

Mathematical Model of Convective Mass Transfer Between Flows of Finely Dispersed Food Media in Adjacent Channels with Permeable Walls

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Abstract: The discussion is concerned with a mathematical model for convective mass transfer between the flows of finely dispersed food media moving in adjacent channels separated by a permeable wall where portions of the fluid phases are exchanged many times between the flows. Numerical solutions are given for a countercurrent flow of a food suspension and a liquid. Equations are derived and curves constructed to show the distribution of the flow velocity and the suspension porosity along the length of the channels as well as the dependence on time of the temperatures of the flows.

Key words: Food dispersion, mass transfer, multiphase flow, food suspension, transport processes, filtration, convective transport, permeable channel, adjacent conduits, mass exchanger

Introduction

Interaction of different phases in food technological apparatus commonly yields hydro- or aerodispersed heterogeneous systems composed of a continuous and a dispersed phases. The output capacity of a unit volume of the reaction space depends on parameters of the intercomponent interaction, interfacial contact area per unit volume, and rates of transfer within phases and exchange between phases. The interfacial area per unit volume is determined by the concentration and dispersity of phases; and the rates of transfer within phases and between phases are governed by, among other things, the particle size in the dispersed phase and hydrodynamics of the flow of the continuous phase about the particles. All other conditions being the same, the smaller the particle size and the higher the velocity of the flow about them, the higher the intensity of a process. However, the known scheme of food technological processes – counterflow of the dispersed and continuous phases – has limitations imposed by the laws of particle motion in a medium. The maximum achievable velocity of flow about particles in counterflow of phases depends on the particle size, difference in density between the particles and the medium, and viscosity of the medium, being small for fine particles of the dispersed phase. At high flow rate of the continuous phase the dispersed phase is carried away from the reaction space and the counterflow of phases is disrupted. This is the reason why, e.g., washing or extraction of a component from a finely dispersed solid phase is commonly carried out by mixing it with an extracting agent and subsequent multistage counterflow mixing – separation of solid and liquid phases in apparatus comprising several mixers, decanters, pumps, and transporters. The essence of the previously proposed methods for carrying out extraction in the solid-liquid system (Aliev and Aliev, 1977), chemical reactions yielding a dispersed solid phase (Aliev, 1987 and Aliev, 1996), sorption and ion exchange in the solid-liquid system (Aliev, 1996 and Aliev, 1996), and heat exchange between media (Aliev, 1984 and Aliev and Aliev, 1986) consists in the use of convective mass and heat exchange between dispersed flows separated by a permeable partition (permeable to continuous, and impermeable to dispersed phases in the flows). Convective mass exchange occurs between the flow of a food suspension of the dispersed solid material and flow of an extracting agent in the course of extraction. In carrying out chemical reactions convective mass exchange occurs between the flow of a sediment suspension, catalyst, or reagent and the flow of a medium to be treated. In performing sorption or ion exchange convective mass exchange occurs between the flow of suspension of sorbent or ion exchanger and flow of a medium to be treated. In the proposed technique, convective heat exchange is carried out between dispersed flows through a permeable partition without mixing of their dispersed phases. The convective mass exchange between food dispersed flows is carried out with a combination of extraction and sorption, when suspension of sorbent or ion exchanger is used as extracting agent. This convective mass exchange is also carried out in performing chemical reactions, sorption or ion exchange, when hydro- or aerodispersed media are the medium to be treated.

For example, it is proposed to carry out extraction as follows (Fig. 1). The solid phase containing a soluble component is mixed in reactor *R* with a part of extracting agent to form a suspension. During the time of suspension residence in reactor *R*, the component is extracted from dispersed particles into the liquid phase. We assumed that volume of particles of a solid phase does not vary at extraction of a component from particles of a solid phase into a liquid phase. Further, continuous counterflow convective mass exchange between the flow of this food suspension and the flow of the main part of the extracting agent is carried out through a permeable

partition in a valve-type pulsating-flow mass exchanger. By the convective mass exchange between flows through a permeable partition is meant, in the given case, exchange of components dissolved in liquid phase without mixing of the dispersed phases of the flows. This process replaces several stages of phase mixing – separation. Component-depleted washed suspension is obtained at the outlet of channel 2 of mass exchanger, and component-enriched extract, at the outlet of channel 1. Compared with multiple mixing – separation of phases, the proposed process has technological advantages: possibility of working with finely dispersed kinetically favorable phases at high counterflow velocities, smaller number of technological procedures and employed equipment units, compactness, and high specific volume output capacity.

The outcome of the process is defined by the achieved degree of component extraction from particles of the solid phase into the liquid continuous phase of food suspension in reactor *R* and also by the degree of convective transfer of the component from the food suspension flow in channel 2 into the flow of a liquid extracting agent in channel 1 of mass exchanger. The processes of phase interaction in stirred displacement reactor have been studied in sufficient detail.

