

## Simulation of Digestive Biscuit's Temperature Changes in Cooling Process Applied in Nane Qodse Razavi Factory

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**Abstract:** This study is produced by investigation of data that gathered from an experimental research. In this study, we study the Biscuit's top surface temperature changes with time, while transfer from oven to packing place by conveyors in Nane Qodse Razavi Factory. The Fourier field equation for transient heat transfer is used as the basis for the deterministic modeling of a single biscuit. According to data taken from factory's quality control laboratory, the humidity inside biscuits change at 0.5% of weight when they move from the oven exit to the packing place. Therefore, mass transmission is ignored. On top of the biscuits we have three heat transfer modes: forced and natural convective and radiative heat transfer. In forced convective heat transfer we have used correlations of flow along a semi infinite plate with arbitrarily specified surface temperature. Under the biscuits because of the high number resulted from dividing thermal resistance of the whole circumference under the biscuit by the thermal resistance of circumference on top of the biscuit, boundary condition of the insulator is used. Experimental results measured by an infrared thermometer. To compare theoretical and experimental data and investigations on correctness of the hypotheses, the graphs of both are drawn in one form. Coefficients of heat transfer and their effect on biscuit's temperature is investigated in different states on conveyor and the results are shown in graphs.

**Key words:** Biscuit cooling, transient heat transfer, conveyor, fourier field equation, temperature dispersion

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### INTRODUCTION

The majority of freshly baked biscuits are cooled before packaging or secondary processing. The normal method of industrial cooling is that at the oven exit, the biscuits are transferred from the oven band to an open conveyor and carried through the factory to cool naturally in ambient air until they are hand-hot or less. In some cases forced air cooling in cooling tunnels is employed to achieve greater control over biscuit temperature at the end of the cooling process. Typical secondary processing involves the deposition of cream, jam or marshmallow on the biscuit or the enrobing of the biscuit with a chocolate or icing coating. Generally speaking, the biscuit should be as cool as possible prior to such processing.

The objective of this study is to quantify the temperature of biscuit and heat transfer mechanisms at different place on the conveyor so that we could understand, which one has the most effect on cooling process and finally optimize the cooling process.

### Theory

**Deterministic cooling model:** To modeling the problem, a digestive biscuit was selected. The biscuit is cylindrical

with a diameter of 64 mm and nominal thickness of 7.5 mm. Top and peripheral surface of the biscuit contact with the surround and air flows on it with the velocity of  $0.15 \text{ m sec}^{-1}$ . The cooling geometry is shown in Fig. 1. The air flow is uni-axial and parallel to the top surface of the biscuit. Radial heat flow within the biscuit is ignored and the problem treated as one dimensional, unsteady state heat flow with a temperature gradient solely in the vertical direction through the biscuit thickness. Thus, the biscuit is approximated as an infinite slab. This approach was found to be valid by comparing the predictions of this restricted model to the full solution for a cylinder consisting of the geometric intersection of an infinite flat plate with an infinite cylinder. The Fourier field equation for an infinite flat plate is:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \quad (1)$$

and can be used to establish temperature as a function of vertical position through the biscuit and time,  $T(x, t)$  once the initial and boundary conditions are known. At the top surface of the biscuit, conductive heat transfer will equal the heat lost by convection and radiation. Note the parameter  $x$  is measured from the center of the biscuit base

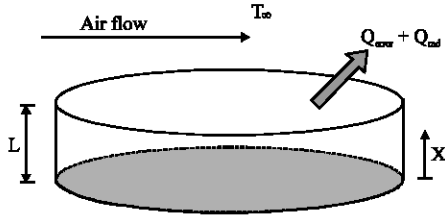


Fig. 1: Biscuit cooling geometry

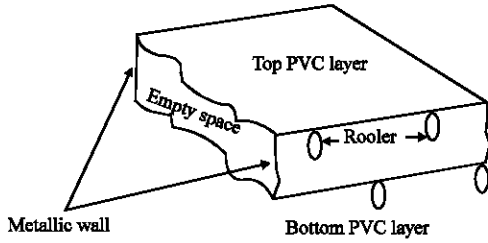


Fig. 2: Structure of a conveyore

(so in effect the biscuit is modeled as one half of an infinite slab that is cooling from both sides). Under the biscuits, because of the high number resulted from dividing thermal resistance of the whole circumference under the biscuit by the thermal resistance of the circumference above the biscuit, boundary condition of the insulator is used.

