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Global Attractivity of a Positive Periodic Solution for Delayed Predator-Prey System with Beddington-De-Angelis Functional Response and Stocking

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Abstract: In this study, we consider a delayed predator-prey system with Beddington-De-Angelis functional response and stocking. By using coincidence degree theory, the existence of positive periodic solutions for above system is established. Then by applying the existence result of positive periodic solutions and constructing a Lyapunov functional, the global attractivity of a positive periodic solution for above system with Beddington-De-Angelis functional response and stocking is established.

Key words: Existence of positive periodic solutins, delayed predator-prey system, global attractivity, Beddington-De-Angelis functional response, stocking coincidence degree theory

INTRODUCTION

We consider the following nonalltononcous predatrrey system with Beddington-De-Angelis functional response and harvest (or stocking).

$$\begin{cases} x_1'(t) = x_1(t)\left[a_1(t) - b_1(t) - \dfrac{s_1(t)x_2(t)}{A_1(t) + x_1(t) + B_1(t)x_2(t)}\right] + h_1(t) \\ x_2'(t) = x_2(t)\left[\begin{array}{c} -a_2(t) - b_2(t)x_2(t) + \\ \dfrac{s_2(t)x_1(t-\Gamma)}{A_1(t) + x_1(t-\Gamma) + B_1(t)x_2(t-\Gamma)} \end{array}\right] + h_2(t) \end{cases} \quad (1)$$

Where:

$x_1(t)$ and $x_2(t)$	=	The population density of prey, predator at time t, respectively
$h_1(t)$ and $h_2(t)$	=	The stocking of prey, predator at time t, respectively
a_i, b_i, $s_i(i\neq n)$	=	Positive continuous w-periodic functions
Γ	=	A positive constant

On the existence and global attractivity of positive periodic solutions to system Eq. (1), few results have been found in the present studies. This motivates us to consider the existence and global attractivity of positive periodic solutions to system Eq. (1). In this study, our purpose is to derive a set of easily verifiable.

Sufficient conditions for the existence and attractivity of a positive periodic solution for system Eq. (1). Existing results on the existence and attractivity of a positive periodic solution in periodic population models often fall into one of the following two categories:The results of the application of the contraction principle or fluctuation principle, which establish both the existence and attractivity of the periodic solutions in periodic population models with time delay (Kilang) (Kuang, 1993); the results of application of becoming Brower fixed point theorem and Lyapunov functional with the results of persistence of positive solution, which first establish the existence of positive periodic solutions in periodic population models with time delays by using the Brower fixed point theorem and the results of persistence of positive solutions, then establish the attractivity of positive periodic solutions in periodic population models by using a Lyapunov functional (Freefman and Peng, 1999; Song and Chen, 1998; HongLiang and Kuiehen, 2000). Though those methods often allow the investigator to address the stability issues of the positive periodic solution of population models, the conditions for the existence part are often unnecessary numerous, tedious, stringent and difficult to satisfy.

In this study, using a similar way to that in Zhang and Wang (2006) and Zhang and Li (2006), we first establish the existence of positive periodic solutions of system Eq. (1) by means of using coincidence degree theory (Mawhin, 1979) and topological degree theory, then establish the attractivity of positive periodic solutions for system Eq. (1) by using a Lyapunov functional.

In this research, by applying Mawhia (1979) continuation theorem, we establish the existence of positive periodic solutions for system Eq. (1). The study shows the results, by constructing a Lyapunov periodic solution of system Eq. (1).

EXISTENCE OF POSITIVE PERIODIC SOLUTIONS

In this study, based on Mawhin's continuation theorem, we will study the existence of at least one positive periodic solution of system Eq. (1). First, we shall make some preparations.

Let X, Z be red Banach spaces, L: Doml⊂X→Z a Fredholm mapping of index Zero and P: X→X, Q: Z→Z continuous projectors such that Imp = Kerl, KerQ = ImL and X = KerL⊕KerP, Z = ImL⊕ImQ. Denote by Lp the restriction of L to Doml⋂KerP, KP: ImL-Kerp⋂Doml the inverse (to Lp) and J: ImQ-KerL an isomorphism of ImQ on to KerL. For convenience of use, we introduce the continuation theorem (Mawhin, 1979) as follows.

