Journal of Modern Mathematics and Statistics 1 (1-4): 46-48, 2007
ISSN: 1994-5388

On the Existence and Uniqueness of Result on the Steady Flow of a Reactive Variable Viscosity Fluid in Cylindrical Pipe with an Isothermal Wall

[1]P.O. Olanrewaju and [2]M.O. Ogundiran
[1]Department of Pure and Applied Mathematic, Lodoke Akintola University of Technology, Ogobomosho, Nigeria
[2] Department of Physical Sceiences, Bells University of Technology, Ota, Nigeria

Abstract: In this study, we investigate the influence of viscous heating parameter on the steady flow of a reactive variable viscous fluid in a cylindrical pipe with an isothermal wall under reasonable assumption. Numerical solutions are constructed for the governing non linear boundary value problem using shooting technique together with Runge-Kutta method and important properties of the temperature field and thermal are critically discussed. Necessary and sufficient conditions for existence and uniqueness were provided for the problem to have physical implications.

Key words: Existence, viscous, cylindrical, isothermal wall, variable

INTRODUCTION

In Petrochemical industries and petroleum refineries, studies related to thermal ignition critically and heat transfer in a reactive variable viscosity fluid are extremely useful in order to ensure safety of life and properties (Bebernes and Eberly, 1989; Bowes, 1984). Thermal ignition occurs when reaction produces heat too rapidly for a stable balance between heat production and heat loss to be preserved. Hence, it is important to know the critical values of the basic physical quantities, such as the ambient temperature surface characteristics, the chemistry of the reacting material and the physical geometry at which hermal ignition occurs (Balakrishman *et al*., 1996; Berbernes and Eberly, 1989; Makinde, 2004, 2005).

Therefore, the concept of thermal critically or non-existence of steady-state solution of non-linear reaction diffusion problems for certain parameter values is very important from the application point of view. This characterizes the thermal stability properties of the material under consideration and onset of thermal runaway phenomenon (Kenneth, 2005).

MATHEMATICAL FORMULATION

The classical formulation of this type of problem was first introduced by Kamenetskii (1969) as shown in Fig. 1. Neglecting the reacting viscous incompressible fluid consumption, the equations for the heat balance and momentum in the original variables together with the boundary conditions can be written as:

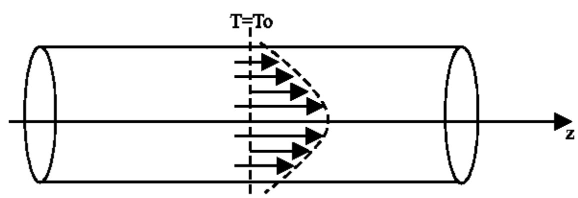

Fig. 1: Geometry of the problem

$$\frac{1}{\bar{r}}\frac{d}{d\bar{r}}\left(\bar{r}\frac{dT}{d\bar{r}}\right) + Q\frac{C_0 A}{K}e^{\frac{-E}{RT}} + \frac{\mu}{K}\left(\frac{\partial \bar{u}}{\partial \bar{r}}\right)^2 = 0 \quad (1)$$

$$\frac{1}{\bar{r}}\frac{d}{d\bar{r}}\left(\mu \bar{r}\frac{d\bar{u}}{d\bar{r}}\right) = -G$$

$$u = 0,\ T = T_0,\ \text{on } r = a \quad (2)$$

$$\frac{dT}{d\bar{r}} = \frac{d\bar{u}}{d\bar{r}} = 0,\ \text{on } \bar{r} = 0 \quad (3)$$

Where, T is the absolute temperature, $\bar{u}$ is the fluid axial velocity, G the constant axial pressure gradient, T_0 the wall reference temperature, K the thermal

Corresponding Auothor: P.O. Olanrewaju, Department of Pure and Applied Mathematics, Lodoke Akintola University of Technology, Ogobomosho, Nigeria

conductivityof the material, Q the heat of reaction, A the rate constant, E the activation energy, R the universal gas constant, Co the initial concentration of the reactant species, a the pipe characteristics radius, ($\bar{z},\bar{r}$) the distance measured in the axial and radial directions, respectively. It is assumed that the dynamic viscosity of the reactive viscous fluid under investigation is temperature dependent, that is:

$$\mu = \mu_0 e^{E/RT} \tag{4}$$

Where, μ_0 is the fluid reference viscosity. The following dimensionless variables are introduced into Eq. 1:

$$\left.\begin{aligned} \theta &= \frac{E(T-T_0)}{RT_0^{\,2}} \\ \varepsilon &= \frac{RT_0}{E} \\ y &= \frac{\bar{r}}{a} \\ \lambda &= \frac{QEAa^2C_0e^{-E/RT}}{T_0^{\,2}Rk} \\ W &= \frac{\mu_0\bar{u}e^{E/RT}}{Ga^2} \\ \beta &= \frac{G^2a^2}{4QC_0A\mu_0} \end{aligned}\right\} \tag{5}$$

and obtain the dimensionless governing equation together with the corresponding boundary conditions as

$$\begin{aligned} &\frac{dW}{dr} = -\frac{r}{2}e^{(\theta/1+\varepsilon\theta)}, \\ &\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta}{dr}\right) + \lambda\left(1+\beta r^2\right)e^{(\theta/1+\varepsilon\theta)} = 0 \end{aligned} \tag{6}$$

$$\frac{d\theta}{dr}(0) = 0,\ \theta(1) = W(1) = 0 \tag{7}$$

where λ, ε, β represent the Kamenetskii parameter, activation energy parameter and the viscous heating parameter, respectively. In the study, Eq. 6 and 7 are solved using both shooting technique with Runge-Kutta method (Olanrewaju *et al.*, 2006).

