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Modeling Horizontal Transmission of HIV/AIDS in an Age-Structured Population

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Abstract: We extended McKendrick-Von-Foerster type age-structured model to formulate a 2-age groups structured for heterosexual transmission of HIV/AIDS, in a proportionate mixing population, with constant per capital force of infection. We derive the governing equations, which describes the dynamics of population and examine the model for local stability of the endemic state. We found that to get a disease-free state we need to maximize the transfer rate between the sexually immature group and sexually matured group and minimize the average time spend as infective in group 2, before developing AIDS.

Key words: Transmission, HIV/AIDS, population, modeling, age-structured, mixing population, Nigeria

INTRODUCTION

HIV-infection, referred to human immunodeficiency virus was first discovered in 1983 in the united states of America as a causative agent of AIDS. Extensive expositions on Biological aspect of HIV/AIDS transmission mechanism is in Hethcote and Ark (1984), Castillo-Chavez *et al.* (1991), Chiang (1980) and Mugisha and Luboobi (2003). Three major modes of transmission of the virus have been identified as responsible for its spread. These includes Heterosexual contact with an infectious person, direct contact with HIV-infected blood or fluid and transmission from mother to her new born, (called mother to child transmission). However, the epidemic is often rooted in economics and cultural attitudes, practices and beliefs. Sub-Sahara Africa is said to be the epicenter of HIV and AIDS epidemic. Studies carried out by UNAIDS shows that there is a pronounced geographical difference in prevalence rates in Kenya, Zambia, Cameroon and Benin republic, which may be due to several other indicators like sexual coercion, STI symptoms and care, age disparity between partners, alcohol and drug consumption, number of sex acts, mobility, sexual partnership networks. However, in some area of the world HIV has spread through heterosexual transmission while in others, HIV became entrenched in certain sub-populations with specific sexual or drug-injecting behaviors that are associated with high virus transmission rates. The interaction between groups with high-risk behaviors and more general population is a major determinant of the rate of spread of the virus. To have a sustained heterosexual epidemic within a general population, there must be a sustained sexual mixing among adults without use of condom. This means each individual infected must have unprotected sex with a minimum of 2 partners, becoming infected by one and passing the infection to at least one other. However, for sustained heterosexual epidemic, significant proportions of the interacting groups must have a number of sexual partners over their lifetime. This research focuses on two age-group structured model for heterosexual mode of transmission of HIV along the line of Mugisha and Luboobi (2003).

Hethcote and Ark (1984), Castillo-Chavez *et al.* (1991) and Inaba (1990), age-structured population model.

MODEL FORMULATION

We considered a continuous age distribution model along the line of Mckendrick-Von-Foerster type age-structured model with the following assumptions:

- The transmission of HIV from an infectious individual of age a at time t, to a susceptible individual of age $\bar{a}$ at time t is through heterosexual mode
- There is random mixing of individuals within the population
- Individuals in the population are subjected to the same natural mortality rate
- HIV-infectious individual who developed AIDS will surely die of AIDS induced death, since HIV/AIDS has no known cure
- AIDS cases has full-blown symptoms and are easily noticeable and not sexually interacted with, as such they don't transmit the virus and don't give birth to new born

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- In each group, the sexual active non-active population compartments satisfy

$$n(t,a) = S(t,a) + I(t,a)$$

Then the dynamics of the population compartments is described by the following equations, along the line of Mugisha and Luboobi (2003), Hethcote and Ark (1984), Inaba (1990) and Castillo-Chavez *et al.* (1991),

$$\frac{\partial S}{\partial t} + \frac{\partial S}{\partial a} = -\mu(a)S(t,a) - \beta(t,a)S(t,a) \quad (1)$$

$$\frac{\partial I}{\partial t} + \frac{\partial I}{\partial a} = \beta(t,a)S(t,a) - (\mu(a) + k(a))I(t,a) \quad (2)$$

$$\frac{\partial A}{\partial t} + \frac{\partial A}{\partial a} = k(a)I(t,a) - (\mu(a) + \alpha(a))A(t,a) \quad (3)$$