Lemma 2.1 let $\Omega \subset$ i,e be an open bounded set and N: X→Z be a continuous operator which is L-compact on $\bar{\Omega} \subset$ i,e; QN: Ω-Z and Kp(I-Q) V: Ω-Z are compact. Assume that

- $\text{For each } \lambda \in (0,1)\ , x \in \alpha\Omega \cap \text{Doml} \quad LX \neq \lambda NX$
- $\text{For each } x \in \alpha\Omega \bigcap \text{KerL}\ , \ QNX \neq 0$
- $\text{Deg}\,A\{JQNX\ , \Omega \bigcap \text{KerL}\ , 0\} \neq 0$

Then the equation LX = NX has at least one solution in $\bar{\Omega} \bigcap \text{DomL}$.

In what follows, we shall use the notations:

$$\bar{f} = \frac{1}{w}\int_0^w f(t)dt\ ,\ f^l = \min_{t\in[0,w]}|f(t)|\ ,\ f^M = \mathop{\text{Max}}_{t\in[0,w]}|f(t)|$$

Where, f is a continuous W-periodic function.

Theorem 2.1 Assume that,

- $a_1^l > \left(\frac{s_1}{B_1}\right)^M$
- $\frac{s_2^l}{b_1^M}\left(a_1^l - (\frac{s_1}{B_1})^M\right) > a_2^M b_2^M \left[A_1^M + \frac{B_1^M}{2b_2^l}(s_3^M + \sqrt{(s_3^M)^2 + 4b_2^l h_2^M}) + \frac{1}{b_1^M}\left(a_1^l - (\frac{s_1}{B_1})^M\right) \right]$

Then system Eq. (1) has at least one positive W-periodic solution.

Proof: Consider the following system:

$$\begin{cases} u_1'(t) = a_1(t) - b_1(t)e^{u_1(t)} - \\ \dfrac{s_1(t)e^{u_2(t)}}{A_1(t) + e^{u_1(t)} + B_1(t)e^{u_2(t)}} + h_1(t)e^{-u_1(t)} \\ u_2'(t) = -a_2(t) - b_2(t)e^{u_2(t)} + \\ \dfrac{s_2(t)e^{u_1(t-\tau)}}{A_1(t) + e^{u_2(t-\tau)} + B_1(t)e^{u_2(t-\tau)}} + h_2(t)e^{-u_2(t)} \end{cases} \quad (2)$$

Where all coefficients in system Eq. (2) are the same as these in system Eq. (1). It is easy to see that if system Eq. (2) has one W-periodic solution $(u_1^*(t), u_2^*(t))^T$, the $(\exp[u_1^*(t)], \exp[u_2^*(t)])^T$ is a positive W-periodic solution of system (1). Therefore, for system Eq. (1) to has at least one positic. W-periodic solution it is sufficient that system Eq. (2) has at least one W-periodic solution. To apply lemma 2.1 to system Eq. (2), we define

$$X = Z = \left\{u(t) = (u_1(t), u_2(t))^T \in (R, R^2) : u(t+w) = u(t)\right\}$$

and

$$\|u\| = \left\|(u_1(t), u_2(t)^T\right\| = \sum_{i=1}^{3} \max_{t\in(0,w)} |u_i(t)|$$

any a ϵX(orZ).

Nu = $(F_1(t), F_2(t))^T$, uϵX Then X and Z are Banach spaces with norm ‖•‖, let

$$F_1(t) = a_1(t) - b_1(t)e^{u_1(t)} - \frac{s_1(t)e^{u_2(t)}}{A_1(t) + e^{u_1(t)} + B_1(t)e^{u_2(t)}} + h_1(t)e^{-u_1(t)}$$

$$F_2(t) = -a_2(t) - b_2(t)e^{u_2(t)} + \frac{s_2(t)e^{u_1(t-\tau)}}{A_1(t) + e^{u_1(t-\tau)} + B_1(t)e^{u_2(t-\tau)}} + h_2(t)e^{-u_2(t)}$$

$$Lu = u' = \frac{du(t)}{dt}\ ,\ Pu = \frac{1}{w}\int_0^w u(t)dt\ ,\ u\in X,\ Qz = \frac{1}{w}\int_0^w z(t)dt \quad z\in Z$$

Then it follows that

$$\text{KerL} = R^{3\to 2}\ ,\ \text{ImL} = \left\{z\in Z : \int_0^w z(t)dt = 0\right\} \text{ is closed in Z.}$$

dim KerL = $R^{3\to 2}$ codim Im L and P, Q are continuous Projectors. Such that Im P = KerL, KerQ = Im L = Im(I-Q). Therefore, L is a Fredholm mapping of index zero.

