^3s^2 -$$
$$88r^3st^2 - 352r^3st - 88r^3s - 88r^3t^2 - 88r^3t + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 + 396r^2st^2 + 396r^2st + 99r^2t^2 -$$
$$528rs^2t^2 - 528rs^2t - 528rst^2 + 924s^2t^2)$$

$$D_{212}^{[0]} = \frac{h^4s^4}{55440r^2t^2}(22r^2s^4 - 88r^2s^3t - 88r^2s^3 + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 - 528r^2st^2 - 528r^2st + 924r^2t^2 -$$
$$24rs^5 + 88rs^4t + 88rs^4 - 88rs^3t^2 - 352rs^3t - 88rs^3 + 396rs^2t^2 + 396rs^2t - 528rst^2 + 7s^6 - 24s^5t - 24s^5 +$$
$$22s^4t^2 + 88s^4t + 22s^4 - 88s^3t^2 - 88s^3t + 99s^2t^2)$$

$$D_{312}^{[0]} = \frac{h^4t^4}{55440r^2s^2}(99r^2s^2t^2 - 528r^2s^2t + 924r^2s^2 - 88r^2st^3 + 396r^2st^2 - 528r^2st + 22r^2t^4 - 88r^2t^3 + 99r^2t^2 -$$
$$88rs^2t^3 + 396rs^2t^2 - 528rs^2t + 88rst^4 - 352rst^3 + 396rst^2 - 24rt^5 + 88rt^4 - 88rt^3 + 22s^2t^4 - 88s^2t^3 +$$
$$99s^2t^2 - 24st^5 + 88st^4 - 88st^3 + 7t^6 - 24t^5 + 22t^4)$$

$$D_{412}^{[0]} = \frac{h^4}{55440r^2s^2t^2}(924r^2s^2t^2 - 528r^2s^2t + 99r^2s^2 - 528r^2st^2 + 396r^2st - 88r^2s + 99r^2t^2 - 88r^2t + 22r^2 -$$
$$528rs^2t^2 + 396rs^2t - 88rs^2 + 396rst^2 - 352rst + 88rs - 88rt^2 + 88rt - 24r + 99s^2t^2 - 88s^2t + 22s^2 -$$
$$88st^2 + 88st - 24s + 22t^2 - 24t + 7)$$

$$D_{512}^{[0]} = \frac{h^3r^3}{27720s^2t^2}(21r^6 - 66r^5s - 66r^5t - 66r^5 + 55r^4s^2 + 220r^4st + 220r^4s + 55r^4t^2 + 220r^4t + 55r^4 -$$
$$198r^3s^2t - 198r^3s^2 - 198r^3st^2 - 792r^3st - 198r^3s - 198r^3t^2 - 198r^3t + 198r^2s^2t^2 + 792r^2s^2t +$$
$$198r^2s^2 + 792r^2st^2 + 792r^2st + 198r^2t^2 - 924rs^2t^2 - 924rs^2t - 924rst^2 + 1386s^2t^2)$$

$$D_{612}^{[0]} = \frac{h^3s^3}{27720r^2t^2}(55r^2s^4 - 198r^2s^3t - 198r^2s^3 + 198r^2s^2t^2 + 792r^2s^2t + 198r^2s^2 - 924r^2st^2 - 924r^2st +$$
$$1386r^2t^2 - 66rs^5 + 220rs^4t + 220rs^4 - 198rs^3t^2 - 792rs^3t - 198rs^3 + 792rs^2t^2 + 792rs^2t - 924rst^2 +$$
$$21s^6 - 66s^5t - 66s^5 + 55s^4t^2 + 220s^4t + 55s^4 - 198s^3t^2 - 198s^3t + 198s^2t^2)$$

