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A Numerical Simulation Model for *in situ* Combustion Processes with Heat Source/Sink and Variable Permeability

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Abstract: This study presents a mathematical model of a constant pressure *in situ* combustion process with heat source and variable permeability using high activation energy asymptotics. Numerical solutions are constructed for the governing equations using thin flame technique together with shooting method technique. The results show that temperature increases as the flame speed decreases and Frank-Kamenetskii number increases.

Key words: *In situ* combustion, permeability, oil recovery, thin flame technique, shooting method

INTRODUCTION

Water and oil which are indispensable to human daily activities exist in the porous media. In order to get water and oil in abundance we dig wells and sink bore-holes. There are light oils and heavy oils. Some areas of the world only have heavy oils exist in the porous media. In such cases, it may become necessary to introduce *in situ* combustion in the processes of oil recovery.

The *in situ* combustion involves burning the heavy hydrocarbons (heavy oils) by introducing heat into the system in the presence of oxygen so that the system yields light oils, water and gas. That is:

$$\text{Heavy oil} + \text{Oxygen} \rightarrow \text{Light oil} + \text{Inert gas} + \text{Water} \qquad (1)$$

In some underground media, the porous rocks and the soil naturally do not have constant permeability distribution especially in layered rocks, due to the formation. Therefore in order to obtain water and oils from the underground sources more cheaply, a better understanding of the flow patterns in the underground media is necessary.

Permeability is the geometrical quality or condition of a medium being permeable. It is the coefficient related to macroscopic parameters that describe the geometrical configuration of the void space and it is the degree or measure of the resistance of the medium to the transport of various extensive quantities such as mass or heat through the medium. Permeability has the dimensions of area and is independent of the fluid properties and is a purely geometric property of the porous medium.

Studies related to modeling and analysing transport phenomena in porous media are continuously being pushed forward.

Adewole and Bello (2004) studied the pressure transient analysis of a horizontal well subject to four vertical well injectors. The study utilized a dimensionless pressure distribution of a horizontal well oil producer, subject to vertical well fluid injectors to identify the possible flow regimes in the horizontal well. The study shows that the number of flow regimes identifiable depends on the permeability distribution and geometry of the reservoir and that more flow regimes may occur if the reservoir length is greater than the breadth and the horizontal permeability is substantially high.

Ayeni (1999) presented a phase plane analysis of a liquid front moving through a hot porous rock. The mechanism by which relatively cold liquid vapourizes as it invades a hot permeable rock and the high rise in the pressure of the system were broadly considered. The occurrence of pressure runaway was discussed analytically in the research.

The theoretical investigation of Minkwycz and Cheng (1976) illustrates the effect of permeability of a porous medium when flows are characterized by Darcy's law.

Barenblatt *et al*. (1990) presented a systematic treatment of the mathematical theory (basic physical concepts and classical models) of fluid flows in natural reservoirs. Concepts of steady and unsteady flows, theory of two phase flow and physio-chemical hydrodynamics of porous media are considered extensively.

The emphasis of this research is to study and model numerically a constant pressure *in situ* combustion processes when there is heat source and variable permeability.

MATHEMATICAL MODEL

For our discussion we assume one liquid phase (heavy oil), one gas phase (oxygen) and one solid phase (coke). Any water present and any other oil components are assumed to be in the gas phase. Following Gerritsen and Kovscek (2005) the stoichiometry of the reactions are modeled as below,

$$HO + O_2 \rightarrow IG + H_2O$$

$$HO \rightarrow LO + C + IG$$

$$C + O_2 \rightarrow IG + H_2O$$

The reaction rates are expressed as:

$$r_2 = P\alpha_2\phi_f S_o \rho_o X_2 Y_4 e^{-E_2/RT}$$

$$r_3 = \alpha_3\phi_f S_o \rho_o X_2 e^{-E_3/RT}$$

$$r_4 = P\alpha_4\phi_v C_c Y_4 e^{-E_4/RT}$$

Following the fraction presence of components in 3 phases as presented in Gerritsen *et al.* (2004) the equations that describe the *in situ* combustion process are:

The oxygen mass balance

$$\phi\frac{\partial}{\partial t}(S_g\rho_g Y_4) - \frac{\partial}{\partial x}\left(\frac{KK_{rg}\rho_g Y_4}{\mu_g}\frac{\partial P_g}{\partial x}\right) = -V\left(A_{g2}r_2 + A_{g4}r_4\right) \quad (2.1)$$