Furthermore, the generalized inverse Kp: ImL-KerpnDoml reads

$$Kp(z) = \int_0^t z(s)ds - \frac{1}{w}\int_0^w \int_0^t z(t)dsdt$$

This

$$QNu = [\frac{1}{w}\int_0^w F_1(s)ds \ , \ \frac{1}{w}\int_0^w F_2(s)ds]^T$$

and

$$Kp(I-Q)Nu = \begin{bmatrix} \int_0^t F_1(s)ds - \frac{1}{w}\int_0^w \int_0^t F_1(s)dsdt + \\ (\frac{1}{2} - \frac{t}{w})\int_0^w F_1(s)ds \\ \int_0^t F_2(s)ds - \frac{1}{w}\int_0^w \int_0^t F_2(s)dsdt + \\ (\frac{1}{2} - \frac{t}{w})\int_0^w F_2(s)ds \end{bmatrix}$$

obviously, QN and Kp(I-Q) N are continuous. It is not difficult to show that $Kp(I-Q)N(\bar{\Omega})$ is compact for any open bounded set $\Omega \subset X$, by using the Arzda-Ascoli theorem. Moreover, $QN(\bar{\Omega})$ is clearly bounded, Thus, N is L-compact on $\bar{\Omega}$ with any open bounded set $\Omega \subset X$.

Corresponding to the operater eqllation $Lx = \lambda Nx$, $\lambda \in (0, 1)$, We have

$$\begin{cases} u_1'(t) = \lambda F_1(t) \\ u_2'(t) = \lambda F_2(t) \end{cases} \tag{3}$$

Assume that $u = u(t) \in X$ is a solution of system Eq. (3) for a certain $\lambda \in (0, 1)$. Since $(u_1(t), u_2(t))^T \in X$, there exist $\xi_i, \eta_i \in [0, w]$ such, that

$$u_i(\xi_i) = \max_{t\in[0,w]} u_i(t) \quad u_i(\eta_i) = \min_{t\in[0,w]} u_i(t) \quad i = 1,2$$

Since, $u_i'(\xi_i) = 0$, $u_i'(\eta_i) = 0$, $i = 1, 2$

Then from this and system Eq. (3), we obtain

$$F_1(\xi_1) = 0 \tag{4}$$

$$F_2(\xi_2) = 0 \tag{5}$$

$$F_1(\eta_1) = 0 \tag{6}$$

$$F_2(\eta_2) = 0 \tag{7}$$

Equation (4) implies $b_1^l e^{u_1(\xi_1)} < a_1^M + h_1^M e^{-u_1(\xi_1)}$, That is $b_1^l e^{2u_1(\xi_1)} < a_1^M e^{-u_1(\xi_1)} + h_1^M$ Then,

$$u_1(\xi_1) < \ln\left[\frac{1}{2b_1^l}\left(a_1^M + \sqrt{(a_1^M)^2 + 4b_1^l h_1^M}\right)\right] \tag{8}$$

Equation (5) implies $b_2^l e^{u_2(\xi_2)} < s_3^M + h_2^M e^{-u_2(\xi_2)}$, That is $b_2^l e^{u_2(\xi_2)} < s_3^M e^{-u_2(\xi_2)} + h_2^M$ Then,

$$u_2(\xi_2) < \ln\left[\frac{1}{2b_2^l}\left(s_3^M + \sqrt{(s_3^M)^2 + 4b_2^l h_2^M}\right)\right] \tag{9}$$

Equation (6) implies $b_1^M e^{u_1(\eta_1)} < a_1^l - \left(\frac{s_1}{B_1}\right)^M$, That is

$$u_1(\eta_1) > \ln\left[\frac{1}{b_1^M}\left(a_1^l - (\frac{s_1}{B_1})^M\right)\right] \tag{10}$$

Equation (7) implies

$$b_2^M e^{u_2(\eta_1)} > \frac{s_2^l e^{u_1(\eta_2-\tau)}}{A_1^M + e^{u_1(\eta_2-\tau)} + B_1^M e^{u_2(\eta_2-\tau)}} - a_2^M$$

$$> \frac{\frac{s_2^l}{b_1^M}(a_1^l - (\frac{s_1}{B_1})^M)}{A_1^M + \frac{B_1^M}{2b_2^l}(s_3^M + \sqrt{(s_3^M)^2 + 4b_2^l h_2^M}) + \frac{1}{b_1^M}(a_1^l - (\frac{s_1}{B_1})^M)} - a_2^M$$