The task to be accomplished consists in establishing the fundamental aspects of convective mass exchange between dispersed flows, in developing calculation procedures, and in designing efficient mass exchangers. Previously, a procedure for calculating the mass-transfer coefficient and a mathematical model of mass-transfer in a shell-and-tube extractor of the "tube-in-tube" type with permeable inner tube and piston pulsator have been reported (Aliev et al., 1980 and Aliev et al., 1981). The present communication is concerned with the case when the exchange between flows separated by a permeable partition in mass exchanger is achieved by creating sign-alternating pressure differential between the flows with the use of more advanced, computer-controlled valve-pulsatory system.

We propose methods for mass and heat transfer between flows of dispersed food media through a convectively permeable surface separating them (which is permeable for fluid phases of flows) (Aliev and Aliev, 1977; Aliev, 1987; Aliev, 1996; Aliev, 1996; Aliev, 1996; Aliev, 1984 and Aliev and Aliev, 1986); they are an alternative to mass transfer processes in heterogeneous liquid- and gas-dispersion systems (Aliev and Aliev, 1977; Aliev, 1987; Aliev, 1996; Aliev, 1996; Aliev, 1996; Aliev, 1984 and Aliev and Aliev, 1986). These methods are conducted by initiating the repeated exchange of portions of fluid (liquid or gaseous) phases between the flows. Our paper is concerned with the case when the initiated mass exchange is created by alternating differences of the pressure between the flows.

The description of a valve-type pulsating-flow mass exchanger with a convectively permeable surface: Mass transfer between the flows of dispersed food media is accomplished, for example, in the valve-type pulsating-flow mass exchanger shown in Fig. 2. The mass exchanger has two adjacent channels 1 and 2 separated by a partition 3 that is permeable for fluid phases of the flows (Aliev and Aliev, 1977; Aliev, 1987; Aliev, 1996; Aliev, 1996; Aliev, 1996; Aliev, 1984 and Aliev and Aliev, 1986). The operation of the mass exchanger is based on two time steps of duration t_s alternating successively in time, depending on the state of valves 4–7. During the first time step, valves 4 and 7 are open and valves 5 and 6 are closed. In this case, the first dispersed food medium with an initial concentration of $c_1(0,t) = c_{1in}$ is conveyed by means of pump 9 through valve 4 to channel 1 and is filtered through the permeable partition creating a flow in channel 2, wherefrom it is discharged at a concentration $c_2(0,t) = c_{2out}$ through open valve 7. During the second time step, valves 4 and 7 are closed and valves 5 and 6 are open. In this case, the second dispersed food medium with an initial concentration of $c_2(l,t) = c_{2in}$ is conveyed by means of pump 10 through valve 6 to channel 2 and filtered through partition 3 creating a flow in channel 1, wherefrom it is discharged at the outlet concentration $c_1(l,t) = c_{1out}$ through open valve 5. Here, c is the concentration in the fluid phase, t is time, and l is the channel length.

Thus, an alternating pressure difference is set up between the channels. With such a pulsating movement of the first and the second dispersed food media in the mass exchanger, portions of their fluid phases are exchanged many times without mixing up with their solid dispersed phases. In the process, countercurrent convective mass transfer takes place between the flows of the first and the second dispersed food media. The result is that the concentration in the food medium c_{2out} at the outlet from the second channel approaches the concentration c_{1in} in the initial food medium, and c_{1out} of the first food medium discharged from the first channel approaches the concentration c_{2in} in the food medium supplied to the inlet of the second channel.

For convective mass transfer between the dispersed food media, this method does not set any limits on the relative velocities of their countercurrent flow, on the differences in the densities of their phases (fluid and solid), and on the solid particle sizes, as distinct from known methods of mass transfer through countercurrent flow of solid particles and the liquid (the gas). The proposed mass transfer of the flows through a convectively permeable surface is much more intensive than through a nonpermeable wall.

The mathematical model: Equations for continuous incompressible physically heterogeneous Newton media are used

to quantitatively describe the processes of mass transfer in dispersed flows (Aliev *et al.*, 1999; Aliev *et al.*, 2001; Aliev *et al.*, 1998; Aliev *et al.*, 2002; Aliev *et al.*, 2001 and Aliev and Aliev, 1997). This description may be used for finely dispersed food systems with a moderate volume fraction (up to 0.25) of the solid dispersed phase (Aliev *et al.*, 1998; Aliev *et al.*, 2002; Aliev *et al.*, 2001 and Aliev and Aliev, 1997). The local velocities of the solid and the fluid phases may be assumed here to be roughly equal, especially when the solid and the fluid phases only slightly differ in their densities (Aliev *et al.*, 1998 and Aliev *et al.*, 2002).