The oil mass balance

$$\phi\frac{\partial}{\partial t}(S_o\rho_o X_2 + S_g\rho_o Y_2) - \frac{\partial}{\partial x}\left(\frac{KK_{ro}\rho_o X_2}{\mu_o}\frac{\partial P_o}{\partial x} + \frac{KK_{rg}\rho_g Y_2}{\mu_g}\frac{\partial P_g}{\partial x}\right) = -V\left(A_{o2}r_2 + A_{o3}r_3\right) \quad (2.2)$$

The coke mass balance

$$\frac{\partial C_c}{\partial t} = V\left(A_{c3}r_3 - A_{c4}r_4\right) \quad (2.3)$$

The energy balance

$$\frac{\partial}{\partial t}(\ell C_p T) - \frac{\partial}{\partial x}\left(\frac{KK_{ro}\rho_o C_{po} T}{\mu_o}\frac{\partial P_o}{\partial x} + \frac{KK_{rg}\rho_g C_{pg} T}{\mu_g}\frac{\partial P_g}{\partial x}\right) = \lambda\frac{\partial^2 T}{\partial x^2} + UA(T_r - T) - V\left(\Delta H_2 r_2 + \Delta H_3 r_3 + \Delta H_4 r_4\right) \quad (2.4)$$

Where E_2 is the activation energy for reaction 2, E_3 is the activation energy for reaction 3, E_4 is the activation energy for reaction 4, R is the ideal gas constant, ϕ_f is the fluid porosity, ϕ_v is the void porosity, ϕ is the porosity, α_2 is the frequency factor for reaction 2, α_3 is the frequency factor for reaction 3, α_4 is the frequency factor for reaction 4, S_g is the gas phase saturation, ρ_g is the gas phase density, Y_4 is the gas phase mole fraction for oxygen component, K is the permeability, K_{rg} is the gas phase relative permeability, A_{g2} is the stoichiometric coefficient for oxygen component in reaction 2, A_{g4} is the stoichiometric coefficient for oxygen component in reaction 4, r_2 is the reaction rate for reaction 2, r_4 is the reaction rate for reaction 4, r_3 is the reaction rate for reaction 3, S_o is the oil phase saturation, ρ_o is the oil phase density, μ_g is the gas phase viscosity, μ_o is the oil phase viscosity, P_g is the gas phase pressure, P_o is the oil phase pressure, X_2 is the oil phase mole fraction for heavy oil component, C_c is the solid phase mole fraction for coke, K_{ro} is the oil phase relative permeability, Y_2 is the gas phase mole fraction for heavy oil component, A_{o2} is the stoichiometric coefficient for heavy oil component in reaction 2, A_{o3} is the stoichiometric coefficient for heavy oil component in reaction 3, A_{c3} is the stoichiometric coefficient for coke in reaction 3, A_{c4} is the stoichiometric coefficient for coke in reaction 4, ρ is the density, C_p is the heat capacity, T is the temperature, C_{pg} is the gas phase heat capacity, C_{po} is the oil phase heat capacity, t is the time, x is the distance, λ is the thermal conductivity, UA is the overall heat transfer coefficient, T_r is the AH_2 heating/cooling temperature, ΔH_3 is the heat of reaction 2, ΔH_3 is the heat of reaction 3 and ΔH_4 is the heat of reaction 4.

We let S_g, S_o, K_{rg}, K_{ro}, ρ_g, ρ_o, C_{pg}, C_{po}, ℓ, C_p, μ_g, μ_o be constants and $S_g = S_o$, $K_{rg} = K_{ro}$, $\rho_g = \rho_o$, $C_{pg} = C_{po}$, $P_g = P_o$, $\mu_g = \mu_o$, $E_2 = E$, $E_3/E = \beta$, $E_4/E = \beta_1$ and by setting $Y_2 = 0$, Eq (2.1-2.4) become

$$\frac{\partial Y_4}{\partial t} - \left(\frac{K_{ro}}{\phi S_o \mu_o}\frac{\partial P_o}{\partial x}\right)\frac{\partial KY_4}{\partial x} = -\frac{VA_{g2}P\phi_f\alpha_2}{\phi} X_2 Y_4 e^{-E/RT} - \frac{VA_{g4}P\phi_v\alpha_4}{\phi S_o \rho_o} C_c Y_4 e^{-\beta_1 E/RT} \quad (2.5)$$