From which, we obtain

$$u_2(\eta_2) > \ln\left\{ \frac{\frac{s_2^l}{b_1^M}(a_1^l - (\frac{s_1}{B_1})^M)}{b_2^M[A_1^M + \frac{B_1^M}{2b_2^l}(s_3^M + \sqrt{(s_3^M)^2 + 4b_2^l h_2^M}) + \frac{1}{b_1^M}(a_1^l - (\frac{s_1}{B_1})^M)]} \right\} - a_2^M \tag{11}$$

From Eq. (8-11), we have for $\forall t \in [0, w]$

$$\ln\left[\frac{1}{b_1^M}\left(a_1^l - (\frac{s_1}{B_1})^M\right)\right] < u_1(t),$$

$$< \ln\left[\frac{1}{2b_1^l}\left(a_1^M + \sqrt{(a_1^M)^2 + 4b_1^l h_1^M}\right)\right]$$

And

$$\ln Q < u_2(t) < \ln\left[\frac{1}{2b_2^l}\left(s_3^M + \sqrt{(s_3^M)^2 + 4b_2^l h_2^M}\right)\right]$$

Clearly,

$$\ln\left[\frac{1}{b_1^M}\left(a_1^l - (\frac{s_1}{B_1})^M\right)\right]$$

$$\ln Q,\ \ln\left[\frac{1}{2b_2^l}\left(s_3^M + \sqrt{(s_3^M)^2 + 4b_2^l h_2^M}\right)\right]$$

$$\ln\left[\frac{1}{2b_1^l}\left(a_1^M + \sqrt{(a_1^M)^2 + 4b_1^l h_1^M}\right)\right]$$

are independent of λ. Now we take

$$\Omega = \{(u_1(t), u_2(t))^T \in X, u_1 \in \left[\begin{array}{l}\ln[\frac{1}{b_1^M}(a_1^l - (\frac{s_1}{B_1})^M],\\ \ln[\frac{1}{2b_1^l}(a_1^M + \sqrt{(a_1^M)^2 + 4b_1^l h_1^M})]\end{array}\right]$$

$$u_2 \in \left[\ln Q, \ln\left[\frac{1}{2b_2^l}(s_3^M + \sqrt{(s_3^M)^2 + 4b_2^l h_2^M}\right]\right]\}$$

This satisfies condition (a) of lemma 2.1

Next, we show that condition (b) of lemma 2.1 holds, i.e., we prove that when $u \in 2\Omega \cap \text{KerL} = 2\Omega \cap R^2$, $QNu \neq (0, 0)^T$. Otherwise, some constant vertor u with $u \in 2\Omega$ satisfies

$$QNu = \begin{bmatrix}\frac{1}{w}\int_0^w F_1(t)dt\\ \frac{1}{w}\int_0^w F_2(t)dt\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}^T$$

Then there exist $t_i \in [o, w]$ $(i = 1, 2)$ such that $Qnu = (F_1(t_1), F_2(t_2))^T = \binom{0}{0}^T$, From $F_1(t_1)$ - 0 and $F_2(t_2)$ - 0.

Following the arguments of Eq. (8-11), we have

$$u_1 \in \left(\ln\left[\frac{1}{b_1^M}\left(a_1^l - \left(\frac{s_1}{B_1}\right)^M\right)\right], \ln\left(\frac{1}{2b_1^l}\left(a_1^M + \sqrt{\left(a_1^M\right)^2 + 4b_1^l h_1^M}\right)\right)\right)$$

$$u_2 \in \left(\ln Q, \ln\left[\frac{1}{2b_2^l}\left(s_3^M + \sqrt{\left(s_3^M\right)^2 + 4b_2^l h_2^M}\right)\right]\right)\frac{n!}{r!(n-r)!}$$

This contradicts the fact that constant vector u satisfies $u \in 2\Omega$.