$$D_{712}^{[0]} = \frac{h^3t^3}{27720r^2s^2}(198r^2s^2t^2 - 924r^2s^2t + 1386r^2s^2 - 198r^2st^3 + 792r^2st^2 - 924r^2st + 55r^2t^4 - 198r^2t^3 + 198r^2t^2 - 198rs^2t^3 + 792rs^2t^2 - 924rs^2t + 220rst^4 - 792rst^3 + 792rst^2 - 66rt^5 + 220rt^4 - 198rt^3 + 55s^2t^4 - 198s^2t^3 + 198s^2t^2 - 66st^5 + 220st^4 - 198st^3 + 21t^6 - 66t^5 + 55t^4)$$

$$D_{812}^{[0]} = \frac{h^3}{27720r^2s^2t^2}(1386r^2s^2t^2 - 924r^2s^2t + 198r^2s^2 - 924r^2st^2 + 792r^2st - 198r^2s + 198r^2t^2 - 198r^2t + 55r^2 - 924rs^2t^2 + 792rs^2t - 198rs^2 + 792rst^2 - 792rst + 220rs - 198rt^2 + 220rt - 66r + 198s^2t^2 - 198s^2t + 55s^2 - 198st^2 + 220st - 66s + 55t^2 - 66t + 21)$$

$$D_{912}^{[0]} = \frac{h^2r^2}{2520s^2t^2}(7r^6 - 20r^5s - 20r^5t - 20r^5 + 15r^4s^2 + 60r^4st + 60r^4s + 15r^4t^2 + 60r^4t + 15r^4 - 48r^3s^2t - 48r^3s^2 - 48r^3st^2 - 192r^3st - 48r^3s - 48r^3t^2 - 48r^3t + 42r^2s^2t^2 + 168r^2s^2t + 42r^2s^2 + 168r^2st^2 + 168r^2st + 42r^2t^2 - 168rs^2t^2 - 168rs^2t - 168rst^2 + 210s^2t^2)$$

$$D_{1012}^{[0]} = \frac{h^2s^2}{2520r^2t^2}(15r^2s^4 - 48r^2s^3t - 48r^2s^3 + 42r^2s^2t^2 + 168r^2s^2t + 42r^2s^2 - 168r^2st^2 - 168r^2st + 210r^2t^2 - 20rs^5 + 60rs^4t + 60rs^4 - 48rs^3t^2 - 192rs^3t - 48rs^3 + 168rs^2t^2 + 168rs^2t - 168rst^2 + 7s^6 - 20s^5t - 20s^5 + 15s^4t^2 + 60s^4t + 15s^4 - 48s^3t^2 - 48s^3t + 42s^2t^2)$$

$$D_{1112}^{[0]} = \frac{h^2t^2}{2520r^2s^2}(42r^2s^2t^2 - 168r^2s^2t + 210r^2s^2 - 48r^2st^3 + 168r^2st^2 - 168r^2st + 15r^2t^4 - 48r^2t^3 + 42r^2t^2 - 48rs^2t^3 + 168rs^2t^2 - 168rs^2t + 60rst^4 - 192rst^3 + 168rst^2 - 20rt^5 + 60rt^4 - 48rt^3 + 15s^2t^4 - 48s^2t^3 + 42s^2t^2 - 20st^5 + 60st^4 - 48st^3 + 7t^6 - 20t^5 + 15t^4)$$

$$D_{1212}^{[0]} = \frac{h^2}{2520r^2s^2t^2}(210r^2s^2t^2 - 168r^2s^2t + 42r^2s^2 - 168r^2st^2 + 168r^2st - 48r^2s + 42r^2t^2 - 48r^2t + 15r^2 - 168rs^2t^2 + 168rs^2t - 48rs^2 + 168rst^2 - 192rst + 60rs - 48rt^2 + 60rt - 20r + 42s^2t^2 - 48s^2t + 15s^2 - 48st^2 + 60st - 20s + 15t^2 - 20t + 7)$$

$$D_{11}^{[1]} = \frac{-h^4r^4}{55440(r-s)^2(r-t)^2(r-1)^2}(14r^6 - 42r^5s - 42r^5t - 42r^5 + 33r^4s^2 + 132r^4st + 132r^4s + 33r^4t^2 + 132r^4t + 33r^4 - 110r^3s^2t - 110r^3s^2 - 110r^3st^2 - 440r^3st - 110r^3s - 110r^3t^2 - 110r^3t + 99r^2s^2t^2 + 396r^2s^2t + 99r^2s^2 + 396r^2st^2 + 396r^2st + 99r^2t^2 - 396rs^2t^2 - 396rs^2t - 396rst^2 + 462s^2t^2)$$