We also believe that within the mass exchanger, the transfer of mass between the solid and the fluid phases and the molecular diffusion are negligible compared with the convective mass transfer in the flows. This has been observed at $Bi = \beta_{SF} \cdot d_S / D_S \gg 1$ and $Pe = u_0 \cdot l / D_F \gg 1$, where β_{SF} is the coefficient of mass transfer between

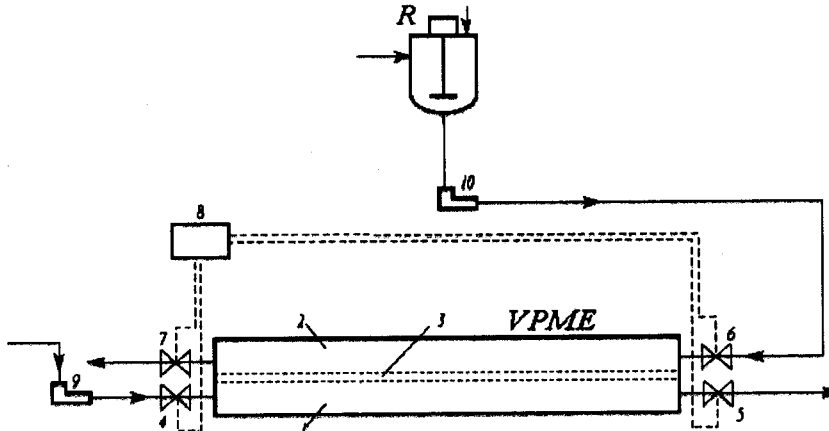


Fig. 1: Schematic of the unit. *R* is reactor; *VPME*, valve-type pulsating-flow mass exchanger; (1, 2) *VPME* channels; (3) permeable partition; (4–7) valves; (8) valve control unit; and (9, 10) pumps.

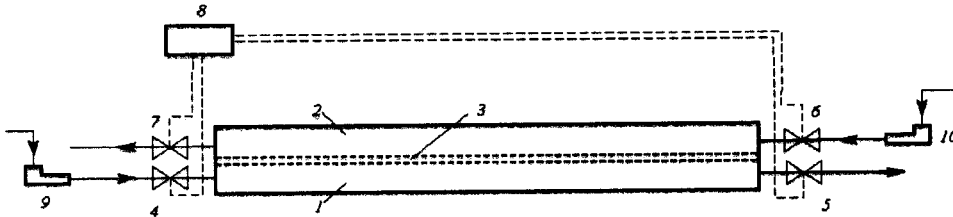


Fig. 2: Schematic diagram of a valve-type pulsating-flow mass exchanger: (1, 2) channels, (3) permeable partition, (4–7) valves, (8) valve control unit, (9, 10) pumps.

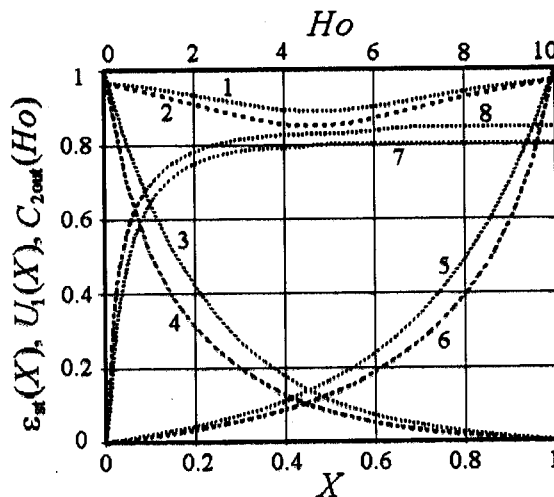


Fig. 3 :Curves showing the mean porosity $\varepsilon_{st}(X)$ (1, 2) and the axial velocity $U_1(X)$ during the first and second time steps (3, 4 and 5, 6, respectively), and $C_{2out}(Ho)$ (7, 8) at $Sr = 0.05$, $l/d_{eq} = 500$, $\varepsilon_{in} = 0.97$, $R_{eff} = 10^6$, $R_{eff2}/R_{eff1} = 1.5$, $\Delta_w = 0.01$, $\rho_s/\rho_f = 1$, and different values of Re_0 : 1, 3, 5, 7 – 1000; 2, 4, 6, 8 – 10000.