There fore, condition(b) of lemmazll holds. Finally, we will prove the condition (c) of lemma, to this end, we define a mapping φ: D_{DM} Lx[0, 1] → x by

$$\varphi(u_1, u_2, u) = \begin{pmatrix} a_1(t_1) - b_1(t_1)e^{u_1} - \\ \dfrac{u\mu s_1(t_1)e^{u_2}}{A_1(t_1) + e^{u_1} + B_1(t_1)e^{u_2}} + \mu h_1(t_1)e^{-u_1} \\ -ua_2(t_2) - b_2(t_2)e^{u_2} + \\ \dfrac{s_2(t_2)e^{u_1}}{A_1(t_1) + e^{u_1} + \mu B(t_2)e^{u_2}} + \mu h_2(t_2)e^{-u_2} \end{pmatrix}$$

Where, $u \in [0, 1]$ is a parameter. We only need to prove that when

$$u \in 2\Omega \cap \text{KerL} = 2\Omega \cap R^2,\ \varphi(u_1, u_2, \mu) \neq (0,0)^T$$

Otherwise, some constant vector u with $u \in 2\Omega$ satisfies

$$\begin{cases} a_1(t_1) - b_1(t_1)e^{u_1} - \\ \dfrac{\mu s_1(t_1)e^{u_2}}{A_1(t_1) + e^{u_1} + B_1(t_1)e^{u_2}} + \mu h_1(t_1)e^{-u_1} = 0 \end{cases} \quad (12)$$

$$\begin{cases} -ua_2(t_2) - b_2(t_2)e^{u_2} + \\ \dfrac{s_2(t_2)e^{u_1}}{A_1(t_1) + e^{u_1} + \mu B(t_2)e^{u_2}} + \mu h_2(t_2)e^{-u_2} = 0 \end{cases} \quad (13)$$

From Eq. (12) and (13), by following the arguments of Eq. (8-11).

And reducing μ-0 or magnifying μ-1, we obtain

$$u_1' \in \left(\ln\left[\frac{1}{b_1^M}\left(a_1^l - \left(\frac{s_1}{B_1}\right)^M\right)\right], \ln\left(\frac{1}{2b_1^l}\left(a_1^M + \sqrt{\left(a_1^M\right)^2 + 4b_1^l h_1^M}\right)\right)\right)$$

$$u_2' \in \left(\ln Q, \ln\left[\frac{1}{2b_2^l}\left(s_3^M + \sqrt{\left(s_3^M\right)^2 + 4b_2^l h_2^M}\right)\right]\right)$$

This contradicts the fact that $u \in 2\Omega$. Hence, when $u \in 2\Omega \cap \text{KerL}$, $\varphi(u_1, u_2, u) \neq (0, 0, 0)$. According to topological degree theory and by taking J = I since KerL=ImQ, we obtain

$$\begin{aligned} &\deg\{JQNu, \Omega \cap \text{KerL}, (0,0)^T\} \\ &= \deg\{\varphi(u_1, u_2, 1), \Omega \cap \text{KerL}, (0,0)^T\} \\ &= \deg\{\varphi(u_1, u_2, 0), \Omega \cap \text{KerL}, (0,0)^T\} \\ &= \deg\left\{\begin{array}{l}(a_1(t_1) - b_1(t_1)e^{u_1}, -b_2(t_2)e^{u_2} + \\ \dfrac{s_2(t_2)e^{u_1}}{A_1(t_1) + e^{u1}})^T, \Omega \cap \text{KerL}, (0,0)^T\end{array}\right\} \end{aligned}$$

Since the system of algebraic equation

$$\begin{cases} a_1(t_1) - b_1(t_1)x = 0 \\ -b_2(t_2)y + \dfrac{s_2(t_2)x}{A_1(t_1) + x} = 0 \end{cases}$$

has a unique solution $(x^*, y^*)^T$ which satisfies: $x^*>0$, $y^*>0$ then $\deg\{JQNu, \Omega \cap \text{KerL}, (0, 0)^T\}$

$$= \deg\left\{\begin{array}{l} a_1(t_1)-b_1(t_1)e^{u_1}, -b_2(t_2)e^{u_2}+ \\ \dfrac{s_2(t_2)e^{u_1}}{A_1(t_1)+e^{u1}}, \Omega\bigcap \mathrm{KerL}, (0,0)^T \end{array}\right\}$$

$$= \mathrm{sign}\begin{vmatrix} -b_1(t_1)x^* & 0 \\ \dfrac{s_2(t_2)A_1(t_2)x^*}{(A_1(t_2)+x^*)^2} & -b_2(t_2)y^* \end{vmatrix}$$

$$= \mathrm{sign}(b_1(t_1)b_2(t_2)x^*y^*) = 1$$

This completes the proof of theorem 2.1.