$$D_{21}^{[1]} = \frac{-h^4s^7}{55440r^2(r-s)^2(r-t)^2(r-1)^2}(88s^2t^2 - 22s^3t^2 + 44rs^2 - 44rs^3 + 12rs^4 + 264rt^2 - 99st^2 + 88s^2t - 88s^3t + 24s^4t - 22s^3 + 24s^4 - 7s^5 - 198rst^2 + 176rs^2t - 44rs^3t + 44rs^2t^2 - 198rst)$$

$$D_{31}^{[1]} = \frac{-h^4t^7}{55440r^2(r-s)^2(r-t)^2(r-1)^2}(88s^2t^2 - 22s^2t^3 + 264rs^2 + 44rt^2 - 44rt^3 + 12rt^4 + 88st^2 - 99s^2t - 88st^3 + 24st^4 - 22t^3 + 24t^4 - 7t^5 + 176rst^2 - 198rs^2t - 44rst^3 + 44rs^2t^2 - 198rst)$$

$$D_{41}^{[1]} = \frac{-h^4}{55440r^2(r-s)^2(r-t)^2(r-1)^2}(12r + 24s + 24t - 99s^2t^2 - 44rs - 44rt - 88st + 44rs^2 + 44rt^2 + 88st^2 + 88s^2t - 22s^2 - 22t^2 - 198rst^2 - 198rs^2t + 264rs^2t^2 + 176rst - 7)$$

$$D_{51}^{[1]}=\frac{-h^3r^3}{13860(r-s)^2(r-t)^2(r-1)^2}(28r^6-77r^5s-77r^5t-77r^5+55r^4s^2+220r^4st+220r^4s+55r^4t^2+$$
$$220r^4t+55r^4-165r^3s^2t-165r^3s^2-165r^3st^2-660r^3st-165r^3s-165r^3t^2-165r^3t+132r^2s^2t^2+$$
$$528r^2s^2t+132r^2s^2+528r^2st^2+528r^2st+132r^2t^2-462rs^2t^2-462rs^2t-462rst^2+462s^2t^2)$$

$$D_{61}^{[1]}=\frac{-h^3s^6}{27720r^2(r-s)^2(r-t)^2(r-1)^2}(198s^2t^2-55s^3t^2+99rs^2-110rs^3+33rs^4+462rt^2-198st^2+$$
$$198s^2t-220s^3t+66s^4t-55s^3+66s^4-21s^5-396rst^2+396rs^2t-110rs^3t+99rs^2t^2-396rst)$$

$$D_{71}^{[1]}=\frac{-h^3t^6}{27720r^2(r-s)^2(r-t)^2(r-1)^2}(198s^2t^2\text{-}55s^2t^3+462rs^2+99rt^2\text{-}110rt^3+33rt^4+198st^2\text{-}198s^2t\text{-}$$
$$220st^3+66st^4\text{-}55t^3+66t^4\text{-}21t^5+396rst^2\text{-}396rs^2t\text{-}110rst^3+99rs^2t^2\text{-}396rst)$$

$$D_{81}^{[1]}=\frac{-h^3}{27720r^2(r-s)^2(r-t)^2(r-1)^2}(33r+66s+66t-198s^2t^2-110rs-110rt-220st+99rs^2+99rt^2+$$
$$198st^2+198s^2t-55s^2-55t^2-396rst^2-396rs^2t+462rs^2t^2+396rst-21)$$

$$D_{91}^{[1]}=\frac{-h^2r^2}{2520(r-s)^2(r-t)^2(r-1)^2}(28r^6-70r^5s-70r^5t-70r^5+45r^4s^2+180r^4st+180r^4s+45r^4t^2+$$
$$180r^4t+45r^4-120r^3s^2t-120r^3s^2-120r^3st^2-480r^3st-120r^3s-120r^3t^2-120r^3t+84r^2s^2t^2+$$
$$336r^2s^2t+84r^2s^2+336r^2st^2+336r^2st+84r^2t^2-252rs^2t^2-252rs^2t-252rst^2+210s^2t^2)$$