$$\frac{\partial X_2}{\partial t}-\left(\frac{K_{ro}}{\phi S_o\mu_o}\frac{\partial P_o}{\partial x}\right)\frac{\partial KX_2}{\partial x}=-\frac{VA_{o2}P\phi_f\alpha_2}{\phi} X_2Y_4e^{-E/RT}-\frac{VA_{o3}P\phi_v\alpha_4}{\phi S_o\rho_o}X_2e^{-\beta E/RT} \quad (2.6)$$

$$\frac{\partial C_c}{\partial t}=VA_{c3}\phi_f S_o\rho_o\alpha_3X_2e^{-\beta E/RT} -VA_{c4}P\phi_v\alpha_4C_cY_4e^{-\beta_1E/RT} \quad (2.7)$$

$$\frac{\partial T}{\partial t}-\left(\frac{2K_{ro}\rho_oC_{po}}{\rho C_p\mu_o}\frac{\partial P_o}{\partial x}\right)\frac{\partial KT}{\partial x}=\frac{\lambda}{\rho C_p}\frac{\partial^2T}{\partial x^2}+\frac{UA}{\rho C_p}(T_r-T)-\frac{V\Delta H_2P\phi_fS_o\rho_o\alpha_2}{\rho C_p}X_2Y_4e^{-E/RT}-\frac{V\Delta H_3\phi_fS_o\rho_o\alpha_3}{\rho C_p}X_2e^{-\beta E/RT}-\frac{V\Delta H_4P\phi_v\alpha_4}{\rho C_p}C_cY_4e^{-\beta_1E/RT} \quad (2.8)$$

with initial conditions

$$Y_4(x,0)=Y_{04}(x),X_2(x,0)=X_{02}(x), C_c(x,0)=C_{0c}(x),T(x,0)=T_0(x) \quad (2.9)$$

and boundary conditions

$$Y_4(0,t)=Y_0,X_2(0,t)=X_0,C_c(0,t)=0, T(0,t)=(\in\theta_0+1)T_0,T(d,t)=T_0 \quad (2.10)$$

We introduce the following dimensionless variables in Eq (2.5-2.10):

$$\theta=\frac{E}{RT_o^2}(T-T_o),\in=\frac{RT_0}{E},m=\frac{1}{\rho C_p},D=-\frac{K_{ro}}{\phi S_o\mu_o}\frac{\partial P_o}{\partial x},$$
$$a=-\frac{VA_{g2}P\phi_f\alpha_2}{\phi}e^{-E/RT_0},a_1=-\frac{VA_{g4}P\phi_v\alpha_4}{\phi S_o\rho_o}e^{-\beta_1E/RT_0},$$
$$a_2=-\frac{VA_{o2}P\phi_f\alpha_2}{\phi}e^{-E/RT_0},a_3=-\frac{VA_{o3}\phi_f\alpha_3}{\phi}e^{-\beta E/RT_0},$$
$$a_4=VA_{c3}\phi_fS_o\rho_o\alpha_3e^{-\beta E/RT_0},a_5=-VA_{c4}P\phi_v\alpha_4e^{-\beta_1E/RT_0},$$
$$n=-\frac{2K_{ro}\rho_oC_{po}}{\rho C_p\mu_o}\frac{\partial P_o}{\partial x},l=-\frac{UA}{\rho C_p},l_1=\frac{UA}{\rho C_p\in T_0},\delta=-\frac{V\Delta H_2P\phi_fS_o\rho_o\alpha_2}{\rho C_p\in T_0}e^{-E/RT_0},$$
$$\delta_1=-\frac{V\Delta H_3\phi_fS_o\rho_o\alpha_3}{\rho C_p\in T_0}e^{-\beta E/RT_0},\ \delta_2=-\frac{V\Delta H_4P\phi_v\alpha_4}{\rho C_p\in T_0}e^{-\beta_1E/RT_0} \quad (2.11)$$

and obtain the dimensionless governing equation together with the corresponding boundary conditions as

$$\frac{\partial Y_4}{\partial t}+D\frac{\partial KY_4}{\partial x}=aX_2Y_4e^{\theta/(1+\in\theta)}+a_1C_cY_4e^{\beta_1\theta/(1+\in\theta)} \quad (2.12)$$

