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Mathematical Study for the Synovial Fluid Flow in Osteoarthritic Knee Joint

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Key words: Synovial fluid, osteoarthritis, fluid flux, hyaluronic acid, synovial cavity and articular cartilage

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Abstract: This article presents a study of synovial fluid flow in the joint cavity and articular cartilage of a diseased human knee joint. The synovial fluid flows due to the pressure gradient and squeezing action and articular cartilages, which are considered as porous materials. In the present paper, the flow of synovial fluid has been addressed primarily by investigating synovial fluid flux in the joint cavity and cartilage. The computational results for pressure gradient-induced synovial fluid flow in the human knee joint have obtained. The results in the form of graphs have shown to comprehend the flow mechanism and the influence of model parameters on fluid flow have investigated. It has also shown that how permeability, viscosity and other critical characteristics affect the synovial fluid flow. This study can assist in gaining essential and substantial insight into the identification of components that may be involved in the development of medical disorders in the human knee joint.

INTRODUCTION

Synovial joints are the most critical aspect of the human body since they are the focal points of the most fundamental activity in humans, which is movement. A synovial joint Fig. 1 can be thought of as a load-bearing system made up of two mated bones that move in tangential and normal directions. The ends of the bones, which are typically spherical in shape, are coated with a soft sponge-like substance called articular cartilage. The joint cavity, the space between these cartilaginous extremities of the bones, is filled with a shear-dependent fluid termed synovial fluid. The synovial joint's behavior is mainly determined by the articular cartilage and synovial fluid properties. The articular cartilage is a smooth, glistening tissue[1-3] Synovial fluid is a thin transparent fluid with a similar composition to plasma. The synovial fluid consists of hyaluronic acid, lubricin and other vital constituents and it also contains some part of glycoproteins and proteinase. Synovial fluid works as a lubricating agent in synovial joints[4]. Lubricin is an essential protein type responsible for the lubricating properties of the synovial fluid[5]. Synovial fluid also provides nutrition to the cells of articular cartilage. The human knee joint is one of the most critical synovial joints in the human body and it is responsible for several activities of humans. The human knee joint consists of three main components, synovial fluid, articular cartilage and synovial membrane. The knee joint is considered to be divided geometrically by cylindrical surfaces. The knee joint is constructed in such a way that it can withstand a load four times the body weight[6]. Synovial fluid Fig. 2 behaves in a non-Newtonian and pseudo-plastic manner. Its viscosity also depends on the concentration of the HA molecules. Normal, healthy synovial fluid is a viscous fluid present in low content in