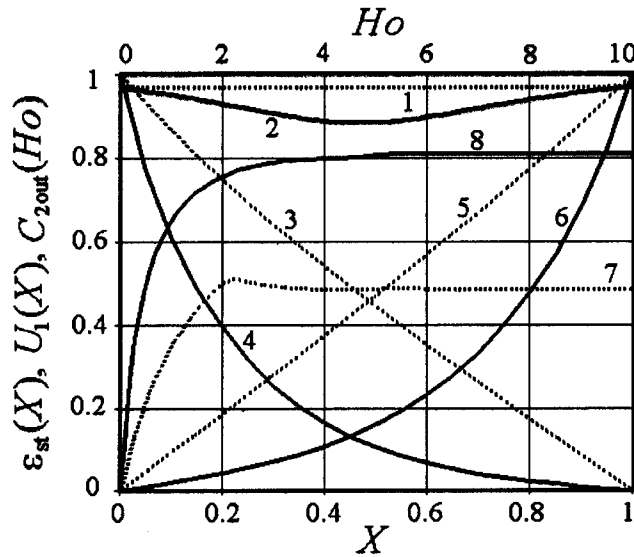


Fig. 4: Curves showing the mean porosity $E_{st}(X)$ (1, 2) and the axial velocity $U_1(X)$ during the first and second time steps (3, 4 and 5, 6, respectively), and $C_{2out}(Ho)$ (7, 8) at $Re_0 = 5000$, $E_{in} = 0.97$, $Sr = 0.05$, $R_{eff} = 10^6$, $R_{eff2}/R_{eff} = 1.5$, $\Delta_w = 0.01$, $\rho_s/\rho_f = 1$, and different values of l/d_{aq} : 1, 3, 5, 7 - 100; 2, 4, 6, 8 - 500.

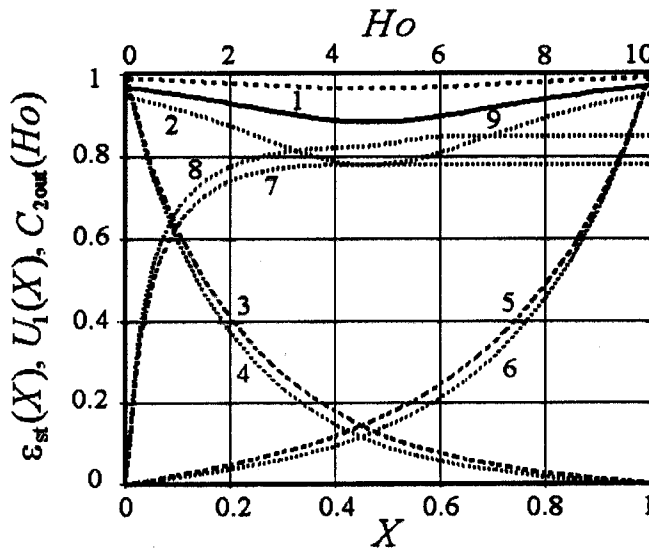


Fig. 5: Curves showing $E_{st}(X)$ (1, 2, 9), $U_1(X)$ during the first and second time steps (3, 4 and 5, 6, respectively), and $C_{2out}(Ho)$ (7, 8) at $Re_0 = 5000$, $l/d_{aq} = 500$, $Sr = 0.05$, $R_{eff} = 10^6$, $R_{eff2}/R_{eff} = 1.5$, $\Delta_w = 0.01$, $\rho_s/\rho_f = 1$, and different values of E_{in} : 1, 3, 5, 7 - 0.99; 4, 6, 8, 9 - 0.95; 2 - 0.97.

dispersed particle of the solid phase and fluid phase, d_s is the typical size of the dispersed particle of the solid phase, and D_s is the diffusion coefficient of the solid phase; u_0 is the mean flow velocity over the sectional area at the inlet to the first channel; D_f is the diffusion coefficient of the fluid phase, respectively.

Accounting for the assumptions made, the set of equations of incompressibility, suspension flow, mass balance for the liquid phase and convective mass transfer in the liquid-phase flow (Aliev *et al.*, 2001) can be written as follows:

$$\oint_S \mathbf{w} \cdot d\mathbf{s} = 0$$

1.

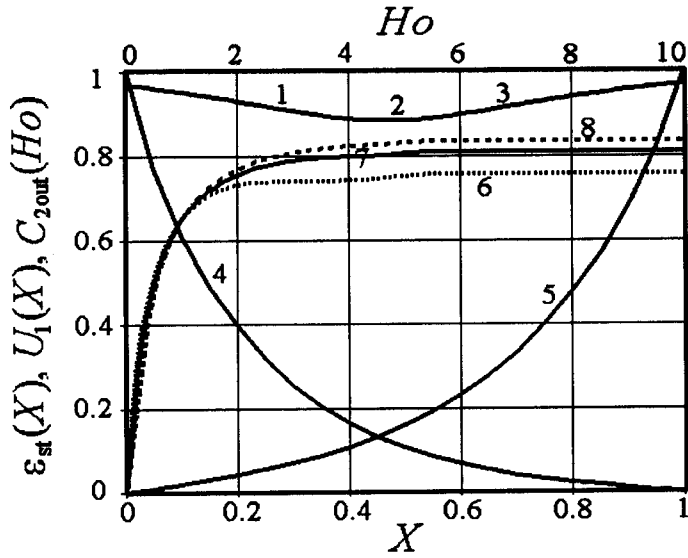


Fig. 6: Curves showing $E_{st}(X)$ (1, 2, 3), $U_1(X)$ during the first and second time steps (4 and 5, respectively) and $C_{2out}(Ho)$ (6, 7, 8) at $Re_0 = 5000$, $l/d_{eq} = 500$, $E_{in} = 0.97$, $R_{eff} = 10^6$, $R_{eff2}/R_{eff} = 1.5$, $\Delta_w = 0.01$, $\rho_S/\rho_F = 1$, and different values of Sr : 1, 4, 5, 6 - 0.01; et al, 4, 5, 7 - 0.05; 3, 4, 5, 8 - 0.1.