$$D_{101}^{[1]}=\frac{-h^2s^5}{2520r^2(r-s)^2(r-t)^2(r-1)^2}(48s^2t^2-15s^3t^2+24rs^2-30rs^3+10rs^4+84rt^2-42st^2+48s^2t-$$
$$60s^3t+20s^4t-15s^3+20s^4-7s^5-84rst^2+96rs^2t-30rs^3t+24rs^2t^2-84rst)$$

$$D_{111}^{[1]}=\frac{-h^2t^5}{2520r^2(r-s)^2(r-t)^2(r-1)^2}(48s^2t^2-15s^2t^3+84rs^2+24rt^2-30rt^3+10rt^4+48st^2-42s^2t-$$
$$60st^3+20st^4-15t^3+20t^4-7t^5+96rst^2-84rs^2t-30rst^3+24rs^2t^2-84rst)$$

$$D_{121}^{[1]}=\frac{-h^2}{2520r^2(r-s)^2(r-t)^2(r-1)^2}(10r+20s+20t-42s^2t^2-30rs-30rt-60st+24rs^2+24rt^2+$$
$$48st^2+48s^2t-15s^2-15t^2-84rst^2-84rs^2t+84rs^2t^2+96rst-7)$$

$$D_{12}^{[1]}=\frac{-h^4r^7}{55440s^2(r-s)^2(s-t)^2(s-1)^2}(88r^2t^2-22r^3t^2+44r^2s-44r^3s+12r^4s-99rt^2+88r^2t-$$
$$88r^3t+24r^4t+264st^2-22r^3+24r^4-7r^5-198rst^2+176r^2st-44r^3st+44r^2st^2-198rst)$$

$$D_{22}^{[1]}=\frac{-h^4s^4}{55440(r-s)^2(s-t)^2(s-1)^2}(33r^2s^4-110r^2s^3t-110r^2s^3+99r^2s^2t^2+396r^2s^2t+99r^2s^2-$$
$$396r^2st^2-396r^2st+462r^2t^2-42rs^5+132rs^4t+132rs^4-110rs^3t^2-440rs^3t-110rs^3+396rs^2$$
$$t^2+396rs^2t-396rst^2+14s^6-42s^5t-42s^5+33s^4t^2+132s^4t+33s^4-110s^3t^2-110s^3t+99s^2t^2)$$

$$D_{32}^{[1]}=\frac{-h^4t^7}{55440s^2(r-s)^2(s-t)^2(s-1)^2}(88r^2t^2\text{-}22r^2t^3+264r^2s+88rt^2\text{-}99r^2t\text{-}88rt^3+24rt^4+44st^2\text{-}$$
$$44st^3+12st^4\text{-}22t^3+24t^4\text{-}7t^5+176rst^2\text{-}198r^2st\text{-}44rst^3+44r^2st^2\text{-}198rst)$$

$$D^{[1]}_{42} = \frac{-h^4}{55440s^2(r-s)^2(s-t)^2(s-1)^2}(24r+12s+24t-99r^2t^2-44rs-88rt-44st+44r^2s+88rt^2+88r^2t+44st^2-22r^2-22t^2-198rst^2-198r^2st+264r^2st^2+176rst-7)$$

$$D^{[1]}_{52} = \frac{-h^3r^6}{27720s^2(r-s)^2(s-t)^2(s-1)^2}(198r^2t^2-55r^3t^2+99r^2s-110r^3s+33r^4s-198rt^2+198r^2t-220r^3t+66r^4t+462st^2\text{-}55r^3+66r^4\text{-}21r^5\text{-}396rst^2+396r^2st\text{-}110r^3st+99r^2st^2$$