Where, S(t, a) I(t, a) and A(t, a) are the population densities of the susceptible, infectious and those who progressed to AIDS, k (a) is the age depended progression rate from HIV to AIDS, μ (α) is the age depended natural mortality rate, β (t, α) is the per capital force of infection,

$$\beta(t,a) = \frac{\int_0^\infty p(a,\bar{a})I(t,\bar{a})d\bar{a}}{\int_0^\infty n(t,a)da} \quad (4)$$

With initial and boundary conditions,

$$S(a,0) = S_0(a), \quad I(a,0) = I_0(a), \quad n(a,0) = n_0(a)$$

$$S(0,t) = \int_0^\infty S(t,a)f(a)da \quad (5)$$

$$I(0,t) = 0 \quad (6)$$

Where, $p(a, \bar{a})$ is the age depended activity level, described as:

$$p(a,\bar{a}) = p(a)p(\bar{a})$$

f (a) is the fertility function. Thus we have,

$$\beta(t,a) = \frac{p_1(a)}{N(t)}\int_0^\infty p_2(\bar{a})I(t,\bar{a})d\bar{a}$$

Where,

$$N(t) = \int_0^\infty n(t,a)da,\ n(t,a) = S(t,a) + I(t,a) + A(t,a)$$

$$S(t) = \int_0^\infty S(t,a)da \quad I(t) = \int_0^\infty I(t,a)da \quad (7)$$

The population fractions of the densities are:

$$s(t,a) = \frac{S(t,a)}{N(t,a)},\ i(t,a) = \frac{I(t,a)}{N(t,a)},\ z(t,a) = \frac{A(t,a)}{N(t,a)} \quad (8)$$

Where, N (t, a) is the population density of age a, at time t.

The equations describing the dynamics of interaction between the host and the infection are:

$$\frac{\partial s(t,a)}{\partial t} + \frac{\partial s(t,a)}{\partial a} = -\beta(t,a)s(t,a)$$
$$\frac{\partial i(t,a)}{\partial t} + \frac{\partial i(t,a)}{\partial a} = \beta(t,a)s(t,a) - k(a)i(t,a)$$
$$\frac{\partial z(t,a)}{\partial t} + \frac{\partial z(t,a)}{\partial a} = k(a)i(t,a)$$

With initial and boundary conditions:

$$s(0,a) = s_0(a) = \frac{S_0(a)}{N(t,a)},\ i(0,a) = i_0(a) = \frac{I(0,a)}{N(t,a)},$$
$$z(0,a) = z_0(a) = \frac{A(0,a)}{N(t,a)}$$
$$s(t,o) = \int_0^\infty \{s(t,a) + i(t,0)\}f(a)da,\ i(t,0) = 0$$

We now use the model to formulate an HIV/AIDS, 2 age-groups transmission model, where the first group I, are the sexually non-active or immature group and the second group II, are the sexually active or matured group. Let the population be divided into 2 age groups by the age intervals, a_{i-1}, a_i, for i = 1, 2, where a_2 is the maximum age. The number of susceptible, infective and AIDS cases in the ith age group (interval) is defined as:

$$S_i(t) = \int_{a_{i-1}}^{a_i} S(t,a)da,\ I_i(t) = \int_{a_{i-1}}^{a_i} I(t,a)da,$$
$$A_i(t) = \int_{a_{i-1}}^{a_i} A(t,a)da \quad (9)$$

Suppose, at the start of the epidemic, the population is at the stationary age distribution, with exponential growth in all the classes as:

$$N(t,a) = e^{r_0 t} B(a)$$

Where, B (a) is the total population at age a and r_0 is the intrinsic rate of population growth at steady age distribution and satisfies the Lotka characteristics equation:

$$\int_0^{\infty} e^{-r_0 a} f(a) l(a) = 1$$

Where, f (a) is fertility function and

$$l(a) = \exp\{-\int_0^a \mu(\sigma) d\sigma\}$$

is the survival function.