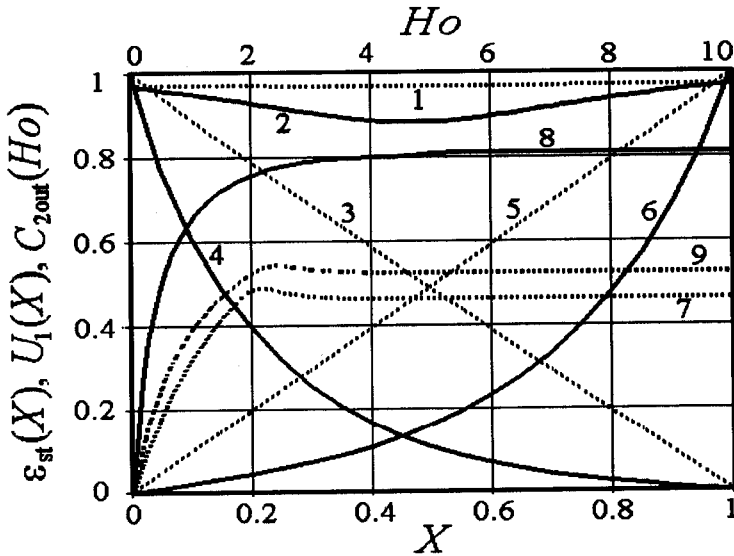


Fig. 7: Curves showing $E_{st}(X)$ (1, 2), $U_1(X)$ during the first and second time steps (3, 4 and 5, 6, respectively), and $C_{2out}(Ho)$ (7, 8, 9) at $Re_0 = 5000$, $l/d_{eq} = 500$, $E_{in} = 0.97$, $Sr = 0.05$, $R_{eff2}/R_{eff} = 1.5$, $\Delta_w = 0.01$, $\rho_S/\rho_F = 1$, and different values of R_{eff} : 2, 3, 5, 7 - 10^6 ; 1, 4, 6, 8 - 10^8 ; 9 - 10^7 .

$$2. \quad \iint_S p_D \cdot \mathbf{w} \cdot (\mathbf{w} \cdot d\mathbf{s}) + \iint_S p \cdot d\mathbf{s} - \iint_S \sigma \cdot d\mathbf{s} = 0$$

$$3. \quad \iiint_\omega \frac{\partial \varepsilon}{\partial t} \cdot d\omega + \iint_S \varepsilon \cdot \mathbf{w} \cdot d\mathbf{s} = 0$$

$$4. \quad \iiint_{\omega} \frac{\partial(\varepsilon \cdot c)}{\partial t} \cdot d\omega + \oint\!\!\!\oint_s \varepsilon \cdot c \cdot w \cdot ds = 0$$

where w , ρ_D , and ε are the velocity, the density, and the porosity of the dispersed media, respectively; ω and s are the arbitrary volume and the enclosed surface confining it, respectively; σ is the frictional stress tensor; p is the pressure. We assume that the fluid phase density ρ_F and viscosity μ_F do not depend on c . In addition, the hydrodynamic relaxation time in going over from one time step to another is negligible compared with t_s .

The equations (1)–(4) can be applied for the suspension flow in the channel (Aliev *et al.*, 2002 and Buevich *et al.*, 1978). Axis x is directed along the channel. Surface s formed by two sectional areas f of the channel and the side surface between them is selected. Integration is performed in (1)–(4) and projections on the x axis are taken. The equations are subdivided into distances dx between the sectional areas, and dx tends to zero. The layer of deposits on the wall is assumed to be thin compared with the equivalent diameter d_{eq} of the channel; therefore, $f = \text{const}$. This is true for suspensions having a low solid-phase content or for a short time step. We assume that $w_x = 0$ on the channel walls (Babenko, 1986 and Bystrov and Mikhailov, 1982); p is constant with respect to f (Gorbis, 1970), c and ε are also constant with respect to f . The latter assumption is partly justified by the fact that alternating cross flows are formed in the mass exchanger. We may write the following, using the coefficients of friction ξ and the flow of momentum β (Loitsynaskii, 1970; Landau and Lifshits, 1988 and Kochenov and Novoselskii, 1967)

$$5. \quad \frac{du}{dx} + \frac{\pi_v}{f} \cdot v = 0$$

$$6. \quad \frac{dp}{dx} + \rho_D \cdot \frac{d}{dx} (\beta \cdot u^2) + e_u \cdot \frac{\pi_f}{f} \cdot \frac{\xi}{8} \cdot \rho_D \cdot u^2 = 0$$

$$7. \quad \frac{\partial \varepsilon}{\partial t} + \frac{\partial}{\partial x} (\varepsilon \cdot u) + \frac{\pi_v}{f} \cdot v = 0$$

$$8. \quad \frac{\partial}{\partial t} (\varepsilon \cdot c) + \frac{\partial}{\partial x} (\varepsilon \cdot c \cdot u) + \frac{\pi_v}{f} \cdot c_\delta \cdot v = 0$$

where π_f and π_v are the perimeter of the channel section and its permeable part, respectively; u is the mean of w_x with respect to f ; v is the mean filtration rate with respect to π_v ; $e_u = u/|u|$.