$$D^{[1]}_{62} = \frac{-h^3s^3}{13860(r-s)^2(s-t)^2(s-1)^2}(55r^2s^4-165r^2s^3t-165r^2s^3+132r^2s^2t^2+528r^2s^2t+132r^2s^2-462r^2st^2-462r^2st+462r^2t^2-77rs^5+220rs^4t+220rs^4-165rs^3t^2-660rs^3t-165rs^3+528rs^2t^2+528rs^2t-462rst^2+28s^6-77s^5t-77s^5+55s^4t^2+220s^4t+55s^4-165s^3t^2-165s^3t+132s^2t^2)$$

$$D^{[1]}_{72} = \frac{-h^3t^6}{27720s^2(r-s)^2(s-t)^2(s-1)^2}(198r^2t^2\text{-}55r^2t^3+462r^2s+198rt^2\text{-}198r^2t\text{-}220rt^3+66rt^4+99st^2\text{-}110st^3+33st^4\text{-}55t^3+66t^4\text{-}21t^5+396rst^2\text{-}396r^2st\text{-}110rst^3+99r^2st^2-$$

$$D^{[1]}_{82} = \frac{-h^3}{27720s^2(r-s)^2(s-t)^2(s-1)^2}(66r+33s+66t-198r^2t^2-110rs-220rt-110st+99r^2s+198rt^2+198r^2t+99st^2-55r^2-55t^2-396rst^2-396r^2st+462r^2st^2+396rst-21)$$

$$D^{[1]}_{92} = \frac{-h^2r^5}{2520s^2(r-s)^2(s-t)^2(s-1)^2}(48r^2t^2-15r^3t^2+24r^2s-30r^3s+10r^4s-42rt^2+48r^2t-60r^3t+20r^4t+84st^2\text{-}15r^3+20r^4\text{-}7r^5\text{-}84rst^2+96r^2st\text{-}30r^3st+24r^2st^2\text{-}84rst)$$

$$D^{[1]}_{102} = \frac{-h^2s^2}{2520(r-s)^2(s-t)^2(s-1)^2}(45r^2s^4-120r^2s^3t-120r^2s^3+84r^2s^2t^2+336r^2s^2t+84r^2s^2-252r^2st^2-252r^2st+210r^2t^2-70rs^5+180rs^4t+180rs^4-120rs^3t^2-480rs^3t-120rs^3+336rs^2t^2+336rs^2t-252rst^2+28s^6-70s^5t-70s^5+45s^4t^2+180s^4t+45s^4-120s^3t^2-120s^3t+84s^2t^2)$$

$$D^{[1]}_{112} = \frac{-h^2t^5}{2520s^2(r-s)^2(s-t)^2(s-1)^2}(48r^2t^2-15r^2t^3+84r^2s+48rt^2-42r^2t-60rt^3+20rt^4+24st^2-30st^3+10st^4-15t^3+20t^4-7t^5+96rst^2-84r^2st-30rst^3+24r^2st^2-84rst)$$

$$D^{[1]}_{122} = \frac{-h^2}{2520s^2(r-s)^2(s-t)^2(s-1)^2}(20r+10s+20t-42r^2t^2-30rs-60rt-30st+24r^2s+48rt^2+48r^2t+24st^2-15r^2-15t^2-84rst^2-84r^2st+84r^2st^2+96rst-7)$$

$$D^{[1]}_{13} = \frac{-h^4r^7}{55440t^2(r-t)^2(s-t)^2(t-1)^2}(88r^2s^2\text{-}22r^3s^2\text{-}99rs^2+88r^2s\text{-}88r^3s+24r^4s+44r^2t\text{-}44r^3t+12r^4t+264s^2t-22r^3+24r^4\text{-}7r^5\text{-}198rs^2t+176r^2st\text{-}44r^3st+44r^2s^2t-198rst)$$

$$D^{[1]}_{23} = \frac{-h^4s^7}{55440t^2(r-t)^2(s-t)^2(t-1)^2}(88r^2s^2-22r^2s^3+88rs^2-99r^2s-88rs^3+24rs^4+264r^2t+44s^2t-44s^3t+12s^4t-22s^3+24s^4-7s^5+176rs^2t-198r^2st-44rs^3t+44r^2s^2t-198rst)$$