The total number of individuals in the age interval (a_{i-1}, a_i) is,

$$N_i(t) = \int_{a_{i-1}}^{a_i} N(a,t) da = e^{r_0 t} \int_{a_{i-1}}^{a_i} B(a) da = e^{r_0 t} \pi_i.$$

Where,

$$\pi_i = \int_{a_{i-1}}^{a_i} B(a) da$$

is the size of the ith age-group at steady state at time t = 0,

Approximated as:

$$\pi_i \approx B(a)(a_i - a_{i-1}), \sum_{i=1}^{2} \pi_i = 1 \qquad (10)$$

within each age group. For $a_{i-1} \le a \le a$, we let $\mu(a) = \mu_i$, $k(a) = k$, $\alpha(a) = \alpha_i$, $\beta(t, a) = \beta_i$, $f(a) = f_i$. The renewal equations for both compartments are reduced to:

$$\begin{aligned} S(0,t) &= \int_0^{\infty} \{S(t,a) + I(t,a)\} f(a) da \\ &= \sum_{i=1}^{2} f_i [S_i(t) + I_i(t)] \quad I(0,t) = 0 \end{aligned} \qquad (11)$$

Where,

$$N(0,t) = e^{r_0 t} B(0) = S(0,t) = e^{r_0 t} \sum_{i=1}^{2} f_i \pi_i, \; i = 1, 2$$

Let, the transfer rate constant between the two age groups be c_i, i = 1, 2. Where, $c_i = 1/a_i - a_{i-1}$. The mode in which an individual in each epidemiological group crosses into another group is described as:

$$\begin{aligned} &S(a_i, t) = c_i S_i(t), \; I(a_i, t) = c_i I_i(t), \\ &A(a_i, t) = c_i A_i(t), \; B(a_i) = c_i \pi_i \end{aligned} \qquad (12)$$

We define the fractions of ith group in the epidemiological classes in line with the representation for the fraction of the population densities as:

$$s_i(t) = \frac{S_i(t)}{N_i(t)} = \frac{\int_{a_{i-1}}^{a_i} S(t,a) da}{\int_{a_{i-1}}^{a_i} n(t,a) da} = \frac{\int_{a_{i-1}}^{a_i} S(t,a) da}{e^{r_0 t} \pi_i} \qquad (13)$$

$$i_i(t) = \frac{I_i(t)}{N_i(t)} = \frac{\int_{a_{i-1}}^{a_i} I(t,a) da}{\int_{a_{i-1}}^{a_i} n(t,a) da} = \frac{\int_{a_{i-1}}^{a_i} I(t,a) da}{e^{r_0 t} \pi_i} \qquad (14)$$

Where, in each group, the following holds,

$$s_i + i_i = 1, \quad \sum_{i=1}^{3} \pi_i = n_0(a)$$

Differentiating the above equation with respect to t, gives the following:

$$s_i'(t) = \frac{S_i'(t)}{e^{r_0 t} \pi_i} - r_0 s_i(t), \quad i_i'(t) = \frac{I_i'(t)}{e^{r_0 t} \pi_i} - r_0 i_i(t)$$

We define the contact or interaction function,

$$p(a, \bar{a}) = p_{ij} \text{ for } a \in [a_{i-1}, a_i) \text{ and } \bar{a} \in [a_{j-1}, a_j)$$

to represent the interaction between susceptible in the ith age group and infective in the jth age group. Also, we assume an interval or class depended force of infection, λ_i in the age interval (a_{i-1}, a_i). Using the representation for the force of infection, in Eq. (4) and (10), we have the class depended force of infection as:

$$\beta_i(t) = \frac{\sum_{k=1}^{2} p_{ik} I_k(t)}{\sum_{i=1}^{2} n_i(t)} = \frac{\sum_{k=1}^{2} p_{ik} e^{r_0 t} \pi_k i_k(t)}{\sum_{i=1}^{2} e^{r_0 t} \pi_i} = \sum_{k=1}^{2} p_{ik} \pi_k i_k(t) \qquad (15)$$