METHOD OF SOLUTION

The non-linear nature of the Eq. 6 and 7 precludes its solution exactly; hence, we employed shooting technique. We let,

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} r \\ W \\ \theta \\ \theta' \end{pmatrix} \tag{8}$$

$$\begin{pmatrix} y_1' \\ y_2' \\ y_3' \\ y_4' \end{pmatrix} = \begin{pmatrix} 1 \\ -\frac{y_1}{2}e^{(y_3/1+\varepsilon y_3)} \\ y_4 \\ -\frac{1}{y_1}y_4 - \lambda\left(1+\beta y_1^{\,2}\right)e^{(y_3/1+\varepsilon y_3)} \end{pmatrix} \tag{9}$$

Satisfying

$$\begin{pmatrix} y_1(0) \\ y_2(0) \\ y_3(0) \\ y_4(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ c \\ \alpha \end{pmatrix} \tag{10}$$

Where, c is a constant and á is the shooting ques value. i.e.

$$\theta'(0) = \alpha \text{ at } \theta(1) = W(1) = 0$$

EXISTENCE AND UNIQUENESS OF SOLUTION

Theorem 1:
Let D denote the region (n (n+1))
Dimensional space,one dimension for t and n dimensions for vector x] $|t-t_0| \le a$, $\|x-x_0\| \le b$.
if

$$\begin{cases} x_1^1 = f_1(x_1,x_2,\dots,xn,t)\ ,\ x_1(t_0) = x_{10} \\ x_2^1 = f_2(x_1,x_2,\dots,xn,t)\ ,\ x_2(t_0) = x_{20} \\ . \\ . \\ . \\ x_n^1 = f_n(x_1,x_2,\dots,xn,t)\ ,\ x_n(t_0) = x_{n0} \end{cases} \tag{10}$$

Then, the system of Eq. 11 has a unique solution of

$$\frac{\partial f_i}{\partial x_j},\quad i,j=1,2,\dots,n \tag{11}$$

are continuous in D.
H1: $\alpha > 0$, $0 \le x_1 \le 1$, $0 \le x_2 \le b$, and
$1 \le x_3 \le L$, where b, l and L are positive constants.

Theorem 2:

IF H_1 holds then problem (1) has a unique solution satisfying (2) and (3).

Proof:

We let;

$$y' = \begin{pmatrix} y_1' \\ y_2' \\ y_3' \\ y_4' \end{pmatrix} = \begin{pmatrix} 1 \\ -\frac{y_1}{2} e^{(y_3/1+\varepsilon y_3)} \\ y_4 \\ -\frac{1}{y_1} y_4 - \lambda(1+\beta y_1{}^2) e^{(y_3/1+\varepsilon y_3)} \end{pmatrix}$$

Where,

$$g_1 = 1, \quad g_2 = -\frac{y_1}{2} e^{(y_3/1+\varepsilon y_3)}$$

$$g_3 = y_4 \quad \text{and} \quad g_4 = -\frac{1}{y_1} y_4 - \lambda(1+\beta y_1{}^2) e^{(y_3/1+\varepsilon y_3)}$$

Clearly $\partial g_i/\partial x_j$ is bounded for i = 1, 2, 3,4.

Thus, g_i, i= 1, 2, 3,4 are lipschitz continuous. Hence there exists a unique solutions of Eq. 1 satisfying Eq. 2 and 3.

REFERENCES

Balakrishnan, E., A. Swift and G.C. Wake, 1996. Critical values for some non-class a Geometries in the thermal ignition theory. Math. Comput. Modelling, 24: 1-10.

Bebernes, J. and D. Eberly, 1989. Mathematical Problems from Combustion Theory. Springer-Verlag, New York.

Bowes, P.C. 1984. Self-heating: Evaluating and Controlling the Hazard, Elsevier Sciences, Amsterdam.

Kamenetskii, D.A., 1969. Diffusion and Heat Transfer in Chemical Kinetics, Plenum Press, New York.

Kenneth, K.K., 2005. Principles of Combustion, John Wiley and Sons, Inc., Hoboken, New Jersey.

Makinde, O.D., 2004. Exothermic explosions in a slab: A case study of series summation Technique. Int. Commun. Heat Mass Trans., 31: 1227-12237.

Makinde, O.D., 2005. Strong exothermic explosions in a cylindrical pipe: A case study of series summation technique. Int. Commun. Heat Mass Trans., 32: 191-195.