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Process and Error Isolation for Arma (1,1) Subsumed in AR (1) Process

Daniel Eni
Department of Mathematics/Statistics, Cross River University of Technology, Calabar, Nigeria

Abstract: In this study, we developed a method which enables us to estimate both the ARMA (1,1) process and the AR(1) error process for a situation where we suspect, from experience that a set of data which can be fitted by ARMA(1,1) model is corrupted by another set of data following AR(1) process. Simulation studies showed that the method performed very well in isolating the set that can be fitted by ARMA(1,1) model from the set that can be fitted by AR(1) model

Key words: ARMA, AR, variance, autocovariance function, parameter estimation

INTRODUCTION

Considered the ARMA (1, 1) process

$$(1-\phi L)f_t = (1-\theta L)a_t \tag{1}$$

Where

- f_t : Is an unobservable process of interest
- α_t : Is a white noise process. ie, it is distributed with zero mean and constant variance σ_α^2 and α_{t-i}, I = 1,2,3 are its values at time t-i
- L : Is a backward shift operator ie
- θ, ϕ : Are weight parameters.

We consider the case where f_t can be estimated only through

$$f_t = g_t - b_t \tag{2}$$

Where

- g_t : Is an observable process
- b_t : Is an error component introduced by faulty measurement or observation processes and is a an autoregressive process of order one ie AR(1).

Substituting (2) into (1), we have

$$(1-\phi L)g_t = (1-\theta L)a_t + (1-\phi L)b_t \tag{3}$$

Since is following AR (1) process, then

$$(1-\alpha L)b_t = e_t$$

$$b_t = \frac{e_t}{(1-\alpha L)} \tag{4}$$

Where

- e_t : Is a white noise process mutually independent of α_t
- α : Is a weight parameters

Substituting (4) into (3), we have the theoretical model

$$(1-\phi L)(1-\alpha L)g_t = (1-\theta L)(1-\alpha L)a_t + (1-\phi L)e_t$$

or

$$g_t = (\phi+\alpha)g_{t-1} - \alpha\phi g_{t-2} + a_t - (\alpha+\theta)a_{t-1} + \alpha\theta a_{t-2} + e_t - \phi e_{t-1} \tag{5}$$

Our interest is to estimate the parameters, θ and ϕ in the modeling of the unobservable process f_t and α in b_t, the error process, through the observe process g_t. The results are then used to isolate both the process of interest f_t and the error process b_t.

The maximum likelihood estimates for the case where both σ_α^2 and σ_b^2 are known (the so called "over verification case") are estimated by Barnet (1967) by directly solving the likelihood equation. Chan and Mak (1979) obtained the maximum likelihood estimates for the case where both σ_α^2 and σ_b^2 are unknown and where the observations are replicated.

Our interest is to use autocovariance function to estimate the parameter values of the real IMA(1)series as well as the parameter value of the AR(1) errors even where the ratio

$$\lambda = \frac{\sigma_a^2}{\sigma_e^2}$$

is unknown. Eni *et al*. (2007a) have used the same method to isolate errors of of AR(1) corrupted with MA(1) process. Also Eni *et al*. (2007b) have considered the case of IMA(1) with white noise. In a similar case, Eni (2006) have considered the case of GARCH (1,1) model with white noise errors using the proposed method.

VARIANCES OF THE WHITE NOISE PROCESSES

Theorem1: The variances σ_α^2 and σ_b^2 of the white noises σ_t and e_t respectively are

$$\sigma_e^2 = \frac{\phi\{v_0-(\phi+\alpha)v_1+\alpha\phi v_2\}\{v_1-(\phi+\alpha)v_0+\alpha\phi v_2\}+(1-\phi\alpha)}{(1-\alpha\phi)\{\alpha\theta(\phi-\theta)-(\alpha+\theta\ \}+\phi\{1-(\phi-\theta)(\alpha+\theta-\phi^2\theta)\ \}}$$

$$\sigma_e^2 = \frac{\{v_0-(\phi+\alpha)v_1+\alpha\phi v_2\}\{\ \alpha\theta(\phi-\theta)-(\alpha+\theta\ \}+\{v_1-(\phi+\alpha)v_0+\alpha\phi v_2\}\{1-(\phi-\theta)(\alpha+\theta-\phi^2\theta)\ \}}{(1-\alpha\phi)\{\alpha\theta(\phi-\theta)-(\alpha+\theta\ \}+\phi\{1-(\phi-\theta)(\alpha+\theta-\phi^2\theta)\ \}}$$