The pressure in the channel may be related to the filtration rate [10]

$$9. \quad v(x) = \frac{p(x) - p_0}{\mu \cdot r_{ef}}$$

$$10. \quad r_{ef} = r_0 \cdot \pi_v \cdot \int_0^\delta \frac{dy}{\pi_v(y)} = r_0 \cdot \pi_v \cdot \Phi$$

where r_{ef} is the filtration resistance; r_0 is the resistivity of the partition; Φ is a dimensionless constant depending on the geometry of the permeable wall.

When going over in (5)–(8) to dimensionless variables, we obtain

$$11. \quad \frac{dU}{dX} + \frac{l}{d_{eq}} \cdot \frac{Re_0}{Re_f} \cdot Eu = 0$$

$$12. \quad \frac{dEu}{dX} + \rho \cdot \frac{d}{dX} (\beta \cdot U^2) + e_u \cdot \frac{l}{d_{eq}} \cdot \frac{\xi}{2} \cdot \rho \cdot U^2 = 0$$

$$13. \quad \frac{\partial \varepsilon}{\partial Ho} + U \cdot \frac{\partial \varepsilon}{\partial X} - \frac{dU}{dX} \cdot (1 - \varepsilon) = 0$$

$$14. \quad \frac{\partial \varepsilon}{\partial Ho} + U \cdot \frac{\partial \varepsilon}{\partial X} - \frac{dU}{dX} \cdot (1 - \varepsilon) = 0$$

where $X = x/l$, $U = u/u_0$, $C(X, Ho) = [c(x, t) - c_{\min}]/(c_{\max} - c_{\min})$, c_{\max} and c_{\min} are arbitrary constants; $Ho = \tau u_0/l$ is the criterion of homochronism; $Eu(X) = [\rho(x) - \rho_0]/(\rho_F \cdot u_0^2)$ is the Euler number; ρ_0 is the pressure outside the channel; $\rho = \rho_D/\rho_F$; $u_0 = u(0)$; $Re_f = f r_0 \cdot \Phi$; $Re_0 = u_0 \cdot d_{eq} \cdot \rho_F / \mu_F$. Concentration C_Δ depends on the sign of $V = v/u_0$. When $V > 0$, $C_\Delta = C$, and when $V < 0$, C_Δ is equal to the concentration of the fluid contained in the partition pores at the channel boundary.

We may consider two adjacent channels separated by a permeable wall (Meerovich and Muchnik, 1986 and Moshev and Ivanov, 1990). Equations (11)–(14) are written for each channel and we consider that the liquid transfer that takes place is the result of the pressure difference in the channels

$$15. \quad \frac{dU_1}{dX} + \frac{f_2}{f_1} \cdot \frac{dU_2}{dX} = 0$$

$$16. \quad \frac{dU_1}{dX} + \frac{l}{d_{eq1}} \cdot \frac{Re_{01}}{Re_{f1}} \cdot Eu_v = 0$$

$$17. \quad \frac{dEu_v}{dX} + \frac{d}{dX} (\rho_1 \cdot \beta_1 \cdot U_1^2 - \rho_2 \cdot \beta_2 \cdot U_2^2) + \frac{e_{u1} \cdot l}{2 \cdot d_{eq1}} \rho_1 \cdot \xi_1 \cdot U_1^2 - \frac{e_{u2} \cdot l}{2 \cdot d_{eq2}} \rho_2 \cdot \xi_2 \cdot U_2^2 = 0$$

$$18. \quad \frac{\partial \varepsilon_1}{\partial Ho} + U_1 \cdot \frac{\partial \varepsilon_1}{\partial X} - \frac{dU_1}{dX} \cdot (1 - \varepsilon_1) = 0$$

$$19. \quad \frac{\partial C_1}{\partial Ho} + U_1 \cdot \frac{\partial C_1}{\partial X} - \frac{1}{\varepsilon_1} \cdot \frac{dU_1}{dX} \cdot (C_{12} - C_1) = 0$$

$$20. \quad \frac{\partial \varepsilon_2}{\partial Ho} + U_2 \cdot \frac{\partial \varepsilon_2}{\partial X} - \frac{dU_2}{dX} \cdot (1 - \varepsilon_2) = 0$$

$$21. \quad \frac{\partial C_2}{\partial Ho} + U_2 \cdot \frac{\partial C_2}{\partial X} - \frac{1}{\varepsilon_2} \cdot \frac{dU_2}{dX} \cdot (C_{21} - C_2) = 0$$