- Global attractivity of a positive periodic solution

In this study by constracting a Lyapunov functional, we derive sufficient conditions for the global attractivity of a positive periodic solution of system Eq. (1).

Lemma 3.1 [Barbalat's Lemma (Song and Chen, 1998), Lemma 1.22]. Let f be a non-negative function defined in [0, +∞] such that f is interrable on [0, +∞] and is uniformly continuous on [0, +∞], Then

$$\lim_{t\to\infty} f(t) = 0$$

Applying Lemma 3.1 to system (1), we can obtain the following theorem.

MAIN THEORY

Theorem 3.1: In addition to the conditions in Theorem 3.1, we assume further that system (1) satisfies

- $b_1^l > \dfrac{s_2^M A_1^M}{(A_1^l)^2} + \dfrac{s_2^M B_1^M + s_1^M}{2(A_1^l)^2 b_2^l}\left(s_3^M + \sqrt{(s_3^M)^2 + 4b_2^l h_2^M}\right)$
- $b_2^l > \dfrac{s_1^M A_1^M}{(A_1^l)^2} + \dfrac{B_1^M + s_1^M}{2(A_1^l)^2 b_1^l}\left(a_1^M + \sqrt{(a_1^M)^2 + 4b_1^l h_1^M}\right)$

Then system Eq. (1) has at least one positive W-periodic solution which attracts all positive solutions of system Eq. (1).

Proof: By theorem 2.1 system Eq. (1) has at least one positive W-periodic solution $(x_1^*(t^1), x_2^*(t_2))^T$.

From this proof of Theorem 2.1, we have for $\forall t \in R$

$$\frac{1}{b_1^M}\left(a_1^l - \left(\frac{s_1}{B_1}\right)^M\right) < x_1^*(t) < \frac{1}{2b_1^l}\left(a_1^M + \sqrt{(a_1^M)^2 + 4b_1^l h_1^M}\right)$$

$$< x_2^*(t) < \frac{1}{2b_2^l}\left(s_3^M + \sqrt{(s_3^M)^2 + 4b_2^l h_2^M}\right)$$

Suppose that $(x_1^*(t^1), x_2^*(t_2))^T$ is a positive solution of system Eq. (1) with the initial conditions

$$\begin{cases} x_1(s) = \varphi_1(s) \ge 0, & s\in[-\tau,0], \quad \varphi_1(0) > 0, \\ x_2(s) = \varphi_2(s) \ge 0, & s\in[-\tau,0], \quad \varphi_2(0) > 0 \end{cases}$$

We define a Lyapunov functional as follows:

$$V(t) = \left|\ln x_1(t) - \ln x_1^*(t)\right| + \left|\ln x_2(t) - \ln x_2^*(t)\right| + (A_1^l)^{-2}\int_{t-\tau}^{t}[s_2^M A_1^M \left|x_1(s) - x_1^*(s)\right| + s_2^M B_1^M x_2^*(s) \left|x_1(s) - x_1^*(s)\right| + B_1^M x_1^*(s)\left|x_2(s) - x_2^*(s)\right|]ds$$

Calculating the upper right derivative D'V(t) of V(t) along the of systeem Eq. (1), we obtain

$$D^{t}V(t) = \mathrm{sign}(x_1(t) - x_1^*(t))\left(\frac{x_1'(t)}{x_1(t)} - \frac{x_1'^*(t)}{x_1^*(t)}\right) + \mathrm{sign}(x_2(t) - x_2^*(t))\left(\frac{x_2'(t)}{x_2(t)} - \frac{x_2'^*(t)}{x_2^*(t)}\right)$$

$$+ \left(s_2^M A_1^M \left|x_1(t) - x_1^*(t)\right| + s_2^M B_1^M x_2^*(t)\left|x_1(t) - x_1^*(t)\right| + B_1^M x_1^*(t)\left|x_2(t) - x_2^*(t)\right|\right)(A_1^l)^{-2}$$

$$-\left((s_2^M A_1^M \left|x_1(t-\tau) - x_1^*(t-\tau)\right| + s_2^M B_1^M x_2^*(t-\tau)\left|x_1(t-\tau) - x_1^*(t-\tau)\right|)(A_1^l)^{-2} + B_1^M x_1^*(t-\tau)\left|x_2(t-\tau) - x_2^*(t-\tau)\right|\right)$$

$$\le -b_1(t)\left|x_1(t) - x_1^*(t)\right| - b_2(t)\left|x_2(t) - x_2^*(t)\right| + D_1(t) + D_2(t) + D_3(t)$$

$$+(A_1^l)^{-2}\left((s_2^M A_1^M + s_2^M B_1^M x_2^*(t))\left|x_1(t) - x_1^*(t)\right| + B_1^M x_1^*(t)\left|x_2(t) - x_2^*(t)\right|\right)$$