For ARMA (1,1) process corrupted with AR (1) errors

Proof:

Multiply (5)by g_t and taking expectations, to get the variance of the process

$$\begin{aligned} v_0 &= (\phi+\alpha)v_1 - \alpha\phi v_2 + \sigma_a^2 - (\alpha+\theta) \\ &E(g_t a_{t-1}) + \phi\theta E(g_t a_{t-2}) + \sigma_e^2 - \theta E(g_t e_{t-1}) \end{aligned} \tag{6}$$

Where

$E(g_t\, g_{t-1}) = v_i$

$E(g_t\, \alpha_t) = \sigma_\alpha^2$: Multiply (5) by α_t and take expectations

$E(g_t\, e_t) = \sigma_\alpha^2$: Multiply (5) by e_t and take expectations

$E(\alpha_t\, e_{t-1}) = v0$: α_t and e_t are mutually independent

Box and Jenkins (1975)

$$E(e_t e_{t-1}) = \begin{cases} 0 & \text{for } i \neq 0 \\ \sigma_a^2 & \text{for } i = 0 \end{cases} \quad \text{by definition} \tag{7}$$

$$E(a_t a_{t-1}) = \begin{cases} 0 & \text{for } i \neq 0 \\ \sigma_a^2 & \text{for } i = 0 \end{cases} \quad \text{by definition}$$

Hamilton (1994) multiplying (5) by α_{t-1}, α_{t-2} and e_{t-1} respectively and taking expectations using the set of Eq. 7, we have

$$E(g_t a_{t-1}) = (\phi-\theta)\sigma_a^2$$

$$E(g_t a_{t-2}) = \phi(\phi-\theta)\sigma_a^2 \tag{8}$$

$$E(g_t e_{t-1}) = \alpha\sigma_e^2$$

substituting (8) into (6), we have

$$\begin{aligned} &v_0 - (\phi+\alpha)v_1 + \alpha\phi v_2 = \\ &\{1-(\phi-\theta)(\alpha+\theta-\phi^2\theta)\ \}\sigma_a^2 + (1-\alpha\phi)\sigma_e^2 \end{aligned} \tag{9}$$

Also multiplying (5) by g_{t-1} and using the set equations in (7) and (8) to take expectations, we have

$$\begin{aligned} &v_1 - (\phi+\alpha)v_0 + \alpha\phi v_2 = \\ &= \{\alpha\theta(\phi-\theta) - (\alpha+\theta\ \}\sigma_a^2 - \phi\sigma_e^2) \end{aligned} \tag{10}$$

Solving (9) and (10) simultaneously for σ_α^2 and σ_e^2 we have the required result as

$$\sigma_a^2 = \frac{\phi\{v_0-(\phi+\alpha)v_1+\alpha\phi v_2\}+\{v_1-(\phi+\alpha)v_0+\alpha\phi v_2\}(1-\phi\alpha)}{(1-\alpha\phi)\{\alpha\theta(\phi-\theta)-(\alpha+\theta\ \}+\phi\{1-(\phi-\theta)(\alpha+\theta-\phi^2\theta)\ \}} \tag{11}$$

$$\sigma_e^2 = \frac{\{v_0-(\phi+\alpha)v_1+\alpha\phi v_2\}\{\ \alpha\theta(\phi-\theta)-(\alpha+\theta\ \}+\{v_1-(\phi+\alpha)v_0+\alpha\phi v_2\}\{1-(\phi-\theta)(\alpha+\theta-\phi^2\theta)\ \}}{(1-\alpha\phi)\{\alpha\theta(\phi-\theta)-(\alpha+\theta\ \}+\phi\{1-(\phi-\theta)(\alpha+\theta-\phi^2\theta)\ \}} \tag{12}$$

As already noted in this study, the model (5) is theoretical since the traditional ARMA model does not make provision of 2 set of white noise errors as we have in (5).