where $Eu_v(X) = [\rho_1(x) - \rho_2(x)]/(\rho_F \cdot u_0^2)$ and the subscript indicates the channel number (1 or 2). The set of equations in (15)–(21) is written in terms of $U_1(X)$, $U_2(X)$, $Eu_v(X)$, $C_1(X, Ho)$, $C_2(X, Ho)$, $\varepsilon_1(X, Ho)$, and $\varepsilon_2(X, Ho)$. The boundary-value problem for the heat transfer of flows is determined by setting the boundary conditions for $x=0$ ($X=0$) $U_1(0)$, $U_2(0)$, $C_1(0, Ho)$, $\varepsilon_1(0, Ho)$, for $x=l$ ($X=1$) $U_1(1)$, $U_2(1)$, $C_2(1, Ho)$, $\varepsilon_2(1, Ho)$ and the initial conditions for $t=0$ ($Ho=0$) $C_1(X, 0)$, $C_2(X, 0)$, $\varepsilon_1(X, 0)$, $\varepsilon_2(X, 0)$.

We have already proposed and tested a similar model for a convective mass transfer in a manifold-drain system (Pavlov *et al.*, 1976) and a heat transfer process with dispersed flows and inert particles (Samarskii and Nikolaev, 1978). The experimental checkout of this model was made in the work (Sergeev *et al.*, 1974).

In practice, $c_1(0, t) = \text{const} = c_{1in} = c_{\max}$ during the first time step and $c_2(l, t) = \text{const} = c_{2in} = c_{\min}$ during the second time step. In the process, the maximal concentration in the mass exchanger $c_{1in} = c_{\max}$ and the minimal concentration $c_{2in} = c_{\min}$. Then, the dimensionless concentration $C(X, Ho) = [c(x, t) - c_{2in}]/(c_{1in} - c_{2in})$, and the boundary conditions will become universal: during the first time step

$$22. \quad C_1(0, Ho) = C_{1in} = \frac{c_1(0, t) - c_{2in}}{c_{1in} - c_{2in}} = \frac{c_{1in} - c_{2in}}{c_{1in} - c_{2in}} = 1$$

and during the second time step

$$23. \quad C_2(l, Ho) = C_{2in} = \frac{c_2(l, t) - c_{2in}}{c_{1in} - c_{2in}} = \frac{c_{2in} - c_{2in}}{c_{1in} - c_{2in}} = 0$$

This means that the outlet dimensionless concentrations $C_1(1, Ho) = C_{1out}(Ho)$ and $C_2(0, Ho) = C_{2out}(Ho)$ do not depend on the inlet concentrations c_{1in} and c_{2in} and, as such, they are universal. That is why the degree of mass transfer characterizes the mass exchanger. It does not depend on the inlet concentrations and characterizes the

mass transfer between the flows

$$24. \quad E = \lim_{Ho \rightarrow \infty} C_{2out}(Ho) = \lim_{t \rightarrow \infty} \frac{c_{2out}(t) - c_{2in}}{c_{1in} - c_{2in}} = \frac{c_{2out}^{st} - c_{2in}}{c_{1in} - c_{2in}}$$

where c_{2out}^{st} is the steady-state value of $c_{2out}(t)$.

Numerical solutions: Square-section adjacent channels having equal sectional areas were calculated. The wall separating the adjacent channels was permeable for the fluid phase and $l/d_{eq1} = l/d_{eq2} = l/d_{eq}$, $f_1 = f_2$. The rate of input of the flows to the first and second channels was the same: the characteristics of these flows are as follows: $\rho_{F1} = \rho_{F2} = \rho_F$, $\mu_{F1} = \mu_{F2} = \mu_F$, $\epsilon_1 = 1$, $\rho_{D1} = \rho_F$, $\rho_1 = \rho_{D1}/\rho_{F1} = 1$, $\mu_1 = \mu_F$, $\epsilon_2 = \epsilon$, $\rho_{D2} = \epsilon \cdot \rho_F + (1 - \epsilon) \cdot \rho_s$, $\rho_2 = \rho_{D2}/\rho_{F2} = \rho$, $\mu_2 = 0.59 \cdot \mu_F \cdot (\epsilon - 0.23)^{-2}$ [15], where ρ_s is the solid-phase density of the suspension, $Sr = t_s \cdot u_0 / l$ is the Strouhal number. During the first time step, the effective dimensionless filtration resistance R_{ef1} is equal to the partition resistance. During the second time step, R_{ef2} is equal to the sum of the resistances of the partition and the deposit. The effective resistance of the deposit is assumed to be equal in the calculations with respect to X (Samarskii and Nikolaev, 1978 and Sergeev *et al.*, 1974). Hence, we may take R_{ef2} to be independent of the coordinates.

The coefficients ξ and β for the given channel were found as a function of the local Reynolds number in this channel (Buevich *et al.*, 1978; Meerovich and Muchnik, 1986 and Moshev and Ivanov, 1990)].