$$D_{33}^{[1]} = \frac{-h^4t^4}{55440(r-t)^2(s-t)^2(t-1)^2}(99r^2s^2t^2 - 396r^2s^2t + 462r^2s^2 - 110r^2st^3 + 396r^2st^2 - 396r^2st + 33r^2t^4 - 110r^2t^3 + 99r^2t^2 - 110rs^2t^3 + 396rs^2t^2 - 396rs^2t + 132rst^4 - 440rst^3 + 396rst^2 - 42rt^5 + 132rt^4 - 110rt^3 + 33s^2t^4 - 110s^2t^3 + 99s^2t^2 - 42st^5 + 132st^4 - 110st^3 + 14t^6 - 42t^5 + 33t^4)$$

$$D_{43}^{[1]} = \frac{-h^4}{55440t^2(r-t)^2(s-t)^2(t-1)^2}(24r + 24s + 12t - 99r^2s^2 - 88rs - 44rt - 44st + 88rs^2 + 88r^2s + 44r^2t + 44s^2t - 22r^2 - 22s^2 - 198rs^2t - 198r^2st + 264r^2s^2t + 176rst - 7)$$

$$D_{53}^{[1]} = \frac{-h^3r^6}{27720t^2(r-t)^2(s-t)^2(t-1)^2}(198r^2s^2 - 55r^3s^2 - 198rs^2 + 198r^2s - 220r^3s + 66r^4s + 99r^2t - 110r^3t + 33r^4t + 462s^2t - 55r^3 + 66r^4 - 21r^5 - 396rs^2t + 396r^2st - 110r^3st + 99r^2s^2t - 396rst)$$

$$D_{63}^{[1]} = \frac{-h^3s^6}{27720t^2(r-t)^2(s-t)^2(t-1)^2}(198r^2s^2 - 55r^2s^3 + 198rs^2 - 198r^2s - 220rs^3 + 66rs^4 + 462r^2t + 99s^2t - 110s^3t + 33s^4t - 55s^3 + 66s^4 - 21s^5 + 396rs^2t - 396r^2st - 110rs^3t + 99r^2s^2t - 396rst)$$

$$D_{73}^{[1]} = \frac{-h^3t^3}{13860(r-t)^2(s-t)^2(t-1)^2}(132r^2s^2t^2 - 462r^2s^2t + 462r^2s^2 - 165r^2st^3 + 528r^2st^2 - 462r^2st + 55r^2t^4 - 165r^2t^3 + 132r^2t^2 - 165rs^2t^3 + 528rs^2t^2 - 462rs^2t + 220rst^4 - 660rst^3 + 528rst^2 - 77rt^5 + 220rt^4 - 165rt^3 + 55s^2t^4 - 165s^2t^3 + 132s^2t^2 - 77st^5 + 220st^4 - 165st^3 + 28t^6 - 77t^5 + 55t^4)$$

$$D_{83}^{[1]} = \frac{-h^3}{27720t^2(r-t)^2(s-t)^2(t-1)^2}(66r + 66s + 33t - 198r^2s^2 - 220rs - 110rt - 110st + 198rs^2 + 198r^2s + 99r^2t + 99s^2t - 55r^2 - 55s^2 - 396rs^2t - 396r^2st + 462r^2s^2t + 396rst - 21)$$

$$D_{93}^{[1]} = \frac{-h^2r^5}{2520t^2(r-t)^2(s-t)^2(t-1)^2}(48r^2s^2 - 15r^3s^2 - 42rs^2 + 48r^2s - 60r^3s + 20r^4s + 24r^2t - 30r^3t + 10r^4t + 84s^2t - 15r^3 + 20r^4 - 7r^5 - 84rs^2t + 96r^2st - 30r^3st + 24r^2s^2t - 84rst)$$