In practice, we have observed (5) as the ARMA (2,2) model below:

$$g_t = \Phi_1 g_{t-1} - \Phi_2 g_{t-2} + \varepsilon_t - \Theta_1\varepsilon_{t-1} + \Theta_2\varepsilon_{t-2} \tag{13}$$

ε_t : Is a white noise process

Comparing the theoretical model (5) and the observable model (13)

$$\Phi_1 = \alpha + \phi \tag{i}$$

$$\Phi_2 = \alpha\phi \tag{ii}$$

$$\begin{aligned} &\varepsilon_t - \Theta_1\varepsilon_{t-1} + \Theta_2\varepsilon_{t-2} = (a_t + e_t) - \\ &\{(\alpha+\theta)a_{t-1} + \phi\, e_{t-1}\} + \alpha\,\theta\, a_{t-2} \end{aligned} \tag{iii}$$

We group the white noise processes in (iii) according to time t-i, i = 0,1,2 to get

$$\varepsilon_t = a_t + e_t \tag{iv}$$

$$\Theta_1\varepsilon_{t-1} = (\alpha+\theta)a_{t-1} + \phi\, e_{t-1} \quad \text{(v)}$$

$$\Theta_2\varepsilon_{t-2} = \alpha\,\theta\, a_{t-2} \quad \text{(vi)}$$

Corollary 1: The variance of the observe process is

$$\sigma_\varepsilon^2 = \frac{A_1\{\alpha\theta(\phi-\theta)-(\alpha+\theta)\} - A_2\{1-(\phi-\theta)(\alpha+\theta-\phi^2\theta)\} + A}{(1-\alpha\phi)\{\alpha\theta(\phi-\theta)-(\alpha+\theta)\} + \phi\{1-(\phi-\theta)(\alpha+\theta-\phi^2\theta)\}}$$

Where

$$A_1 = v_0 - (\phi+\alpha)v_1 + \alpha\phi v_2$$

$$A_2 = v_1 - (\phi+\alpha)v_0 + \alpha\phi v_2$$

$$A = A_1\phi + A_2(1-\alpha\phi)$$

Proof:
Consider from (iv)

$$\varepsilon_t\varepsilon_t = (a_t+e_t)(a_t+e_t) = a_t a_t + 2a_t e_t + e_t e_t$$

Taking expectations, we have

$$\sigma_U^2 = \sigma_a^2 + \sigma_e^2$$

Substituting the results Eq. 11 and 12, we have the required result as

$$\sigma_e^2 = \frac{A_1\{\alpha\theta(\phi-\theta)-(\alpha+\theta)\} - A_2\{1-(\phi-\theta)(\alpha+\theta-\phi^2\theta)\} + A}{(1-\alpha\phi)\{\alpha\theta(\phi-\theta)-(\alpha+\theta)\} + \phi\{1-(\phi-\theta)(\alpha+\theta-\phi^2\theta)\}} \quad (14)$$

Where

$$A_1 = v_0 - (\phi+\alpha)v_1 + \alpha\phi v_2$$

$$A_2 = v_1 - (\phi+\alpha)v_0 + \alpha\phi v_2$$

$$A = A_1\phi + A_2(1-\alpha\phi)$$

PARAMETER ESTIMATIONS

The parametersΦ_1, Φ_2, Θ_1 and Θ_2 can be estimated using maximum likelihood estimate technique. While v_0, v_1 and v_2 are computed from the set of data $\{g_{t}\}_{i = 0, 1,\dots N}$To do this, We follow Box and Jenkins (1976) to compute the variance s using the formula

$$v_i = \frac{1}{N}\sum_{t=1}^{N-1}(g_t-\mu)(g_{t-i}-\mu) \quad (15)$$

Where

$$i = 0,1,2$$

$$\mu = \frac{1}{N}\sum_{t=1}^{N} g_t$$

N is the total number of data points

Our objective is to estimate α, ϕ and θ found in Eq. 5.

Substitution of $\alpha = \Phi_1 - \phi$ into (ii) will result into the quadratic equation

$$\phi^2 - \phi\,\Phi_1 + \Phi_2 = 0 \quad \text{(vii)}$$

The results of (vii) will be substituted into (i) to obtain the values of α. This will give two pairs of ϕ, α results. However, we consider the fact that Eq. 11 and 12 must be positive and recommend the choice of the ϕ, α pair that will make both equations positive.