$$\frac{\partial X_2}{\partial t}+D\frac{\partial KX_2}{\partial x}=a_2X_2Y_4e^{\theta/(1+\in\theta)}+a_3X_2e^{\beta\theta/(1+\in\theta)} \quad (2.13)$$

$$\frac{\partial C_c}{\partial t}=a_4X_2e^{\beta\theta/(1+\in\theta)}+a_5C_cY_4e^{\beta_1\theta/(1+\in\theta)} \quad (2.14)$$

$$\frac{\partial\theta}{\partial t}+n\frac{\partial K\theta}{\partial x}=m\lambda\frac{\partial^2\theta}{\partial x^2}+l\theta+l_1(T_r-T_0)+\delta X_2Y_4e^{\theta/(1+\in\theta)}+\delta_1X_2e^{\beta\theta/(1+\in\theta)}+\partial_2C_cY_4e^{\beta_1\theta/(1+\in\theta)} \quad (2.15)$$

$$Y_4(0,t)=Y_0,X_2(0,t)=X_0,C_c(0,t)=0,\theta(0,t)=\theta_o,\theta(1,t)=0, \quad (2.16)$$

Where,

T_o, θ, ϵ and δ, δ_1, δ_2 represent the wall temperature, the dimensionless temperature, the dimensionless activation energy and Frank-Kamenetskii parameters, respectively.

We consider the permeability to be varied with space and time and to be of the form

$$K(x,t)=x-ct \quad (2.17)$$

Then we obtain

$$\frac{\partial Y_4}{\partial t}+D(x-ct)\frac{\partial Y_4}{\partial x}+DY_4= aX_2Y_4e^{\theta/(1+\in\theta)}+a_1C_cY_4e^{\beta_1\theta/(1+\in\theta)} \quad (2.18)$$

$$\frac{\partial X_2}{\partial t}+D(x-ct)\frac{\partial X_2}{\partial x}+DX_2= a_2X_2Y_4e^{\theta/(1+\in\theta)}+a_3X_2e^{\beta\theta/(1+\in\theta)} \quad (2.19)$$

$$\frac{\partial C_c}{\partial t}=a_4X_2e^{\beta\theta/(1+\in\theta)}+a_5C_cY_4e^{\beta_1\theta/(1+\in\theta)} \quad (2.20)$$

$$\frac{\partial\theta}{\partial t}+n(x-ct)\frac{\partial\theta}{\partial x}+n\theta=m\lambda\frac{\partial^2\theta}{\partial x^2}+l\theta+l_1(T_r-T_0)+\delta X_2Y_4e^{\theta/(1+\in\theta)}+\delta_1X_2e^{\beta\theta/(1+\in\theta)}+\delta_2C_cY_4e^{\beta_1\theta/(1+\in\theta)} \quad (2.21)$$

MATERIALS AND METHODS

We apply a moving grid

$$\varepsilon = x - vt \tag{3.1}$$

and using chain rule and since similarity solution exist if v = c, Eq. (2.18-2.21) become

$$\frac{dY_4}{d\varepsilon} = -\left(\frac{D}{D\varepsilon - v}\right)Y_4 + \left(\frac{a}{D\varepsilon - v}\right) X_2Y_4e^{\theta/(1+\epsilon\theta)} + \left(\frac{a_1}{D\varepsilon - v}\right)C_cY_4e^{\beta_1\theta/(1+\epsilon\theta)} \tag{3.2}$$

$$\frac{dX_2}{d\varepsilon} = -\left(\frac{D}{D\varepsilon - v}\right)X_2 + \left(\frac{a_2}{D\varepsilon - v}\right) X_2Y_4e^{\theta/(1+\epsilon\theta)} + \left(\frac{a_3}{D\varepsilon - v}\right)X_2e^{\beta\theta/(1+\epsilon\theta)} \tag{3.3}$$

$$\frac{dC_c}{d\varepsilon} = \left(\frac{a_4}{v}\right)X_2e^{\beta\theta/(1+\epsilon\theta)} + \left(\frac{a_5}{v}\right)C_cY_4e^{\beta_1\theta/(1+\epsilon\theta)} \tag{3.4}$$

