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Effect of Resistance Parameter on Uniform and Non-uniform Portion of Artery for Non-Newtonian Fluid Model of Blood Flow Through an Arterial Stenosis

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Abstract: An investigation has been done for the resistance to flow across mild stenosis situated symmetrically on steady blood flow through arteries with uniform or non-uniform cross-section by assuming the blood to be Non-Newtonian, incompressible and homogeneous fluid. An analytical solution for Power law fluid has been obtained. The study reveals that as the height of the stenosis increases in uniform or non-uniform portion of the artery, the resistance parameter also increases.

Key words: Power law fluid, wall parameter, blood flow, arterial stenosis

INTRODUCTION

Stenosis could affect one or more segments of the human cardiovascular system. Studies on initiation and growth of stenosis (atherosclerotic plaques) in the human cardiovascular system have been carried out from several view-points. Arteriosclerosis is a common disease, which severely influences human health.

It has been found that the initiation and localization of arteriosclerosis is closely related to local hemodynamic factors. Due to these serious consequences, attention has been given in studies of blood flow in stenotic region under different conditions. By assuming the artery to be circularly cylindrical in shape (Mishra, 2003; Mishra and Panda, 2005c) discussed the characteristics of blood in stenosed artery and the stenosis to be symmetric about the axis of artery. Mishra and Panda (2005a) studied the flow of blood in human body under axisymmetric peristalsis and also performed an *in vitro* treatment for the flow of blood in a stenosed artery assuming the blood to be Newtonian (Mishra and Panda, 2005b). Large number of researchers viz. Smith *et al.* (2002), Siouffi *et al.* (1984), Tu and Deville (1996), Misra *et al.* (1993), Tu *et al.* (1992), Misra and Chakravarty (1986), Siouffi *et al.* (1984), Young and Tsai (1973a, b) and Verma and Mishra (2005) have contributed a lot in developing a mathematical model for blood flow in atherosclerosis.

By assuming that blood to be Non-Newtonian, incompressible and homogeneous fluid, cylindrical polar co-ordinate is used, with the axis of symmetry of artery taken as Z axis. The stenoses are mild and the motion of the fluid is laminar and steady. The inertia term is neglected, as the motion is slow. No body force acts on the fluid and there is no slip at the wall.

Development of the model: The constitutive relationship for the power fluid is given by the relationship

$$\tau = (\mu e)^n \ (n<1) \tag{1}$$

Where:
τ = Stress tensor

$$e = \text{Strain rate} = -(du/dr) \tag{2}$$

Where:
u = Velocity of fluid
r = Radius of the artery
μ = Viscosity of blood

For the steady flow through circular artery

$$\tau = \frac{r\frac{dp}{dz}}{2} = \frac{rG}{2} \tag{3}$$

Where:
G = Pressure gradient = dp/dz

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From Eq. (1-3), we have
Let, n = 1/3

$$\left(-\frac{du}{dr}\right) = \frac{1}{\mu}\left(\frac{rG}{2}\right)^3$$

On integration,

$$u = -\left(\frac{G}{2\mu}\right)^3\left(\frac{r^4}{4}\right) + C \quad (4)$$

Where, C is constant of integration

Applying the boundary condition that u = 0 at r = R, we have

$$C = \left(\frac{G}{2\mu}\right)^3\left(\frac{R^4}{4}\right)$$

Substituting the value of C in Eq. (4),

$$u = \left(\frac{G}{2\mu}\right)^3 \frac{\left(R^4 - r^4\right)}{4} \quad (5)$$

The core velocity is given by

$$u_c = \left(\frac{G}{2\mu}\right)^3 \frac{\left(R^4 - R_c^4\right)}{4} \quad (6)$$

The flow rate through the artery is the sum of the flow through the core region and that in the peripheral region.

$$Q = Q_{core} + Q_{peripheral} \quad (7)$$

$$Q_{core} = u_c \pi R_c^2 = \left(\frac{G}{2\mu}\right)^3 \frac{\left(R^4 R_c^2 - R_c^6\right)}{4} \quad (8)$$

The flow through the peripheral region can be obtained as:

$$Q_{peripheral} = \int_{R_c}^{R} 2\pi urdr$$

$$= \frac{2\pi}{4}\left(\frac{G}{2\mu}\right)^3 \int_{R_c}^{R} \left(R^4 - r^4\right) rdr$$

Integrating the above expression and simplifying, we get:

$$Q_{peripheral} = \frac{2\pi}{4}\left(\frac{G}{2\mu}\right)^3\left[\frac{R^6}{3} - \frac{R^4 R_c^2}{2} + \frac{R_c^6}{6}\right] \quad (9)$$

From Eq. (7-9) the expression for the flow through the tube as:

$$Q = \frac{2\pi}{4}\left(\frac{G}{2\mu}\right)^3\left[\frac{2R^6}{3} - \frac{2R_c^6}{3}\right] \quad (10)$$

Putting the value of G = dp/dz in Eq. (10),we get,

$$Q = \frac{2\pi}{4}\left(\frac{dp/dz}{2\mu}\right)^3\left[\frac{2R^6}{3} - \frac{2R_c^6}{3}\right] \quad (11)$$

$$\frac{dp}{dz} = \left[48\frac{\mu^3 Q}{\pi}\right]\left[2R^6 - 2R_c^6\right]^{-1} \quad (12)$$

$$\lambda = \frac{dp}{Q} = 48\frac{\mu^3}{\pi}\int_0^1 \left[2R^6 - 2R_c^6\right]^{-1} dz \quad (13)$$