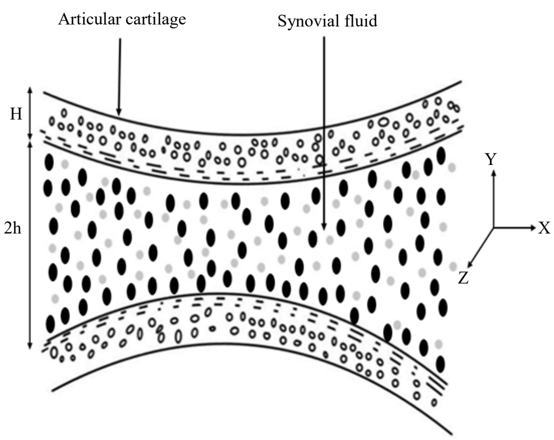

Fig. 1: Geometry of the synovial joint

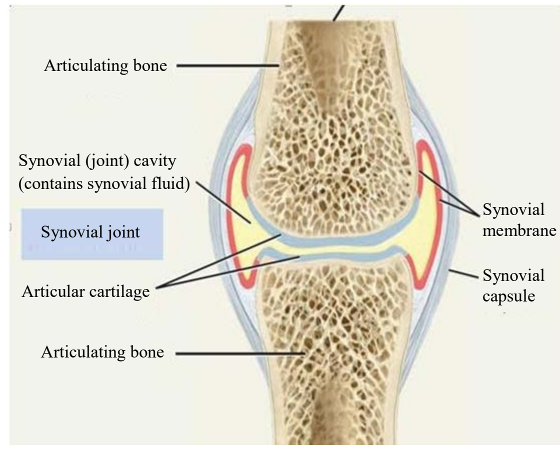

Fig. 2: Synovial Joint

all synovial joints with non-Newtonian behavior. However, when a joint is wounded or infected, the amount of fluid in the joint may increase. As with any viscous material, synovial fluid resists shear stresses. The synovial fluid's viscosity decreases inversely with the joint velocity or rate of shear. That is, at high rates of shear, it becomes less viscous[7]. Under pathological conditions, the synovial fluid loses its non-Newtonian behavior and becomes Newtonian[8]. When bones come together during motion, water and other low molecular weight molecules present in synovial fluid flow through the pores of the articular cartilage, but the HA molecules remain, increasing the concentration of these molecules in the synovial fluid and hence its viscosity. Osteoarthritis is a degenerative joint disease that affects a wide range of species. It is characterized by increasing loss of joint function, discomfort, cartilage breakdown, subchondral bone remodeling, capsule thickening and increased synovial effusion. Osteoarthritis (OA) is the most prevalent kind of joint disease in humans, horses and dogs. While the pathogenesis of OA varies between species and between people within a species, several aspects of the disease's pathophysiology are similar[9]. The surface of the cartilage can become rough and broken in a degenerative joint and the synovial fluid might lose its non-Newtonian action[10,11]. The pores of cartilage may enlarge, allowing hyaluronic acid molecules to flow through. The permeability of the articular cartilage is a significant factor for the identification of degenerative disease in the synovial joints[12]. The primary goal of mathematical modeling of joints is to enable us to determine the contribution of each of these aspects to the joint's functioning. Synovial fluid is often explored for diagnostic and research reasons since it is considered to represent the inflammatory condition in the synovial joint. Synovial fluid is often viscous and non-homogeneous because of its complex nature, particularly the existence of HA. Recently, Brouwers et al. studied that synovial fluid must be treated with hyaluronidase for accurate identification of inflammatory cells and soluble mediators[13]. Hydration lubrication has developed as a new perspective for lubrication in aquatic and biological environments, accounting for the exceptionally low friction associated with joint articular cartilage lubrication. Lin *et al*. studied the articular cartilage boundary lubrication by HA molecules. Cartilage's structure and mechanical properties are closely linked to the tribological properties of cartilage. The types of lubrication in cartilage go beyond the traditional fluid film or boundary lubrication methods. Cartilage is a pale connective tissue that is made and kept up by cells called chondrocytes. It is quite hydrated and has a porous. As we get older, the thickness of the articular cartilage layer changes. When situations are normal, articular cartilage is very smooth and wears very little over a long period of time[14]. Biomedical engineering develops novel medical systems with the objective of enhancing the quality of life, particularly for artificial synovial fluid and artificial joints. However, more advancements in the field of joint replacement and the formation of artificial synovial fluid are required to evaluate possible novel treatment options. Without a doubt, one of the most difficult issues is the biochemical and biomechanical changes in the synovial fluid and the wear and degeneration of articular cartilage produced by the sliding surfaces rubbing against one another. This deterioration may increase if the synovial fluid loses its properties and the natural joint's lubricating characteristics are disrupted.

The current article examines the impact of viscosity of the synovial fluid, permeability of articular cartilage and other properties of the synovial joint on synovial fluid flow using Navier Stokes and Brinkman equations. This investigation believed to aid in understanding the process of synovial fluid flow, better understanding the pathological conditions of the synovial joints and the involvement of long-chain hyaluronic acid molecules in synovial fluids.

MATERIALS AND METHODS

The flow of synovial fluid in a diseased human knee joint is addressed in the current study. In the present proposed problem, we have considered the flow of synovial fluid in human knee joints due to the pressure gradient. The articular cartilage has been considered a porous substance. The articular cartilage length is substantially greater than the breadth, resulting in a human knee joint shape comparable to the parallel plate. The human knee joint can be split into the fluid film and the articular cartilage regions. The symmetric case of the human knee joint has been used to calculate the velocity in both regions. The flow of the synovial fluid in the joint cavity and articular cartilages is governed by the Navier-Stokes Equations and Brinkman equation, respectively[15]. We have assumed synovial fluid to be Newtonian, incompressible, non-conducting and non-magnetic and the flow of synovial fluid to be steady and laminar. Thus, the fluctuation of any synovial fluid parameter with time is zero. Furthermore, we assumed that the pressure fluctuation throughout the fluid layer thickness is zero and there are no external forces present. Some external forces are always there, but their influence is insignificant and no-slip boundary conditions are assumed at the cartilage surfaces. By making all of these assumptions, Brinkman's equation and the Navier-Stokes equation are reduced to the following equations, which regulate the flow of synovial fluid in the diseased human knee joint:

$$\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial p}{\partial x}=0 \quad (1)$$

$$\mu'\frac{\partial^2 v}{\partial y^2}-\frac{\mu}{\varphi}v-\frac{\partial p}{\partial x}=0 \quad (2)$$