When i =1, the force of infection in group I, given by,

$$\beta_1(t) = \sum_{k=1}^{2} p_{1k} I_k(t) \pi_k = p_{11} i_1(t) \pi_1 + p_{12} i_2(t) \pi_2$$

For i = 2, we have the force of infection in group 2,

$$\beta_2(t) = \sum_{k=1}^{2} p_{2k} I_k(t) \pi_k = p_{21} i_1(t) \pi_1 + p_{22} i_2(t) \pi_2$$

Since, there is no sexual interaction among individual in group I and between individuals in Group 1 and 2 and so all terms p_{11}, p_{12}, p_{21}. We have $\beta_1(t) = 0$,

$$\lambda_2(t) = p_{22} i_2(t) \pi_2$$

Integrating the Eq. (23) with respect to a over (a_{i-1}, a_i) and using Eq. (9) gives:

$$S(t,a_i) - S(t,a_{i-1}) + \frac{dS_i(t)}{dt} = -\lambda_i \int_{a_{i-1}}^{a_i} S(t,a)da - \mu_i \int_{a_{i-1}}^{a_i} S(t,a)da$$

$$S(t,a_i) - S(t,a_{i-1}) + \frac{dS_i(t)}{dt} = -\lambda_i S_i(t) - \mu_i S_i(t) \tag{16}$$

Thus, for i = 1, we have,

$$S(t,a_1) - S(t,0) + \frac{dS_1(t)}{dt} = -\lambda_1 S_1(t) - \mu_1 S_1(t) \tag{17}$$

Using Eq. (11-13) in Eq. (16)
We have the following:

$$c_1 S_1(t) - \sum_{i=1}^{2} f_i(S_1(t) + I_1(t)) + \frac{dS_1(t)}{dt} = -\lambda_1 S_1(t) - \mu_1 S_1(t)$$

$$c_1 s_1(t)\pi_1 - \pi_2 f_2(s_2(t) + i_2(t)) + \pi_1 \frac{ds_1(t)}{dt} + \pi_1 s_1(t) = \lambda_1 \pi_1 s_1(t) - \mu_1 \pi_1 s_1(t)$$

$$\frac{ds_1(t)}{dt} = \frac{\pi_2}{\pi_1} f_2(s_2(t) + i_2(t)) - (r_0 + c_1 + \mu_1) s_1(t)$$

i = 2, Eq. (16), reduces it to the form,

$$S(t,a_2) - S(t,a_1) + \frac{dS_2(t)}{dt} = -\lambda_2 S_2(t) - \mu_2 S_2(t)$$

Using Eq. (12-14) into Eq. (17), we get the following:

$$c_2 S_2(t) - c_1 S_1(t) + \frac{dS_2(t)}{dt} = -\lambda_2 S_2(t) - \mu_2(t),$$

this gives,

$$c_2 \pi_2 s_2(t) - c_1 \pi_1 s_1(t) + \pi_2 \frac{ds_2(t)}{dt} + r_0 \pi_2 s_2(t) = -\lambda_2 \pi_2 s_2(t) - \mu_2 \pi_2 s_2(t)$$

Thus, we have,

$$\frac{ds_2(t)}{dt} = c_1 \frac{\pi_1}{\pi_2} s_1(t) + \lambda_2 s_2(t) - (r_0 + c_2 + \mu_2) s_2(t)$$

Integrating Eq. (2) w. r. t a and using we get:

$$\frac{dI_1(t)}{dt} + I(t,a_i) - I(t,a_{i-1}) = \beta_i S_i(t) - \mu_i I_i(t) - k_i I_i(t) \tag{18}$$

Thus, i = 1, gives the equations:

$$\frac{dI_1(t)}{dt} + I(t,a_1) - I(t,a_0) = \beta_1 S_1(t) - \mu_1 I_1(t) - k_1 I_1(t)$$

$$\frac{dI_1(t)}{dt} + c_1 I_1(t) = \beta_1 S_1(t) - \mu_1 I_1(t) - k_1 I_1(t)$$

$$\frac{dI_1(t)}{dt} + c_1 I_1(t) = \beta_1 S_1(t) - \mu_1 I_1(t) - k_1 I_1(t)$$

Using Eq. (13) and (14), we have

$$\frac{di_1(t)}{dt} = \beta s_1(t) - (r_0 + + c_1 + \mu_1 + k_1) i_1(t)$$

When i = 2, in Eq. (18) gives,

$$\frac{dI_2(t)}{dt} + I(t,a_2) - I(t,a_1) = \beta_2 S_2(t) - \mu_2 I_2(t) - k_2 I_2(t)$$