Proof of this idea is given:

Thermal resistance of the circumference above the biscuit:

$$R_{s2} = \frac{1}{h_{s2} \times A} = \frac{1}{14 \times A} = 0.071/A \quad (2)$$

Where:

$\overline{h_{s2}}$  = Mean total heat transfer coefficient along the conveyore above the biscuit

Thermal resistance of the whole circumference under the biscuit

Considering Fig. 2, it's apparent that the circumference under the biscuits is composed of respectively:

- Contact surface with roughness about 1 mm
- Top PVC layer with thickness about 2 mm
- Empty space with thickness about 20 cm
- Bottom PVC layer with thickness about 2 mm
- Surrounding air below bottom PVC layer

Heat transfer in horizontal enclosed spaces involves two distinct situations. If the upper plate is maintained at a higher temperature than the lower plate,

the lower-density fluid is above the higher-density fluid and no convection currents will be experienced. In this case, the heat transfer across the space will be by conduction alone.

Thermal contact resistance:

$$R_c = \frac{1}{h_c \times A} \quad (3)$$

That:

$$h_c = \frac{1}{L_g} \times \left( \frac{A_c}{A} \frac{2 \times k_b \times k_p}{k_b + k_p} + \frac{A_v}{A} \times k_f \right) \quad (4)$$

Where,  $A_c$ ,  $A_v$ ,  $L_g$ ,  $k_b$ ,  $k_p$ ,  $k_f$ ,  $A$  are respectively contact area, void area, thickness of the void space, thermal conductivity of the fluid, which fill the void space, total cross-sectional area of the bars.

$$h_c = \frac{1}{1 \times 10^{-3}} \times \left( \frac{1}{2} \frac{2 \times 0.21 \times 0.19}{0.21 + 0.19} + \frac{1}{2} \times 0.029 \right) = 114.25 \quad (5)$$

$$R_c = \frac{1}{114.25 \times A} = 0.00875/A \quad (6)$$

Thermal resistance of top PVC layer:

$$R_{p1} = \frac{\Delta x_p}{K_p \times A} = \frac{2 \times 10^{-3}}{0.19 \times A} = 0.0105/A \quad (7)$$

Thermal resistance of horizontal enclosed space:

$$R_a = \frac{H_{em}}{K_a \times A} = \frac{20 \times 10^{-2}}{0.029 \times A} = 6.895/A \quad (8)$$

Thermal resistance of bottom PVC layer:

$$R_{p1} = \frac{\Delta x_p}{K_p \times A} = \frac{2 \times 10^{-3}}{0.19 \times A} = 0.0105/A \quad (9)$$

Thermal resistance of the circumference under the bottom PVC layer:

$$R_{s2} = \frac{1}{h_{s2} \times A} = \frac{1}{10 \times A} = 0.1/A \quad (10)$$

So, total thermal resistance becomes:

$$R_t = R_c + R_{p1} + R_a + R_{p2} + R_{s2} = (0.00875 + 0.0105 + 6.895 + 0.0105 + 0.1)/A = 7.02475/A \quad (11)$$

Therefore, by dividing thermal resistance of the whole circumference under the biscuit by the thermal resistance of circumference above the biscuit, we obtain:

$$\frac{R_t}{R_{s1}} = \frac{7.02475/A}{0.071/A} = 98.873 \quad (12)$$

as seen the obtained number shows the huge heat resistance of the total ambience under the biscuit compared with the heat resistance above the biscuits. So, the assumption of having an isolator under the biscuits seems logical.

Thus, the initial and boundary conditions will be:

$$-k \frac{\partial T(0,t)}{\partial x} = 0 \quad (13)$$

$$-k \frac{\partial T(L,t)}{\partial x} = h[T(L,t) - T_\infty] \quad (14)$$

$$T_i = f(x) \quad (15)$$

The solution to this thermal system is supplied by the following infinite series:

$$T - T_\infty = \sum_{n=1}^{\infty} \frac{2 \left( \int_0^L (f(x) - T_\infty) \cos(\lambda_n x/L) dx \right) \cos(\lambda_n x/L)}{\lambda_n + \sin(\lambda_n) \cos(\lambda_n)} \exp(-\lambda_n^2 Fo) \quad (16)$$

$f(x)$  is temperature distribution in the biscuit.