Equation 1 is the reduced form of the Navier-Stokes equation by applying our assumptions and Eq. 2 is the Brinkman equation that is an extension of Darcy's law for fluid flow through the porous medium.

Where, u is the velocity of the synovial fluid across the fluid film present in the joint cavity and is the velocity of synovial fluid in articular cartilage. The symbols μ, μ' and φ denote the viscosity, apparent viscosity of the synovial fluid and the permeability of articular cartilage. The boundary conditions for the synovial fluid flowing through the fluid film present in the synovial cavity and articular cartilage are given by:

$$\frac{\partial u}{\partial y}=0 \text{ at } y=0 \quad (3)$$

$$v=0 \text{ at } y=h+H \quad (4)$$

The no-slip boundary condition is responsible for the boundary condition given in Eq. 4. Since the articular cartilage is stationary, therefore the velocity of the synovial fluid at y = h+H must be equal to zero. The synovial fluid flowing along porous articular cartilage will experience shear stress at that point. Due to the no-slip condition, the synovial fluid velocity at the interface must be equal to the velocity of synovial fluid in articular cartilage. The shear stress in synovial fluid due to its Newtonian and laminar behavior is proportional to the strain rate in the fluid, where the viscosity is the proportionality constant. Because shear stresses at the boundary interaction are the same, we get the following matching condition for shear stresses:

$$\mu\frac{\partial^2 u}{\partial y^2}=\mu'\frac{\partial^2 v}{\partial y^2} \text{ at } y=h \quad (5)$$

Given our assumption of the no-slip boundary condition at interface, the velocity of synovial fluid in the fluid film and articular cartilage at the interface, where synovial fluid film zone and articular cartilage meet each other, must be the same. As a result, the matching conditions for velocities at the interface are as follows:

$$u=v=V \text{ at } y=h \quad (6)$$

where, v is the velocity of the synovial fluid at the interface, where articular cartilage and fluid film meets.

Now solving Eq. 1 with the help of boundary condition given in Eq. 3 and matching condition given in Eq. 6, we obtained the expression for the velocity of the synovial fluid in the fluid film, which is given by:

$$u=\frac{1}{\mu}\frac{\partial p}{\partial x}\left(y^2-h^2\right)+V \quad (7)$$

Now we solve the Eq. 2 using the boundary condition given by Eq. 4 and matching condition provided by Eq. 6, we get the velocity of the synovial fluid in articular cartilage, which is given by the expression as follows:

$$v=\left[\left(\frac{\varphi}{\mu}\frac{\partial p}{\partial x}\frac{1}{\sinh\sinh(MH)}\right)\sinh M(h+H-y)-\sinh M(h-y)-\sinh MH)\right]+\left[\frac{V\sinh M(h+H-y)}{\sinh MH)}\right] \quad (8)$$

Using the matching condition for stresses at the interface given in Eq. 5, we obtain the velocity of the synovial fluid at the interface. Therefore, the expression for the velocity of the synovial fluid at the interface is given as follows:

$$V=\frac{\varphi}{\mu}\frac{\partial P}{\partial x}\tanh(MH)\left(\tanh\frac{MH}{2}-Mh\right) \tag{9}$$

Therefore the final expression for the velocities of the synovial fluid in fluid film and articular cartilage are as follows:

$$u=\frac{1}{\mu}\frac{\partial p}{\partial x}\left(y^2-h^2\right)+\left[\frac{\varphi}{\mu}\frac{\partial P}{\partial x}\tanh(MH)\left(\tanh\frac{MH}{2}-Mh\right)\right] \tag{10}$$

$$v=\left[\left(\frac{\varphi}{\mu}\frac{\partial p}{\partial x}\frac{1}{\sinh\sinh(MH)}\right)\sinh M(h+H-y)-\sinh M(h-y)-\sinh MH)\right]+\left[\frac{V\sinh M(h+H-y)}{\sinh MH)}\right] \tag{11}$$