This reduces to:

$$\frac{di_2(t)}{dt} = c_1 \frac{\pi_1}{\pi_2} i_1(t) - \beta_2 s_2(t) - (r_0 + c_2 + \mu_2 + k_2) i_2(t)$$

The two-age groups HIV/AIDS epidemic models are:

$$\frac{ds_1(t)}{dt} = \frac{\pi_2}{\pi_1} f_2(s_2(t) + i_2(t)) - (r_0 + c_1 + \mu_1) s_1(t) \tag{19}$$

$$\frac{di_1(t)}{dt} = \beta_1 s_1(t) - (r_0 + c_1 + \mu_1 + k_1) i_1(t) \tag{20}$$

$$\frac{ds_2(t)}{dt} = c_1 \frac{\pi_1}{\pi_2} s_1(t) + \beta_2 s_2(t) - (r_0 + c_2 + \mu_2) s_2(t) \tag{21}$$

$$\frac{di_2(t)}{dt} = c_1\frac{\pi_1}{\pi_2}i_1(t) + \beta_2 s_2(t) - (r_0 + c_2 + \mu_2 + k_2)i_2(t) \quad (22)$$

MODEL ANALYSIS

We assume that the infection rates in the sexually active population, β_2 is a constant. Using $s_1 = 1\text{-}i_1$, $s_2 = 1\text{-}i_2$, the system of Eq. (19-22) are reduced to a two dimensional system in terms of population fractions of infectives:

$$\frac{di_1(t)}{dt} = \beta_1(1 - i_1(t)) - (r_0 + c_1 + \mu_1 + k_1)i_1(t) \quad (23)$$

$$\frac{di_2(t)}{dt} = c_1\frac{\pi_1}{\pi_2}i_1(t) + \beta_2(1 - i_2(t)) - (r_0 + c_2 + \mu_2 + k_2)i_2(t) \quad (24)$$

Using wolfram resaerch mathematica software, we find solutions for, $i_1(t)$, $i_2(t)$ from Eq. (23-24), using $i_1(0)$, = $i_2(0) = 0$,

$$i_1(t) = \phi - \phi' e^{-(a+\beta_1)t}, \quad \phi = \frac{1}{a+\beta_1}, \quad \phi' = \frac{\beta_1}{a+\beta_1}, \; a = r_0 + c_1 + \mu_1 + k_1. \quad (25)$$

$$i_2(t) = -\psi + \psi' e^{-(a+\beta_1)t} - \psi'' e^{-(b+\beta_2)t}, \quad \psi = \frac{S}{p}, \quad \psi' = \frac{D'}{p}, \quad \psi'' = \frac{T}{p}$$

$$s_1(t) = \alpha_1 e^{-\frac{1}{2}(c-\sqrt{c^2-4D})t} + \alpha_2 e^{-\frac{1}{2}(c+\sqrt{c^2-4D})t} - \frac{A}{(1-c)(a+\beta_1)+D}e^{-(a+\beta_1)t} + \frac{B}{(1-c)(b+\beta_2)+D}e^{-(b+\beta_2)t}$$

$$s_2(t) = -\frac{2kM}{c-\sqrt{c^2-4D}-k}e^{-\frac{1}{2}(c-\sqrt{c^2-4D})t} - \frac{2M}{c+\sqrt{c^2-4D}-k}e^{-\frac{1}{2}(c+\sqrt{c^2-4D})t} - \frac{A}{(1-c)[(a+\beta_1)+D](a+\beta_1-\eta_1)}e^{-(a+\beta_1)t} - \frac{B}{(1-c)[(a+\beta_2)+D](a+\beta_2-\eta_1)}e^{-(b+\beta_2)t} + \alpha_3$$