Where,

$$D_1(t)=\begin{cases}\dfrac{s_1(t)x_2^*(t)}{A_1(t)+x_1^*(t)+B_1(t)x_2^*(t)}-\dfrac{s_1(t)x_2(t)}{A_1(t)+x_1(t)+B_1(t)x_2(t)} & x_1(t)>x_1^*(t)\\ \dfrac{s_1(t)x_2(t)}{A_1(t)+x_1(t)+B_1(t)x_2(t)}-\dfrac{s_1(t)x_2^*(t)}{A_1(t)+x_1^*(t)+B_1(t)x_2^*(t)} & x_1(t)<x_1^*(t)\\ 0 & x_1(t)=x_1^*(t)\end{cases}$$

$$D_2(t)=\begin{cases}\dfrac{-h_1(t)}{x_1^*(t)}+\dfrac{h_1(t)}{x_1(t)} & x_1(t)>x_1^*(t)\\ \dfrac{-h_1(t)}{x_1(t)}+\dfrac{h_1(t)}{x_1^*(t)} & x_1(t)<x_1^*(t)\\ 0 & x_1(t)=x_1^*(t)\end{cases},\quad D_3(t)=\begin{cases}\dfrac{-h_2(t)}{x_2^*(t)}+\dfrac{h_2(t)}{x_2(t)} & x_1(t)>x_1^*(t)\\ \dfrac{-h_2(t)}{x_2(t)}+\dfrac{h_2(t)}{x_2^*(t)} & x_1(t)<x_1^*(t)\\ 0 & x_1(t)=x_1^*(t)\end{cases}$$

Here are the following three cases to consider for $D_1(t)$: (i) If $x_1(t) > x_1^*(t)$, then

$$\begin{aligned}D_1(t)&=\frac{-s_1(t)A_1(t)(x_2(t)-x_2^*(t))+s_1(t)x_2^*(t)x_1(t)-s_1(t)x_2(t)x_1^*(t)}{(A_1(t)+x_1^*(t)+B_1(t)x_2^*(t))(A_1(t)+x_1(t)+B_1(t)x_2(t))}\\ &=\frac{-s_1(t)A_1(t)(x_2(t)-x_2^*(t))-s_1(t)x_1^*(t)(x_2(t)-x_1(t))+s_1(t)x_2^*(t)(x_1(t)-x_1^*(t))}{(A_1(t)+x_1^*(t)+B_1(t)x_2^*(t))(A_1(t)+x_1(t)+B_1(t)x_2(t))}\\ &\le\left((s_1^M A_1^M+s_1^M x_1^*(t))\left|x_2(t)-x_2^*(t)\right|+s_1^M x_2^*(t)\left|x_1(t)-x_1^*(t)\right|\right)(A_1^l)^{-2}\end{aligned}$$

- If $x_1(t)$, then

$$\begin{aligned}D_1(t)&=\frac{s_1(t)A_1(t)(x_2(t)-x_2^*(t))-s_1(t)x_2^*(t)x_1(t)+s_1(t)x_2(t)x_1^*(t)}{(A_1(t)+x_1^*(t)+B_1(t)x_2^*(t))(A_1(t)+x_1(t)+B_1(t)x_2(t))}\\ &<\left((s_1^M A_1^M+s_1^M x_1^*(t))\left|x_2(t)-x_2^*(t)\right|+s_1^M x_2^*(t)\left|x_1(t)-x_1^*(t)\right|\right)(A_1^l)^{-2}\end{aligned}$$

- If $x_1(t) = x_1^*(t)$, then $D_1(t) = 0$. Hence, we have

$$D_1(t)<\left((s_1^M A_1^M+s_1^M x_1^*(t))\left|x_2(t)-x_2^*(t)\right|+s_1^M x_2^*(t)\left|x_1(t)-x_1^*(t)\right|\right)(A_1^l)^{-2}$$