The inlet dimensionless temperatures in the first and second channels are equal 1 and 0. The following equations were used for the countercurrent pi-network:

$$25. \quad U_1(0) = 1, \quad U_2(0) = -1, \quad U_1(1) = 0, \quad U_2(1) = 0, \quad C_1(0, Ho) = 1$$

$$26. \quad U_1(0) = 0, \quad U_2(0) = 0, \quad U_1(1) = 1, \quad U_2(1) = -1, \quad C_2(1, Ho) = 0, \quad \epsilon(1, Ho) = \epsilon_{in}$$

$$27. \quad C_1(X, 0) = 1, \quad C_2(X, 0) = 0, \quad \epsilon(X, 0) = \epsilon_{in}$$

where (25) and (26) are the boundary conditions for the first and the second time steps and (27) are the initial conditions. In the calculations made using (15)–(21) for similar square-section channels with boundary conditions (25) and (26) for any of the time steps we have $U_1(X) = -U_2(X)$. In view of this, the hydrodynamic part of (15)–(21) was solved by an iterative method with respect to U_1 , U_2 , Eu_v . Each iteration was calculated using the orthogonal matrix method (Yeroshenko and Zaichik, 1984). The found functions $U_1(X)$, $U_2(X)$, $Eu_v(X)$ were used in the concentration and porosity calculations from (15)–(21).

In the course of the calculations, changes in the positions of the interfaces of the fluids at concentrations $C_1(X, Ho)$ and $C_2(X, Ho)$ inside the permeable partition were taken into account for every point of the apparatus. The displacement of the interface in time ΔHo referred to the wall thickness δ_w is equal to $V \cdot \Delta Ho / (\Delta_w \cdot D_{eq})$, where $\Delta_w = \delta_w / d_{eq}$, $D_{eq} = d_{eq} / l$.

Calculations were first made using the equations for the first time step with the initial conditions (27), and then for the second time step. After this, the calculations were repeated in the same sequence. The initial concentration distribution for each next time step was assumed to be the final distribution of the preceding one.

Figures 3–7 show the distributions $\epsilon_{st}(X)$ and $U_1(X)$ in the first and second time steps and also the function $C_{2out}(Ho)$ for different values of Re_0 (Fig. 3), l/d_{eq} (Fig. 4), ϵ_{in} (Fig. 5), $Sr = t_s \cdot u_0 / l$ (Fig. 6), and R_{ef1} (Fig. 7). Here, ϵ_{st} is the mean porosity during the first and the second time steps under steady-state operating conditions. The figures show that the typical time variable of the process is the residence time l/u_0 of the fluid in the channel ($Ho \sim 1$). The results obtained show that $C_{2out}(Ho)$ grows with an increase in l/d_{eq} , Re_0 , and Sr as well as with a decrease in ϵ_{in} , Δ_w , and R_{ef1} . The fact is that the greater the filtration rate gradient $|dV/dX|$, the higher is the steady-state value $C_{2out}(Ho)$. The data calculated on the model predict the highest values of $|dV/dX|$ for large numbers of Re_0 , l/d_{eq} and small values of ϵ_{in} and R_{ef1} . Calculations have shown that a change in the ratio ρ_s/ρ_F between 0.5 and 2.0 does not influence the hydraulic and mass-transfer characteristics of the analyzed system. A reduction in the porosity was noted in the middle part of the channel under all the considered operating conditions. In this case the degree of thickening of the suspension grows with $|dV/dX|$.

The obtained mathematical model makes it possible to design miscellaneous valve-type pulsating-flow mass exchangers more thoroughly, including calculations of the main parameters (Re_0 , l/d_{eq} , and R_{ef1}) required to ensure the desired degree of mass transfer of the flows. The values of l/d_{eq} and, in the final analysis, the length l of the mass exchanger are found for given E , d_{eq} , ϵ_{in} , and R_{ef1} and Re_0 calculated with respect to the volumetric flow and the sectional area of the channel on the basis of the developed model using the KRT computer code.

Conclusions

We can obtain longitudinal distributions of velocities, pressures, porosities, and concentrations for countercurrent flow of a food liquid and a food suspension in the channels of a valve-type pulsating-flow mass exchanger with a convectively permeable mass-transfer surface using the mathematical model of convective mass transfer between

the flows of finely dispersed food media when solving the problem of transient convective mass transfer between food suspension and food liquid flows moving in adjacent channels separated by a permeable partition when creating repeated exchange of fluid-phase portions between the flows.

The higher the longitudinal gradient of the rate of filtration and the lower the ratio of the volume of the permeable partition to the pulse volume, the higher is the efficiency of convective mass transfer between the flows in the adjacent permeable channels.

Thickening of the suspension is possible in adjacent permeable channels in the event of a pulsating flow. This thickening is expected to be a maximum in the middle part of the channel and its degree will increase with the longitudinal gradient of the filtration rate.

When the fluid flows in adjacent permeable channels, the longitudinal gradient of the filtration rate will increase with an increase in the inlet Reynolds number, a decrease in the effective dimensionless resistance of the permeable partition, and with an increase in the ratio of the channel length to its equivalent diameter.

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