$$\frac{d^2\theta}{d\varepsilon^2} = \left(\frac{n\varepsilon - v}{m\lambda}\right)\frac{d\theta}{d\varepsilon}\left(\frac{1}{m\lambda}\right) + \left(\frac{n-1}{m\lambda}\right) \theta - \left(\frac{l_1}{m\lambda}\right)(T_r - T_0) - \left(\frac{\delta}{m\lambda}\right)X_2Y_4e^{\theta/(1+\epsilon\theta)} - \left(\frac{\delta_1}{m\lambda}\right)X_2e^{\beta\theta/(1+\epsilon\theta)} - \left(\frac{\delta_2}{m\lambda}\right)C_cY_4e^{\beta_1\theta/(1+\epsilon\theta)} \tag{3.5}$$

$$Y_4(0) = 0.126, X_2(0) = 0.4, C_c(0) = 0, \theta(0) = \theta_0, \theta(1) = 0 \tag{3.6}$$

where v is the flame velocity.

We solve Eq. (3.2-3.6) by shooting method technique and transform the four system of differential equations into a system of 6 Eq. To achieve the 6 Eq. we let

$$T_1 = \varepsilon, T_2 = Y_4, T_3 = X_2, T_4 = C_c, T_5 = \theta, T_6 = \theta'$$

Thus (3.2-3.5) become

$$T_1' = 1 \tag{3.7}$$

$$T_2' = -\left(\frac{D}{DT_1 - v}\right)T_2 + \left(\frac{a}{DT_1 - v}\right)T_3T_2e^{T_5/(1+\epsilon T_5)} + \left(\frac{a_1}{DT_1 - v}\right)T_4T_2e^{\beta_1T_5/(1+\epsilon T_5)}$$
$$T_3' = -\left(\frac{D}{DT_1 - v}\right)T_3 + \left(\frac{a_2}{DT_1 - v}\right)T_3T_2e^{T_5/(1+\epsilon T_5)} + \left(\frac{a_3}{DT_1 - v}\right)T_3e^{\beta T_5/(1+\epsilon T_5)} \tag{3.8}$$

$$T_4' = \left(\frac{a_4}{v}\right)T_3e^{\beta T_5/(1+\epsilon T_5)} + \left(\frac{a_5}{v}\right)T_4T_2e^{\beta_1T_5/(1+\epsilon T_5)} \tag{3.9}$$

$$T_5' = T_6 \tag{3.10}$$

$$T_6^{¢} = \left(\frac{nT_1 - v}{m?}\right)T_6 + \left(\frac{n-1}{m?}\right)T_5 - \left(\frac{l_1}{m?}\right)(T_r - T_0) - \left(\frac{d}{m?}\right)T_3T_2e^{T_5/(1+îT_5)} - \left(\frac{d_1}{m?}\right)T_3e^{\beta T_5/(1+îTT_5)} - \left(\frac{d_2}{m?}\right)T_4T_2e^{\beta_1T_5/(1+îT_5)} \tag{3.11}$$

The initial conditions are

$$T_1(0) = 0, T_2(0) = 0.126, T_3(0) = 0.4, T_4(0) = 0, T_5(0) = \theta_0, T_6(0) = \gamma \tag{3.12}$$

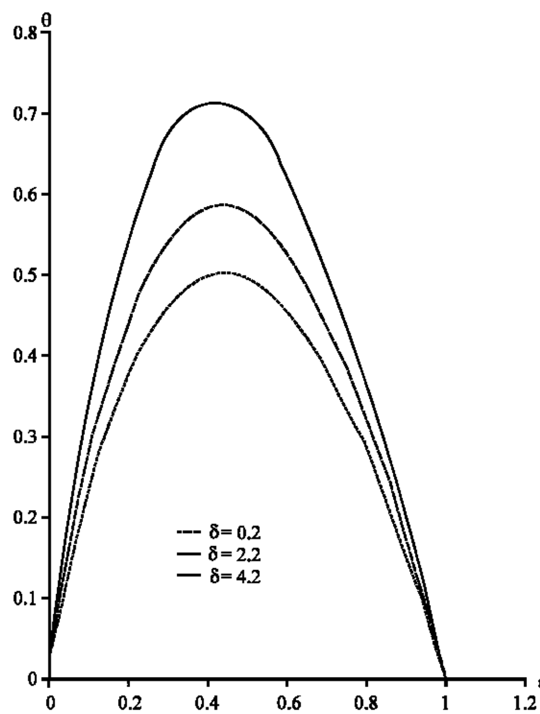

Fig. 1: Temperture profile for various values of δ

Here $T_1 = \varepsilon$ and when, $\varepsilon = 0$, $T_1 = 0$. Also $T_6(0) = \gamma$ is guessed and it is changed until $T_5(1)$ as given by (3.6).