There are 3 cases to consider for $D_2(t)$:

- If $x_1(t)>x_1^*(t)$, then $D_2(t)<\dfrac{h_1(t)}{x_1^*(t)}-\dfrac{h_1(t)}{x_1^*(t)}=0$

- If $x_1(t)<x_1^*(t)$, then $D_2(t)=\dfrac{h_1(t)}{x_1^*(t)}-\dfrac{h_1(t)}{x_1(t)}<\dfrac{h_1(t)}{x_1^*(t)}-\dfrac{h_1(t)}{x_1^*(t)}=0$

- If $x_1(t)=x_1^*(t)$, then $D_2(t)=0$

Hence, $D_2(t)\le 0$, Similarly, $D_3(t)\le 0$. Therefore,

$$D^+V(t) \le -b_1^l\left|x_1(t)-x_1^*(t)\right| - b_2^l\left|x_2(t)-x_2^*(t)\right| + \frac{s_2^M A_1^M + s_2^M B_1^M x_2^*(t)}{(A_1^l)^2}\left|x_1(t)-x_1^*(t)\right| + \frac{B_1^M x_1^*(t)}{(A_1^l)^2}\left|x_2(t)-x_2^*(t)\right|$$
$$+\Big((s_1^M A_1^M + s_1^M x_1^*(t))\left|x_2(t)-x_2^*(t)\right| + s_1^M x_2^*(t)\left|x_1(t)-x_1^*(t)\right|\Big)(A_1^l)^{-2} < -[b_1^l - \frac{s_2^M A_1^M}{(A_1^l)^2} - \frac{s_2^M B_1^M + s_1^M}{2(A_1^l)^2 b_2^l}(s_3^M + \sqrt{(s_3^M)^2 + 4b_2^l h_2^M})]$$
$$\left|x_1(t)-x_1^*(t)\right| - [b_2^l - \frac{s_2^M A_1^M}{(A_1^l)^2} - \frac{B_1^M + s_1^M}{2(A_1^l)^2 b_1^l}(a_3^M + \sqrt{(a_3^M)^2 + 4b_1^l h_1^M})]\left|x_2(t)-x_2^*(t)\right|, t \ge 0$$

It follows from condition (i) and (ii) in Theorem 3.1 that there exists $\alpha^*>0$ such that

$$D^+V(t) < -\alpha^*\left|x_1(t)-x_1^*(t)\right| - \alpha^*\left|x_2(t)-x_2^*(t)\right| \quad ,t \ge 0 \quad (14)$$

Integrating on both sides of in equality Eq. (14) leads to

$$V(t) + \alpha^*\int_0^t(\left|x_1(s)-x_1^*(s)\right| + \left|x_2(s)-x_2^*(s)\right|)ds \le V(0) < +\infty, \ t>0$$

Which implies

$$\left|x_1(t)-x_1^*(t)\right| \in L'[0,+\infty) \ , \left|x_2(t)-x_2^*(t)\right| \in L'[0,+\infty)$$

$$\left|\ln x_1(t) - \ln x_2(t)\right| < V(t) \le V(0) < +\infty, \qquad t>0 \quad (15)$$

$$\left|\ln x_2(t) - \ln x_2^*(t)\right| < V(t) \le V(0) < +\infty, \qquad t>0 \quad (16)$$

From the boundedness of $x_1^*(t)$ and $x_2^*(t)$ and inequality Eq. (15) and (16), it follows that $x_1(t)$ and $x_2(t)$ are bounded for $t \ge 0$. Form the boundedness of $x_1(t)$ and $x_2(t)$ and system Eq. (1), it follows that $x_1(t)$, $x_1^*(t)$, $x_2(t)$, $x_2^*(t)$ and $(x_1(t)-x_1^*(t))'$, $(x_2(t)-x_2^*(t))'$ remain bounded on $[0, +\infty]$. Hence, $x_1(t)-x_1^*(t)$ and $x_2(t)-x_2^*(t)$ are uniformly continuous. By lemma 3.1, it follows that

$$\lim_{t\to\infty}(x_1(t)-x_1^*(t)) = 0 \text{and} \lim_{t\to\infty}(x_2(t)-x_2^*(t)) = 0$$

This implies that system Eq. (1) has a positive W-periodic solution which attracts all positive solutions of system Eq. (1). The proof is finished.

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