At any instant, the synovial fluid flux across a cartilage and fluid film is the amount of fluid that passes through the cartilage and fluid film in a flow per unit time. It is obtained by integrating the velocity across the cartilage and fluid film. Therefore, as a result, synovial fluid flux is determined by integrating velocity with respect to from zero to a final value The synovial fluid flux is calculated as follows:

$$Q=\int_0^h u\,dy+\int_h^{h+H} v\,dy \tag{12}$$

We acquire the final expression for the synovial fluid flow in articular cartilage and synovial fluid film existing in the joint cavity by integrating the values of u and v along with film thickness and cartilage thickness obtained from the Eq. 10 and 11. The expression for the synovial fluid flux is as follows:

$$Q=-\frac{\partial P}{\partial x}\left[\frac{1}{M^3\mu'}\left(\tanh MH\left(Mh-\tanh\frac{MH}{2}\right)^2+MH+2\tanh\frac{MH}{2}\right)+\frac{h^3}{3\mu}\right] \tag{13}$$

RESULTS AND DISCUSSION

We have obtained the computational results using the proposed model for synovial fluid flow in the joint cavity and articular cartilage by applying the suitable physiological parameters. The Navier-Stokes and Brinkman equation for the flow of synovial fluid in the synovial cavity and articular cartilage have considered for this research work. We have shown the obtained results and the effect of some significant parameters on flow through graphs. The variation of flux with permeability, film thickness and cartilage thickness for different parameters have been shown in the graphs. We have used MATLAB to obtain the numerical data for viscosity, flow rate, articular cartilage permeability, fluid film thickness and synovial fluid flux. In order to produce the varied synovial fluid flow patterns, all parameter values were obtained from the study literature[1,16,17]. It is critical to establish the configuration of the flux profile because the flux profile may provide a complete description of the flow field. Figure 3-4 shows the flux variation with permeability and film thickness and Fig. 5 shows the flux variation with cartilage thickness for different parameters.

In Figure 3, we have shown the variation of the flux with permeability and the synovial fluid flux decreases with permeability generally for every parameter. By observing the sub figures of the same figure, it is noticed that the fluid flux increases for viscosity, pressure gradient and film thickness while decreases for apparent viscosity. The synovial fluid flux is directly proportion to the viscosity of the synovial fluid, the thickness of the fluid film and the pressure gradient in the fluid film. It is inversely proportional to the apparent viscosity of the synovial fluid. Therefore, it could be concluded that the synovial fluid flux increases when the pressure gradient in fluid film, the viscosity of the synovial fluid and the thickness of the fluid film increases. The synovial fluid flux decreases when the apparent viscosity of the synovial fluid increases.

Figure 4 shows the variation of flux with the fluid film thickness present in the joint cavity for different values of viscosity, permeability, apparent viscosity and pressure gradient. The flux increases with the fluid film thickness in general for every parameter. The sub figures of Fig. 4 show that the flux increases with permeability and pressure gradient and it decreases with viscosity and apparent viscosity. In this case, the synovial fluid flux is directly proportional to the articular cartilage permeability and the fluid film thickness. At the same time, the synovial fluid flux is inversely proportional to the viscosity and apparent viscosity of the synovial fluid. Based on these observations and variation profiles of synovial fluid flux shown in the figures, it may be concluded that the synovial fluid flux increases when the pressure gradient in fluid film and the permeability of the articular cartilage increases. The synovial fluid flux decreases with an increase in the viscosity and apparent viscosity of the synovial fluid. In Fig. 5 we have shown the variation of flux with the thickness of articular cartilage for different values of viscosity of the synovial fluid. No changes can be seen in flux with cartilage thickness for other parameters. The only parameter that affects the flux along the articular cartilage thickness is the viscosity of the synovial fluid. The flux is linearly related tothe articular cartilage thickness and flux is