$$D_{103}^{[1]} = \frac{-h^2s^5}{2520t^2(r-t)^2(s-t)^2(t-1)^2}(48r^2s^2 - 15r^2s^3 + 48rs^2 - 42r^2s - 60rs^3 + 20rs^4 + 84r^2t + 24s^2t - 30s^3t + 10s^4t - 15s^3 + 20s^4 - 7s^5 + 96rs^2t - 84r^2st - 30rs^3t + 24r^2s^2t - 84rst)$$

$$D_{113}^{[1]} = \frac{-h^2t^2}{2520(r-t)^2(s-t)^2(t-1)^2}(84r^2s^2t^2 - 252r^2s^2t + 210r^2s^2 - 120r^2st^3 + 336r^2st^2 - 252r^2st + 45r^2t^4 - 120r^2t^3 + 84r^2t^2 - 120rs^2t^3 + 336rs^2t^2 - 252rs^2t + 180rst^4 - 480rst^3 + 336rst^2 - 70rt^5 + 180rt^4 - 120rt^3 + 45s^2t^4 - 120s^2t^3 + 84s^2t^2 - 70st^5 + 180st^4 - 120st^3 + 28t^6 - 70t^5 + 45t^4)$$

$$D_{123}^{[1]} = \frac{-h^2}{2520t^2(r-t)^2(s-t)^2(t-1)^2}(20r + 20s + 10t - 42r^2s^2 - 60rs - 30rt - 30st + 48rs^2 + 48r^2s + 24r^2t + 24s^2t - 15r^2 - 15s^2 - 84rs^2t - 84r^2st + 84r^2s^2t + 96rst - 7)$$

$$D_{14}^{[1]} = \frac{-h^4r^7}{55440(r-1)^2(s-1)^2(t-1)^2}(-7r^5 + 24r^4s + 24r^4t + 12r^4 - 22r^3s^2 - 88r^3st - 44r^3s - 22r^3t^2 - 44r^3t + 88r^2s^2t + 44r^2s^2 + 88r^2st^2 + 176r^2st + 44r^2t^2 - 99rs^2t^2 - 198rs^2t - 198rst^2 + 264s^2t^2)$$

$$D_{24}^{[1]} = \frac{-h^4 s^7}{55440(r-1)^2(s-1)^2(t-1)^2}(-22r^2s^3 + 88r^2s^2t + 44r^2s^2 - 99r^2st^2 - 198r^2st + 264r^2t^2 + 24rs^4 - 88rs^3t - 44rs^3 + 88rs^2t^2 + 176rs^2t - 198rst^2 - 7s^5 + 24s^4t + 12s^4 - 22s^3t^2 - 44s^3t + 44s^2t^2)$$

$$D_{34}^{[1]} = \frac{-h^4 t^7}{55440(r-1)^2(s-1)^2(t-1)^2}(-99r^2s^2t + 264r^2s^2 + 88r^2st^2 - 198r^2st - 22r^2t^3 + 44r^2t^2 + 88rs^2t^2 - 198rs^2t - 88rst^3 + 176rst^2 + 24rt^4 - 44rt^3 - 22s^2t^3 + 44s^2t^2 + 24st^4 - 44st^3 - 7t^5 + 12t^4)$$

$$D_{44}^{[1]} = \frac{-h^4}{55440(r-1)^2(s-1)^2(t-1)^2}(462r^2s^2t^2 - 396r^2s^2t + 99r^2s^2 - 396r^2st^2 + 396r^2st - 110r^2s + 99r^2t^2 - 110r^2t + 33r^2 - 396rs^2t^2 + 396rs^2t - 110rs^2 + 396rst^2 - 440rst + 132rs - 110rt^2 + 132rt - 42r + 99s^2t^2 - 110s^2t + 33s^2 - 110st^2 + 132st - 42s + 33t^2 - 42t + 14)$$

$$D_{54}^{[1]} = \frac{-h^3 r^6}{27720(r-1)^2(s-1)^2(t-1)^2}(-21r^5 + 66r^4s + 66r^4t + 33r^4 - 55r^3s^2 - 220r^3st - 110r^3s - 55r^3t^2 - 110r^3t + 198r^2s^2t + 99r^2s^2 + 198r^2st^2 + 396r^2st + 99r^2t^2 - 198rs^2t^2 - 396rs^2t - 396rst^2 + 462s^2t^2)$$