Only 10 terms of this series solution will be taken. Because, the subsequent terms in the expansion are neglected, as biscuit top surface temperature is of interest, the solution will be evaluated at  $L$ .

The models utilizes a number of constants including product thermal diffusivity,  $\alpha$ , the Fourier number,  $Fo$  and Biot number,  $Bi$  which are required in the solution. These are defined as:

$$\alpha = \frac{k}{\rho C_p}, \quad Fo = \frac{\alpha t}{L^2}, \quad Bi = \frac{hL}{k} \quad (17)$$

The constants  $\lambda_n$  can be found from the implicit equations:

$$\lambda_n \times \tan \lambda_n = Bi \quad (18)$$

According to data taken from factory's quality control laboratory, the humidity inside biscuits change at 0.5%, when they move from the oven exit to the packing place. Therefore, it is implicit in the above model that

certain other phenomena that may occur during food cooling (in addition to temperature reduction) such as moisture evaporation/condensation, shrinkage and compositional change do not take place for this system. Biscuit weight and dimensions were routinely measured, before and after cooling and confirmed the validity of this assumption.

Noted that the coefficient,  $h$ , that used in the above formula is combination of these 3 terms:

$$h = h_{rad} + h_n + h_f \quad (19)$$

where,  $h_{rad}$ ,  $h_n$ ,  $h_f$  are radiative and natural convection and forced convection heat transfer coefficient's that expressed as:

**Forced convective heat transfer coefficient estimation:**

As said before, the sample is on a conveyor and move with it, for this case we use correlations of flow along a semi infinite plate with arbitrarily specified surface temperature. The correlation that we used for this case is (Kays and Crawford, 2005):

If the surface temperature is expressed by:

$$T_s = T_\infty + A + \sum_{n=1}^{\infty} B_n \times (lend - x)^n \quad (20)$$

So,

$$h_f = \frac{(0.332 \times k_a \times Pr^{1/3} \times Re^{1/2} \times (\sum_{n=1}^{\infty} n \times B_n) \times \frac{4}{3} \times (lend - x)^n \times \beta_n + A)}{(lend - x) \times (A + \sum_{n=1}^{\infty} B_n \times (lend - x)^n)} \quad (21)$$

Where, the beta function:

$$\beta_n = \frac{\Gamma(\frac{4}{3} \times n) \times \Gamma(\frac{2}{3})}{\Gamma(\frac{4}{3} \times n + \frac{2}{3})} \quad (22)$$

In above correlations  $lend$  is length of conveyor. Noted that In this problem we use 3 terms of this series.

$\Gamma(n)$  is Gama function. Note that  $A$ ,  $\beta_n$  are coefficients that we should choose arbitrarily.

**Natural convective heat transfer coefficient estimation:**

For this we used the correlation (Holman, 1986a):

For above the biscuit:

$$\overline{Nuf} = C(Gr_f \times Pr_f)^m \quad (23)$$

Where:

$$\overline{Nuf} = \frac{\bar{h} \times (\ell)}{K_a} \quad (24)$$

That:

$\ell$  = Distance fluid particle travels in boundary layer

Where, the subscript f indicates that the properties in the dimensionless groups are evaluated at the film temperature:

$$T_f = \frac{T_\infty + T_w}{2} \quad (25)$$

In the above correlation  $K_a = 0.029 \text{ W/m/K}$ .

**Radiative heat transfer coefficient estimation:** The correlation that used for this coefficient is:

$$h_{rad} = 0.85 \times 5.67 \times 10^{-8} \times 1 \times ((T + 273)^2 + (T_s + 273)^2) \times ((T + 273) + (T_s + 273)) \quad (26)$$

## MATERIALS AND METHODS

**Experimental research:** A digestive biscuit was chosen for investigation; with a characteristic diameter of 65 mm, thickness of 7.5 mm and weight of 9 g. A representative dough composition for such biscuit consists of 100 g of flour, 20 g of fat, 26 g of sugar and 20 g of water.

Two distinct programs of experimental research were carried out to quantify input data for the modeling approaches. The first program involved determining the magnitudes (both mean and dispersion) of the intrinsic biscuit properties (density and thickness). The second program were conducted to establish the mean and dispersion of the biscuit temperature at different place on conveyor and to validate the experimental data with the theoretical solutions.