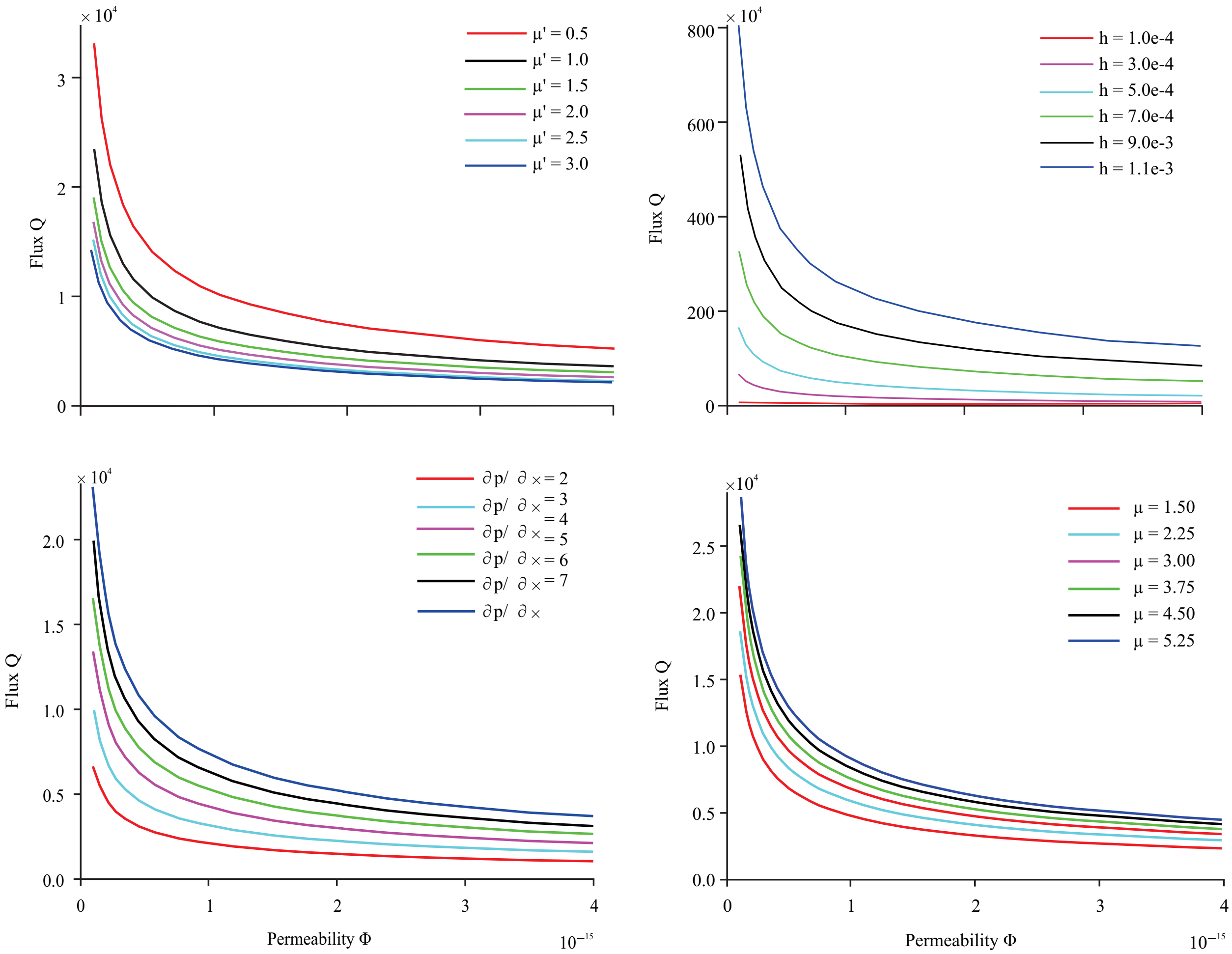

Fig. 3: Variation of flux with permeability for viscosity, apparent viscosity, pressure gradient and film thickness