With the parameter values ϕ, α known, the only unknown parameter in Eq. 5 is θ. To obtain θ, we consider the theorem below

Theorem 2: The parameter θ can be estimated using the iterative formula below

$$\theta_{i+1} = \theta_i - \left\{ \frac{\left(B_1\begin{bmatrix}(1-\alpha\phi)\{\alpha\theta(\phi+\theta)-(\alpha+\theta)\} + \\ \phi\{1-(\phi-\theta)(\alpha+\theta-\phi^2\theta)\}\end{bmatrix} - B_2\begin{bmatrix}A_1\{\alpha\theta(\phi-\theta)-(\alpha+\theta)\} - \\ A_2\{1-(\phi-\theta)(\alpha+\theta-\phi^2\theta)\} + A\end{bmatrix}\right)}{\left(B_1\begin{bmatrix}(1-\alpha\phi)\{\alpha(\phi-2\theta)-1\} + \\ \phi\{\alpha-\phi+\phi^3+2\theta(1-\phi^2)\}\end{bmatrix} - B_2\begin{bmatrix}(A_1\{\alpha(\phi-2\theta)-1\} - \\ A_2\{\alpha-\phi+\phi^3+2\theta(1-\phi^2)\}\end{bmatrix}\right)} \right\}$$

Where

$$B_1 = v_0 - \Phi_1 v_1 + \Phi_2 v_2$$

$$B_2 = \{1-\Theta_1(\Phi_1-\Theta_1) + \Theta_2[\Phi_1(\Phi_1-\Theta_1) - \Phi_2 + \Theta_2]\}$$

$$A_1 = v_0 - (\phi+\alpha)v_1 + \alpha\phi v_2$$

$$A_2 = v_1 - (\phi + \alpha)v_0 + \alpha\phi v_2$$

$$A = A_1\phi + A_2(1 - \alpha\phi)$$

Moreover, the stating point of iteration is

$$\theta_0 = \Theta_1 - (\alpha + \phi)$$

Proof: We multiply (13) by g_t and take expectation to get

$$v_0 = \Phi_1 v_1 - \Phi_2 v_2 + \sigma_\varepsilon^2 - \Theta_1 E(g_t\varepsilon_{t-1}) + \Theta_2 E(g_t\varepsilon_{t-2}) \quad (16)$$

Where

$$E(g_t g_{t-i}) = v_i \qquad i = 1,2$$

$$E(g_t\varepsilon_t) = \sigma_\varepsilon^2 \quad (17)$$

$$E(\varepsilon_t\varepsilon_{t-i}) = \begin{cases} \sigma_e^2 & \text{for } i = 0 \\ 0 & \text{otherwise} \end{cases}$$

Again, multiplying (13) by ε_{t-1}, ε_{t-2}, respectively and taking expectation using (17), we get

$$E(g_t\varepsilon_{t-i}) = (\Phi_1 - \Theta_1)\sigma_e^2 \quad (18)$$

$$E(g_t\varepsilon_{t-2}) = \left\{\Phi_1(\Phi_1 - \Theta_1) - \Phi_2 + \Theta_2\right\}\sigma_e^2$$

Substituting (18) into (16), we have

$$B_1 = B_2\sigma_e^2 \quad (19)$$

Where

$$B_1 = v_0 - \Phi_1 v_1 + \Phi_2 v_2$$
$$B_2 = \left\{1 - \Theta_1(\Phi_1 - \Theta_1) + \Theta_2\left[\Phi_1(\Phi_1 - \Theta_1) - \Phi_2 + \Theta_2\right]\right\}$$

Substituting (14) into (19), we have

$$B_1\left[(1-\alpha\phi)\{\alpha\theta(\phi-\theta) - (\alpha+\theta)\} + \phi\{1 - (\phi-\theta)(\alpha+\theta-\phi^2\theta)\}\right] = B_2\left[A_1\{\alpha\theta(\phi-\theta) - (\alpha+\theta)\} - A_2\{1-(\phi-\theta)(\alpha+\theta-\phi^2\theta)\} + A\right]$$

Or

$$f(\theta) = B_1\begin{bmatrix}(1-\alpha\phi)\{\alpha\theta(\phi+\theta) - (\alpha+\theta)\} + \\ \phi\{1-(\phi-\theta)(\alpha+\theta-\phi^2\theta)\}\end{bmatrix} - B_2\begin{bmatrix}A_1\{\alpha\theta(\phi-\theta) - (\alpha+\theta)\} \\ -A_2\{1-(\phi-\theta)(\alpha+\theta-\phi^2\theta)\} + A\end{bmatrix} \quad (20)$$