Temperatures were measured with an infrared thermometer with accuracy about  $\pm 0.1^\circ$  reading temperature  $+1^\circ\text{C}$ .

**Density measurement:** Biscuit bulk density was measured by determining the sample mass and volume. The volume was determined by measuring the external dimensions (diameter and thickness) assuming a constant shape. At least 20 replicates were measured and the results are shown in Table 1.

**Thermal conductivity calculation (Tavakoulipour, 1997a):** An estimation of Biscuit thermal conductivity can be calculated from the product composition using a weighted average of the component conductivities:

$$k = 0.25 \times M_c + 0.155 \times M_p + 0.16 \times M_f + 0.135 \times M_a + 0.58 \times M_m \quad (27)$$

Where, M is the mass fraction of the component.

Table 1: Density values for digestive 19% fat biscuits

Biscuit type	Number of replicates	Density ( $\text{kg m}^{-3}$ )
Digestive 19% fat	20	551.52 $\pm$ 2.2

Table 2: Mass fraction for digestive biscuits

Component	Mass fraction (%)
Fat	19
Protein	8
Carbohydrate	69.97
Ash	0.03
Moisture	3

According to data taken from factory's quality control laboratory Mass fraction are given in Table 2. So, the Biscuit thermal conductivity become  $k = 0.21$ .

**Heat capacity calculation:** Heat capacity can be estimated using the composition of the product by this correlation (Tavakoulipour, 1997b):

$$C_p = 1.424 \times M_c + 1.549 \times M_p + 1.675 \times M_{fat} + 0.837 \times M_{ash} + 4.187 \times M_m \quad (28)$$

Where, M is the mass fraction of the component that are given in Table 2. So, the Heat capacity become  $C_p = 1586 \text{ kJ/kg/k}$ .

## RESULTS AND DISCUSSION

To solve Eq. 5, the length of conveyor is divided into several segments. In each segment convective and radiative heat transfer coefficients are assumed to be constant. Noted that by increasing the number of segment, the accuracy of solution improved. The biscuit's temperature at the beginning of each segment's is considered as initial temperature and temperature at the end of each segment is considered as desired temperature.

The temperature at the end of each segment is considered as initial temperature for next segment. Experimental temperatures are gathered by measuring the biscuit's top surface. To comparison, these 2 diagrams are depicted in Fig. 2. Noted that the distance is measured from oven output.

As distinct from the Fig. 2, theoretical diagram has a good accordance with the experimental diagram. This accordance proofs the assumptions. In following we study the variation of convective and radiative heat transfer coefficients along the conveyor, from oven toward packing place. These variations are depicted in Fig. 3-5.

As shown in Fig. 3-4, the amount of natural convective and radiative heat transfer coefficients decreased along the conveyor from oven to packing place. The reason of this phenomenon is by decreasing

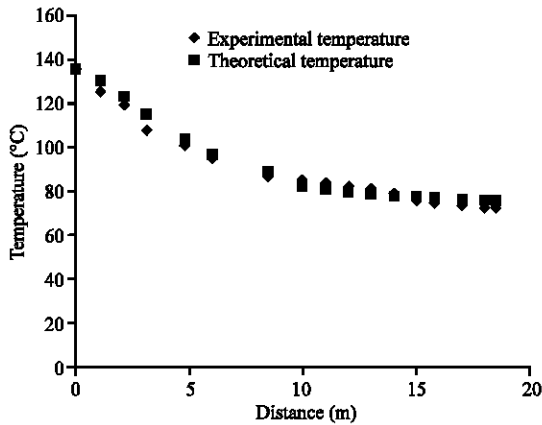


Fig. 3: Comparison of experimental and theoretical data at different place on conveyor