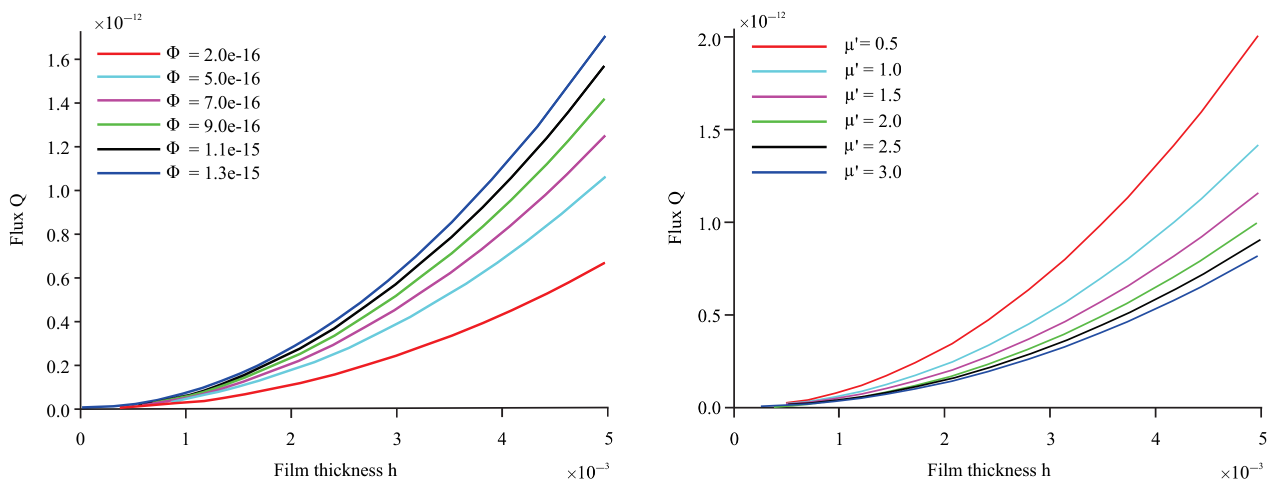

Fig. 4: Continued

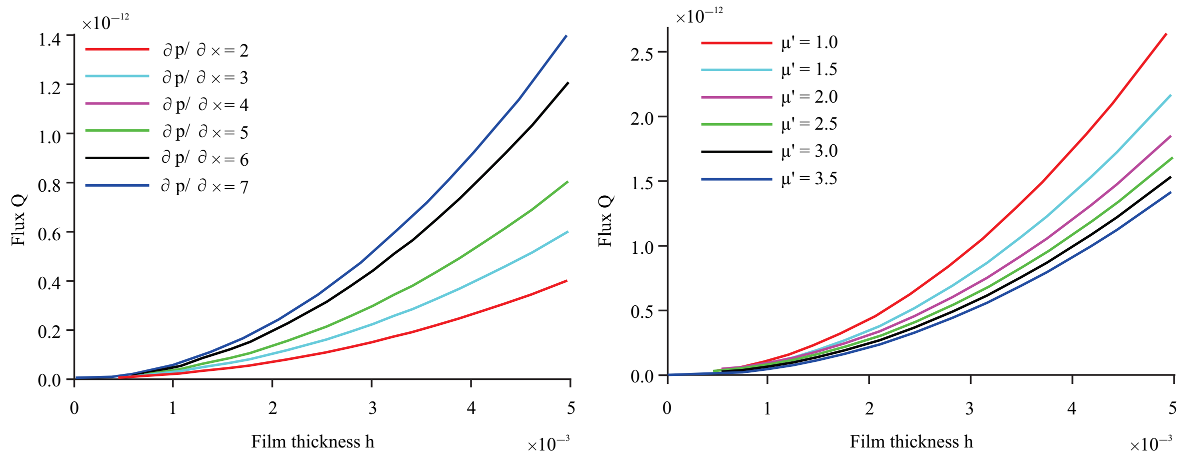

Fig. 4: Variation of flux with film thickness for viscosity, apparent viscosity, pressure gradient and permeability