Our objective is to estimate the parameter θ. However, Eq. 20 is non-linear and can be solved by the Newton-Raphson process. In this case, the θ_{i+1} solution may be obtained from the i^{th} approximation according to

$$\theta_{i+1} = \theta_i - \left\{\frac{\left(B_1\begin{bmatrix}(1-\alpha\phi)\{\alpha\theta(\phi+\theta) - (\alpha+\theta)\} + \\ \phi\{1-(\phi-\theta)(\alpha+\theta-\phi^2\theta)\}\end{bmatrix} - B_2\begin{bmatrix}A_1\{\alpha\theta(\phi-\theta) - (\alpha+\theta)\} - \\ A_2\{1-(\phi-\theta)(\alpha+\theta-\phi^2\theta)\} + A\end{bmatrix}\right)}{\left(B_1\begin{bmatrix}(1-\alpha\phi)\{\alpha(\phi-2\theta) - 1\} + \\ \phi\{\alpha-\phi+\phi^3+2\theta(1-\phi^2)\}\end{bmatrix} - B_2\begin{bmatrix}(A_1\{\alpha(\phi-2\theta) - 1\} - \\ A_2\{\alpha-\phi+\phi^3+2\theta(1-\phi^2)\}\end{bmatrix}\right)}\right\} \quad (21)$$

Where

$$B_1 = v_0 - \Phi_1 v_1 + \Phi_2 v_2$$

$$B_2 = \left\{1 - \Theta_1(\Phi_1 - \Theta_1) + \Theta_2\left[\Phi_1(\Phi_1 - \Theta_1) - \Phi_2 + \Theta_2\right]\right\}$$

$$A_1 = v_0 - (\phi + \alpha)v_1 + \alpha\phi v_2$$

$$A_2 = v_1 - (\phi + \alpha)v_0 + \alpha\phi v_2$$

$$A = A_1\phi + A_2(1 - \alpha\phi)$$

To get a starting point for the iteration (21) is $\theta_0 = \Theta_1$-(α + ϕ) this is obtained by inspecting v. The denominator is the derivative of Eq. 21 with respect to θ. We expect the values of θ to be such that f(θ) at the point of convergence.

PROCESS AND ERROR ISOLATIONS

With the parameters, α and ϕ as well as the autocovariances, v_0, v_i and v_2 known, we can use Eq. 18 and 19 to estimate σ_α^2 and σ_e^2. Hence we can generate normal random processes with mean zero and variance σ_α^2 and with mean zero and variance σ_e^2 to represent the white noise processes α_t and e_t, respectively. We can do this by using the random number generator of any software package like MATLAB, for example.

With our knowledge of ϕ and θ together with the white noise process α_t, we can now estimate the process of interest f_t using Eq. 1 which results into the recursion below.

$$f_1 = f_1$$
$$f_2 = \phi\, f_1 - \theta\, a_1 + a_2$$
$$f_3 = \phi^2 f_1 - \phi\,\theta\, a_1 + (\phi - \theta) a_2 + a_3$$
$$\vdots$$
$$f_t = \phi^{t-1} f_1 - \phi^{t-2}\theta\, a_1 + \sum \phi^{t-(i+1)}(\phi - \theta) a_i + a_t,\ t = 1,2,... \quad (22)$$

In addition, with our knowledge of α together with the white noise process e_t, we can estimate the error process b_t using Eq. 6. This will result to the recursion below.

$$b_1 = e_1$$
$$b_2 = e_2 + \alpha e_1$$
$$\vdots$$
$$b_t = e_t + \alpha\ e_{t-1} + \alpha^2 e_{t-2} + \cdots + \alpha^{t-1} e_1$$
$$= \sum_{i=0}^{t-1} \alpha^i e_{t-i}, \quad t = 1,2,... \quad (23)$$

ILLUSTRATION

We use the NORMRND facility in MATLAB5 (1999) to simulate 1400 data points each of α_t and e_t. α_t is generated with mean 0 and variance 3.30 while e_t is generated with mean 0 and variance 1.15. To make α_t and e_t white noise, the means must be zeros while the variances can be any suitable positive values and they must be normally distributed. From Eq. 2 and 6, we ensure stationarity by choosing ϕ =0.45, θ and α = 0.35 (Box and Jenkins, 1976).

We use this to simulate 1,400 data points each of the following.

- f_t (Assumed unobserved) = 0.45 f_t-0.25α_t + α_t, t = 1, 2... 1400 (see Eq. 1)

The values of the process f_t was obtained using recursion (22)

- The AR(1) errors $b_t = 0.35b_{t-1} + e_t$, t=1,2,...1400 (see Eq. 4)

The values of the error process b_t was obtained using recursion (23)

- The observed value g_t is the sum of (1) and (2).