Where, a_1, a_2, a_3 are the constant of integration,

$$S = (a+b)c\beta_1 - (c\beta_1^2 + a\beta_2 + ab\beta_2 - 2a\beta_1\beta_2 + b\beta_1\beta_2 + c\beta_1\beta_2 - \beta_1^2\beta_2 + \beta_1\beta_2^2)$$

$$D' = \beta_1 c(b+\beta_2)$$

$$T = ac\beta_1 - c\beta_1^2 + a^2\beta_2 - ab\beta_2 + 2a\beta_1\beta_2 - b\beta_1\beta_2 + \beta_1^2\beta_1 - a\beta_2^2 - \beta_1\beta_2^2$$

$$p = (b+\beta_2)(a^2 - ab - 2a\beta_1 - b\beta_1 + \beta_1^2 - a\beta_2 - \beta_1\beta_2)$$

$$b = r_0 + c_2 + \mu_2 + k_2, \qquad c = c_1\frac{\pi_1}{\pi_2}$$

$$c = 2r_0 + c_1 + c_2 + \mu_1 + \mu_2 - \beta_2, \; B = \frac{\pi_2}{\pi_1}f_2, \quad D = B\left(\frac{r_0 + c_1 + \mu_1}{B} - M\right), \; M = c_1\frac{\pi_1}{\pi_2}$$

$$k = r_0 + c_2 + \mu_2 - \beta_2, \; A = r_0 + c_1 + \mu_1$$

The limiting population size in each of the epidemiological classes is:

$$i_1(t) = \phi_1 = \frac{1}{a+\beta_1} = \frac{1}{r_0 + c_1 + \mu_1 + k_1 + \beta_1}$$

$$i_2(t) = -\psi + \psi' = \frac{D' - S}{p} = \frac{(b+\beta_1+\beta_2)\beta_1 c + ((b+c-2a-\beta_1+\beta_2)\beta_1)\beta_2}{(b+\beta_2)[(1-b-2\beta_1-\beta_2)a - (\beta_1+b+\beta_2)\beta_1]}$$

$$s_1(t) = 1 - \frac{1}{a+\beta_1} = 1 - \frac{1}{r_0 + c_1 + \mu_1 + k_1 + \beta_1}$$

$$s_2(t) = 1 - \frac{(b+\beta_1+\beta_2)\beta_1 c + (b+c-2a-\beta_1+\beta_2)\beta_1)\beta_2}{(b+\beta_2)[(1-b-2\beta_1-\beta_2) - (\beta_1+b+\beta_2)\beta_1]}$$

Provided:

$$(1-b-2\beta_1-\beta_2)a > (\beta_1+b+\beta_2)\beta_1$$

Since, in the sexually immature group, the force of infection is zero, means that there will be no infection in this group, everybody is susceptible and so $i_1(t) = 0$. We would have a disease-free sexually immature group, since vertical transmission of the infection is avoidable, if infectious mothers are given sufficient medical guide and advice.

STEADY STATE SOLUTION

Let M(i_1*, i_2*), be steady state solution of the system. Substituting these into the governing Eq. (23) and (24) gives, the following:

$$0 = \beta_1(1-i^*) - (r_0 + c_1 + \mu_1 + k_1)i_1^* \quad (26)$$

$$0 = c_1\frac{\pi_1}{\pi_2}i_1^* + \beta_2(1-i_2^*) - (r_0 + c_2 + \mu_2 + k_2)i_2^* \quad (27)$$

Solving theses equations gives, the steady solutions to the system:

$$i_1^* = \frac{\beta_1}{\beta_1 + r_0 + c_1 + \mu_1}, \quad (28)$$

$$i_2^* = \frac{1}{\beta_2 + r_0 + c_2 + \mu_2 + k_2}\{\frac{c_1\beta_1 \pi_1/\pi_2}{\beta_1 + r_0 + c_1 + \mu_1 + k_1} + \beta_2\} \quad (29)$$

Thus, in order to have a disease-free State we need to have, $\beta_1 = \beta_2 = 0$
Where,

$$i_1^* = \frac{\beta_1}{\beta_1 + r_0 + c_1 + \mu_1}$$

is the fraction of the immature group infectives who survived the infection before showing full-blown AIDS symptoms. Similarly,

$$\frac{1}{\beta_2 + r_0 + c_2 + \mu_2 + k_2}$$

is the average length of time an infectious individual adult in group 2, can stay in the group. $i_1^* = 0$, since $\beta_1 = 0$, no vertical transmission is assumed.