Where, λ= Resistance to flow at the wall for the flow of blood

$$\lambda_0 = \frac{dp}{Q} = 48\frac{\mu^3}{\pi}\int_0^1 \left[2R_1^6 - 2R_c^6\right]^{-1} dz \quad (14)$$

Where, λ_0 = Resistance to flow at the wall for the flow of blood in uniform portion of artery

$$\lambda' = \frac{\lambda}{\lambda_0}$$

Where, λ' = Resistance parameter

$$= \left[\frac{R_1^4}{8} + \frac{R_c^4}{24} - \frac{R_c R_1^3}{6}\right]\int_0^1 \left[\frac{R^4}{8} + \frac{R_c^4}{24} - \frac{R_c R^3}{6}\right]^{-1} dz \quad (15)$$

Assume, k = 1 (stenosis in non uniform portion), (n - k) = 1 (number of stenosis in uniform portion). The surface of stenosis as obtained is:

$$R(Z) = R_{s1}(z) = R_1; 0 \le z \le d_1 \text{ and } d_1 + L_1 \le z \le l_1$$

$$R(z) = R_{s1}(z) = R_1 - \delta S_1/2.$$

$$\left[1 + \cos\left(\frac{2\pi}{L_1}\right)\left(z - d_1 - L_1/2\right)\right]; d_1 \le z \le d_1 + L_1 \quad (16)$$

$$R(z) = R_{s2}(z) = R_2(z); l_1 \le z \le d_2 \text{ and } d_2 + L_2 \le z \le 1$$

$$R(z) = R_{s2}(z) = R_2(z) - \delta S_2/2.$$

$$\left[1 + \cos\left(2\pi/L_2\right)\left(z - d_2 - L_2/2\right)\right]; d_2 \le z \le d_2 + L_2$$

Table 1: Variation of λ' against $\delta S'_2$ for K= -0.001, 0, 0.001

	λ'		
$\delta S'_2$	K = -0.001	K = 0	K = 0.001
0.027	5.150996	5.0028200003	4.854644
0.034	5.151416	5.0032400003	4.855064
0.040	5.151776003	5.003600003	4.855424003
0.046	5.152136	5.0039600003	4.855784
0.053	5.152556003	5.0043800003	4.856204003
0.060	5.152976003	5.0048000003	4.856624003
0.067	5.153126001	5.005200003	4.856927003

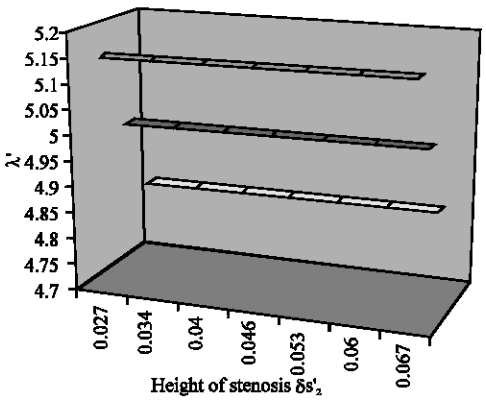

Fig. 1: Variation of λ' against $\delta S'_2$ for various value of K

To observe explicity the effect of various parameters resistance to the flow, the following function has been assumed for the artery radius for the portion of the artery which is non uniform.

$$R(z) = R_1 e^{K(z-l_1)^2}; \; l_1 \le z \le l \tag{17}$$

Where, K is the wall exponent parameter.
Using Eq. (17) and (16) in (15) and integrating ,we have,

$$\lambda' = (1 - L_1') + L_1'\left[1 + 6\delta S_1' + R_c^6\right] + L_2'\left[1 + 6\delta S_2' + R_c^6\right] + \left(1 + R_c^6\right)\left(1 - l_1' - L_2'\right) - 2K\left(1 - l_1'\right)^3$$

Numerical computation for theoretical analysis: In order to get a physiological insight into the effect of stenosis on the resistance to flow, for pulmonary artery, the following values are taken:

- Length of the 1st stenosis, L_1 = 0.05 cm
- Length of the 2nd stenosis, L_2 = 0.05 cm
- Height of the 1st stenosis δS_1 = 0.03 cm
- Height of the 2nd stenosis δS_2 = 0.03 (Initially) cm
- Length of the artery, l = 5 cm
- Radius of core region = 0.02 cm
- Length of the uniform portion of the artery, l_1 = 4.0 cm
- Radius of the uniform portion of the artery, R_1 = 1.5 cm
- Position of 2nd stenosis d_2 = 5 cm

From Table 1 and Fig. 1 we see that in cases the divergence of artery (K<0), uniform portion of the artery (K = 0), the convergence of artery (K>0) the resistance parameter λ' increases as the height of the stenosis in the non-uniform portion of the artery increases.

Nomenclature:

ρ	=	Density of blood
μ	=	Viscosity of blood
P	=	Pressure
R_1	=	Radius of uniform portion of artery
R(z)	=	Radius of obstructed portion of artery
$R_{sn}(z)$	=	Radius of obstructed portion due to the nth stenosis of artery
δS_n	=	Amplitude of nth stenosis.
L_n	=	Length of nth stenosis
d_n	=	Location of nth stenosis
l_1	=	Length of uniform portion of artery
l	=	length of artery
K	=	Wall exponent parameter
λ	=	Resistance to flow at the wall for the flow of blood
λ_0	=	Resistance to flow at the wall for the flow of blood in uniform portion of artery.
λ' (λ/λ_0)	=	Resistance parameter
Z'	=	(Z/l), d_n' = (d_n/l), L_n' = (L_n/l), l_1' = (l_1/l), $\delta\delta_n'$ = ($\delta\delta_n/R_1$), R'(z)/ R_1), R_1' = R_1/l

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