We discarded the first 200 data points to avoid initialization problems. This leaves us with 1200 data points for our analyses. However, due to space limitations, only ten data points from t = 201-t = 210 are shown in Table 1 for f_t and g_t as simulated processes.

Table 1: Simulated processes f_t and g_t with isolated processes $\hat{f}_t$ and $\hat{b}_t$
simulated processesestimated processes

f_t	g_t	$\hat{f}_t$	$\hat{b}_t$
1.235634	2.28663	1.4976	0.763661
3.021413	3.26811	2.8945	0.275617
1.694441	2.425333	1.23471	0.910201
2.951804	3.262341	2.8144	0.932351
2.976411	2.426314	2.7925	-0.32110
1.324601	1.06741	1.1443	-0.19316
4.853211	2.964353	5.312612	-2.46321
4.901011	2.43903	4.772	-2.52942
2.614231	1.85221	2.5324	-1.37421
2.984682	2.14632	3.17811	-1.02170

Our objective is to isolate the process f_t (Assumed unobserved or unknown) from the observed process g_t. We also isolate the error process b_t from g_t. We compute the first three autocovariances of the observe process g_t using formula (15) to obtain the rsesult v_0 = 1.9671, v_1 = 1.7469 and v_2 = 0.8712.

The Mcleod and Sales (1983) maximum likelihood estimates facilities in STATISTICA (1995) was used to obtain the following parameter values in g_t modeled after Eq. 13. The parameter values are

$$\Phi_1 = \alpha + \phi = 0.789$$
$$\Phi_2 = \alpha\phi = 0.1554$$
$$\Theta_1 = 0.99$$
$$\Theta_2 = 0.094$$

and solving using (vii) to get

ϕ = 0.4097 and α = 0.3793

We use the iterative formula (21) with θ_0 = 0.201 as starting value to estimate the parameter value θ. The iteration converges after four attempts to θ = 0.237.

We substitute ϕ = 0.4097, α = 0.3793 and θ = 0.237 together with v_0 = 1.9671, v_1 = 1.7469 and v_2 = 0.8712 into Eq. 11 and 12 to estimate the variances $\hat{\sigma}_e^2$ and $\hat{\sigma}_a^2$ of the white noise processes e_t and a_t respectively. We obtained the results $\hat{\sigma}_e^2$ = 1.41 and $\hat{\sigma}_a^2$ = 2.31.

Finally, we used the NORMRND facility in MATLAB5 (1999) to estimate the white noise process e_t distributed with mean = 0 and $\hat{\sigma}_a^2$ =1.41 as well as the process a_t distributed with mean = 0 and $\hat{\sigma}_e^2$ = 2.31. We then isolate the f_t by modeling it as

$$\hat{f}_t = 0.4097\hat{f}_{t-1} - 0.3793a_{t-1} + a_t$$

The values of the process f_t is obtained using the recursion

$$f_t = \phi^{t-1} f_1 - \phi^{t-2}\theta\, a_1 + \sum \phi^{t-(i+1)}(\phi - \theta) a_i + a_t,$$

t =1,2....1400 as in (22)

Ten values from t=201 to 210 are recorded in Table 1 as $\hat{f}_t$.

We also isolate the error process b_t by modeling it as

$$\hat{b}_t = e_t + 0.3793b_{t-1} \quad t=1,2....1400$$

The values of the error process b_t is obtained using the recursion formula

$$\hat{b}_t = \sum_{i=0}^{t-1} \alpha^i e_{t-1}, \quad t = 1,2,...\text{as in (23)}$$

Ten values from t = 201 to 210 are recorded in Table 1 as $\hat{b}_t$.

Examining Table1, we notice that the isolated process $\hat{f}_t$ is very close to the true (simulated) process f_t. Additionally, we observe that, the sum of the isolated processes $\hat{f}_t$ and $\hat{b}_t$ is close to the observe process g_t

This shows that the process developed in this study has been able to isolate the true ARMA process as well as the AR noise process.

CONCLUSION

We showed how autocovariance functions can be used to estimate the variances of the white noises that characterize an ARMA (1, 1) process subsumed in AR(1) errors. This was used to develop an iteration formula that can be used to estimate the parameters of both the ARMA (1, 1) model and the AR(1) errors. We also showed how the results obtained can be used to isolate both an unobservable process of interest known to follow ARMA (1,1) process and the AR(1) process that disturbs it. We performed simulation studies to demonstrate our findings. The studies showed that our method performed very well.

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