Using, the assumptions of sexually immature for group 1 and 2, the only HIV-infection transmitting group is 2 and so Eq. (29) reduces to,

$$i_2^* = \frac{1}{\beta_2 + r_0 + c_2 + \mu_2 + k_2}\beta_2 \quad (30)$$

Which we define as the fraction of adult infective who survived the adult group before showing full-blown AIDS symptoms.

Theorem: The endemic steady state is locally asymptotically if:

$$(\beta_1 + r_0 + c_1 + \mu_1 + k_1)(\beta_2 + r_0 + c_2 + \mu_2 + k_2) > c_1\frac{\pi_1}{\pi_2} \quad (31)$$

Proof: We define the Jacobian matrix of the system as,

$$\begin{pmatrix} -(\beta_1 + r_0 + c_1 + \mu_1 + k_1, & 0 \\ c_1\frac{\pi_1}{\pi_2} & -(\beta_2 + r_0 + c_2 + \mu_2 + k_2) \end{pmatrix}$$

This matrix has trace = -{$(\beta_1 + \beta_2) + (c_1 + c_2) + (\mu_1 + \mu_2) + (k_1 + k_2)$}, less than zero and determinant =

$$(\beta_1 + r_0 + c_1 + \mu_1 + k_1)(\beta_2 + r_0 + c_2 + \mu_2 + k_2) - c_1\frac{\pi_1}{\pi_2}$$

which is positive only when

$$(\beta_1 + r_0 + c_1 + \mu_1 + k_1)(\beta_2 + r_0 + c_2 + \mu_2 + k_2) > c_1\frac{\pi_1}{\pi_2}$$

THE BASIC REPRODUCTIVE NUMBER

The basic reproductive number is defined as the average number of secondary infectious cases produced by a typical infectious individual during his/her infectious entire life time when introduced into a totally susceptible population. This is obtained using the next generator operator by Diekmann (Heffernan *et al*., 2005):

$$R_0 = \frac{c_1\pi_1}{\pi_2[(r_0 + c_1 + \mu_1 + k_1) + (r_0 + c_2 + \mu_2 + k_2)]} \quad (32)$$

From the interpretation of R_0 in Heffernan *et al.* (2005) and Castillo-Chavez *et al.* (1991), if R_0>1, then the endemic steady state is stable and HIV/AIDS epidemic will persist, in the population and however, if R_0>1, then the disease-free steady state is locally asymptotically stable and the epidemic will die out. This is possible if the transfer rate between the group I and II satisfies:

$$c_1 < \frac{\pi_2}{\pi_1 - \pi_2}\{(r_0 + \mu_1 + k_1) + (r_0 + c_2 + \mu_2 + k_2)\}, \pi_1 > \pi_2 \quad (33)$$

CONCLUSION

In this study, we have formulated a deterministic dynamic model for transmission of HIV/AIDS, in a two age-group population, assuming constant force of infection. We obtained analytical expression for the proportion of the population in each epidemiological classes and examine the governing equations for local stability of the endemic state. We found that the endemic steady state is locally asymptotically stable if Eq. (31) holds and a disease-free steady state is realized if Eq. (33) is satisfied. To control the spread of the disease and achieve its total elimination we need to minimize the transfer rate between group 1 and 2, so that individuals spend more time in the sexually immature group and less time as an infected adult in group 2. This will increase the population of the immature group and gradually eliminate the infectious population in group 2, with both groups 1 and 2 reduced to susceptible population.

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