$$D_{64}^{[1]} = \frac{-h^3 s^6}{27720(r-1)^2(s-1)^2(t-1)^2}(-55r^2s^3 + 198r^2s^2t + 99r^2s^2 - 198r^2st^2 - 396r^2st + 462r^2t^2 + 66rs^4 - 220rs^3t - 110rs^3 + 198rs^2t^2 + 396rs^2t - 396rst^2 - 21s^5 + 66s^4t + 33s^4 - 55s^3t^2 - 110s^3t + 99s^2t^2)$$

$$D_{74}^{[1]} = \frac{-h^3 t^6}{27720(r-1)^2(s-1)^2(t-1)^2}(-198r^2s^2t + 462r^2s^2 + 198r^2st^2 - 396r^2st - 55r^2t^3 + 99r^2t^2 + 198rs^2t^2 - 396rs^2t - 220rst^3 + 396rst^2 + 66rt^4 - 110rt^3 - 55s^2t^3 + 99s^2t^2 + 66st^4 - 110st^3 - 21t^5 + 33t^4)$$

$$D_{84}^{[1]} \frac{-h^3}{13860(r-1)^2(s-1)^2(t-1)^2}(462r^2s^2t^2 - 462r^2s^2t + 132r^2s^2 - 462r^2st^2 + 528r^2st - 165r^2s + 132r^2t^2 - 165r^2t + 55r^2 - 462rs^2t^2 + 528rs^2t - 165rs^2 + 528rst^2 - 660rst + 220rs - 165rt^2 + 220rt - 77r + 132s^2t^2 - 165s^2t + 55s^2 - 165st^2 + 220st - 77s + 55t^2 - 77t + 28)$$

$$D_{94}^{[1]} = \frac{-h^2 r^5}{2520(r-1)^2(s-1)^2(t-1)^2}(-7r^5 + 20r^4s + 20r^4t + 10r^4 - 15r^3s^2 - 60r^3st - 30r^3s - 15r^3t^2 - 30r^3t + 48r^2s^2t + 24r^2s^2 + 48r^2st^2 + 96r^2st + 24r^2t^2 - 42rs^2t^2 - 84rs^2t - 84rst^2 + 84s^2t^2)$$

$$D_{104}^{[1]} = \frac{-h^2 s^5}{2520(r-1)^2(s-1)^2(t-1)^2}(-15r^2s^3 + 48r^2s^2t + 24r^2s^2 - 42r^2st^2 - 84r^2st + 84r^2t^2 + 20rs^4 - 60rs^3t - 30rs^3 + 48rs^2t^2 + 96rs^2t - 84rst^2 - 7s^5 + 20s^4t + 10s^4 - 15s^3t^2 - 30s^3t + 24s^2t^2)$$

$$D_{114}^{[1]} = \frac{-h^2 t^5}{2520(r-1)^2(s-1)^2(t-1)^2}(-42r^2s^2t + 84r^2s^2 + 48r^2st^2 - 84r^2st - 15r^2t^3 + 24r^2t^2 + 48rs^2t^2 - 84rs^2t - 60rst^3 + 96rst^2 + 20rt^4 - 30rt^3 - 15s^2t^3 + 24s^2t^2 + 20st^4 - 30st^3 - 7t^5 + 10t^4)$$

$$D_{124}^{[1]} = \frac{-h^2}{2520(r-1)^2(s-1)^2(t-1)^2}(210r^2s^2t^2 - 252r^2s^2t + 84r^2s^2 - 252r^2st^2 + 336r^2st - 120r^2s + 84r^2t^2 - 120r^2t + 45r^2 - 252rs^2t^2 + 336rs^2t - 120rs^2 + 336rst^2 - 480rst + 180rs - 120rt^2 + 180rt - 70r + 84s^2t^2 - 120s^2t + 45s^2 - 120st^2 + 180st - 70s + 45t^2 - 70t + 28)$$

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