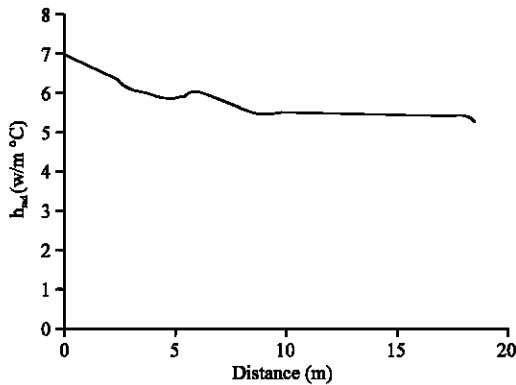


Fig. 4: Radiative heat transfer coefficient at different place on conveyor

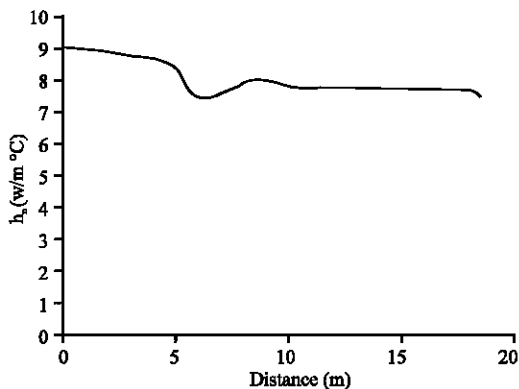


Fig. 5: Natural convection heat transfer coefficient at different place on conveyor

the biscuit top surface temperature along the conveyor, difference between the biscuit temperature and surround temperature is decreased.

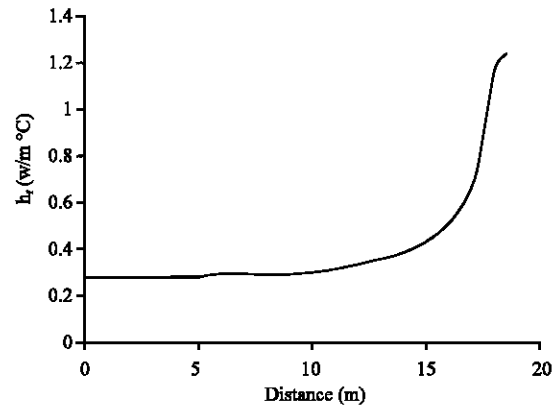


Fig. 6: Force convection heat transfer coefficient at different place on conveyor

In Fig. 5, the forced convective heat transfer coefficient is studied. As shown in the Fig. 6, the amount of forced convective heat transfer coefficient increased along the conveyor from oven to packing place. The reason of this phenomenon is this, one of the main parameter's in forced convective heat transfer mechanism is velocity of air. In this study, velocity of air is about  $0.15 \text{ m sec}^{-1}$  that is small, the other parameter is distance from the place that boundary layer is began. In this case the distance is length of conveyor, as said before has a quantity about 20 m that is too much versus amount of other parameter applied in Eq. 10.

### CONCLUSION

By considering Fig. 5 and 6, we noticed that the effect of forced convective heat transfer mechanism on biscuit's cooling process versus other heat transfer mechanism is very small. Therefore, if we want to increase the rate of cooling, to get less time for the biscuit's to be cooled, one way is to increase the amount of forced convective heat transfer mechanism, one way is to increase the velocity of air, as said before the velocity of air is about  $0.15 \text{ m sec}^{-1}$  that is small, so by increasing the velocity of air we can increase the forced convective heat transfer mechanism, the other choice is to change the direction of blowing air on the conveyor, in current mode the direction of air flow is parallel to conveyor, a suitable solution is to use air that flows on the conveyor vertically, by applying this method, we can get relatively high quantity for forced convective heat transfer mechanism. The correlation that used for this case is (Holman, 1986b):

$$\overline{\text{Nu}}_d = 2 \times 0.332 \times \text{Re}^{\frac{1}{2}} \times \text{Pr}^{\frac{1}{3}} \quad (29)$$

In Eq. 29 characteristic dimension is diameter of biscuit.

**Nomenclature:**

- $A_c$  = Contact area
- $A_v$  = Void area
- $Bi$  = Biot number (dimensionless)
- $Fo$  = Fourier number (dimensionless)
- $H$  = Surface heat transfer coefficient ( $W/m^2K$ )
- $H_{en}$  = Thickness of the enclosure
- $K$  = Thermal conductivity ( $W/m/K$ )
- $k_a$  = Air thermal conductivity
- $L$  = Thickness (m)
- $L_g$  = Thickness of the void space
- $Pr$  = Prandtl number
- $\rho$  = Density ( $kg\ m^{-3}$ )
- $Re$  = Reynolds number
- $T$  = Temperature ( $^{\circ}C$ )
- $\alpha$  = Thermal diffusivity ( $m^2\ sec^{-1}$ )

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