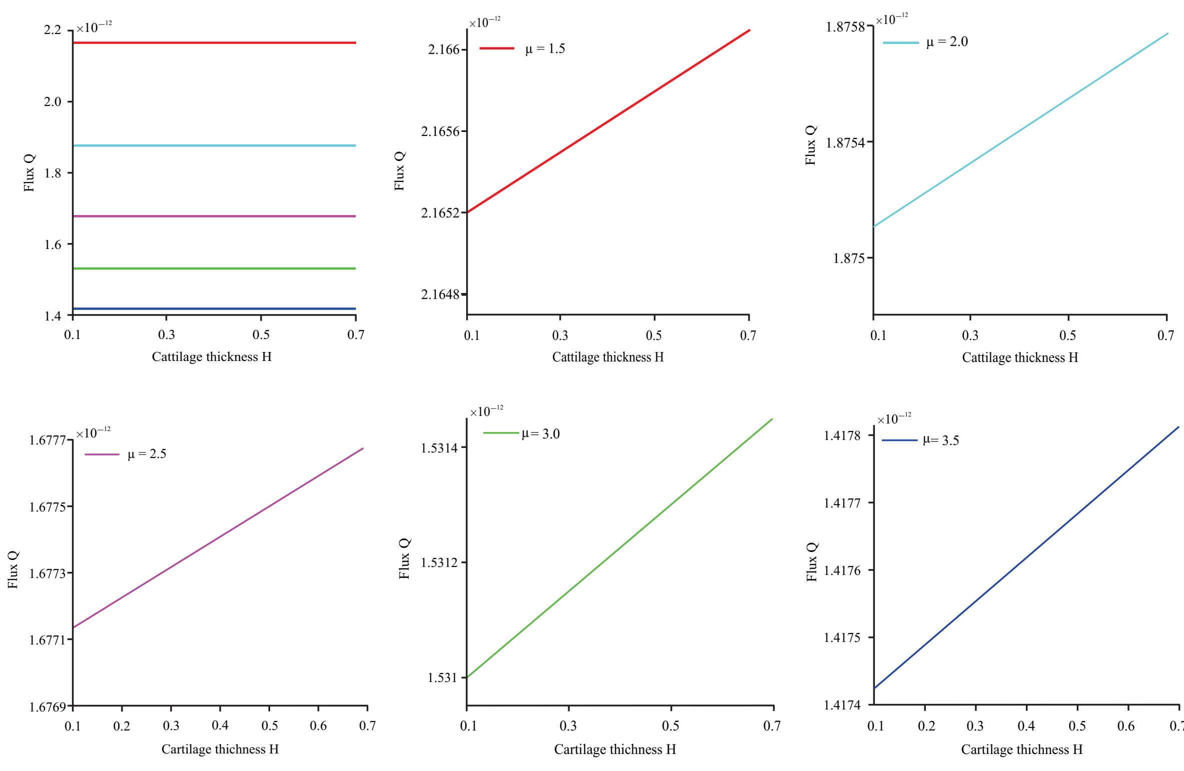

Fig. 5:Variation of flux with cartilage thickness for viscosity

more when the viscosity of the synovial fluid is less. When the viscosity of the synovial fluid is less, than the synovial flux is more in articular cartilages and the synovial fluid flux increases with the viscosity of the synovial fluid. Therefore, it could be concluded that the synovial fluid flux increases in articular cartilage when the viscosity of the synovial fluid increases.

CONCLUSION

The main objective of this work was to construct a mathematical model to explain the flow of synovial fluid in the human knee joint in diseased conditions. The computational results for synovial fluid flow in the human knee joint generated due to pressure gradient have obtained. The effect of viscosity, permeability, pressure gradient, apparent viscosity and film thickness on the flow of synovial fluid in the pathological human knee joint have shown. The flow of synovial fluid has investigated predominantly by analyzing synovial fluid flux in the synovial cavity and articular cartilage. The acquired data have shown in the form of plots for the better understand the flow mechanism and the effect of model parameters on fluid flow have explored. The synovial fluid flux increases when the thickness of the synovial fluid film between the articular cartilages increases and it decreases when the permeability of the articular cartilage increases. It indicates that when the synovial fluid's viscosity decreases in diseased conditions, the synovial fluid flows more freely during the squeezing process and a large amount of synovial fluid enters into the articular cartilage because of decrease viscosity. Due to this, the amount of synovial fluid that remained in the cavity becomes much less in amount. The synovial knee joint becomes less lubricated prone to damage the articular cartilage by rubbing each other. The permeability of articular cartilage also decreases in diseased conditions. It has also observed that the flow of synovial fluid decreases with an increase in the permeability of articular cartilage. This study can help to get an essential and significant insight concerning the identification of the factors that might be involved in building up the medical issues in the human knee joint. This study may also enrich our present understanding of the role of the flow of synovial fluid in the development of joint disease. Moreover, this type of study may also help to alleviate the designation of therapeutic methods to understand the pathological states of the human knee joint.

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