We solve (3.7-3.12) by Runge-Kutta method of order four; that is where

$$Z = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{pmatrix}$$

and

$$F(Z) = \begin{pmatrix} 1 \\ -\left(\frac{D}{DT_1 - v}\right)T_2 + \left(\frac{a}{DT_1 - v}\right) \\ T_3T_2e^{T_5/(1+\epsilon T_5)} + \left(\frac{a_1}{DT_1 - v}\right)T_4T_2e^{\beta_1 T_5/(1+\epsilon T_5)} \\ -\left(\frac{D}{DT_1 - v}\right)T_3 + \left(\frac{a_2}{DT_1 - v}\right) \\ T_3T_2e^{T_5/(1+\epsilon T_5)} + \left(\frac{a_3}{DT - v_1}\right)T_3e^{\beta T_5/(1+\epsilon T_5)} \\ \left(\frac{a_4}{v}\right)T_3e^{\beta T_5/(1+\epsilon T_5)} + \left(\frac{a_5}{v}\right)T_4T_2e^{\beta_1 T_5/(1+\epsilon T_5)} \\ T_6 \\ \left(\frac{1}{m\lambda}\right)\begin{bmatrix} (nT_1 - v)T_6 + (n-1)T_5 - \\ l_1(T_r - T_0) - \delta T_3T_2e^{T_5/(1+\epsilon T_5)} \\ -\delta_1 T_3e^{\beta T_5/(1+\epsilon T_5)} - \delta_2 T_4T_2e^{\beta_1 T_5/(1+\epsilon T_5)} \end{bmatrix} \end{pmatrix}$$

$$F_1 = hF(Z_n), F_2 = hF\left(Z_n + \frac{1}{2}F_1\right),$$

$$F_3 = hF\left(Z_n + \frac{1}{2}F_2\right), F_4 = hF(Z_n + F_3)$$

The numerical results are presented in Fig. 1 and 2.

RESULTS AND DISCUSSION

The results obtained in this reserch are as presented on the graphs displayed in Fig. 1 and 2.

Our result in Fig. 1 shows that for a variable permeable medium, the temperature increases as the Frank-Kamenetskii parameter δ increases and this is as result of increase in heat of reaction.

Our result in Fig. 2 shows that as the flame speed increases the temperature of a variable permeable medium decreases and this is contrary to what was obtained in a constant permeability system.

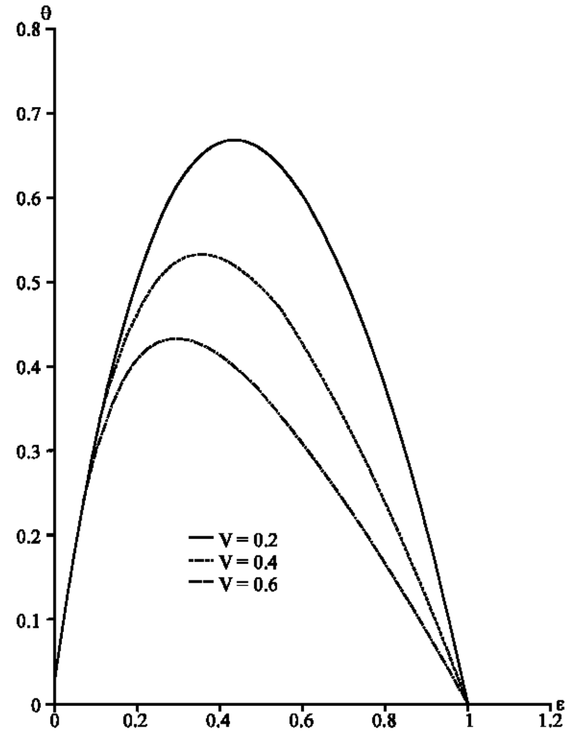

Fig. 2: Temperture profile for various values of v

The significance of the effects of the reaction parameter δ is most envisaged in the processes of recovery of heavy oils from porous rocks in which *in situ* combustion is necessary. When the heat reaction is high, the rate of conversion of heavy oils into light oils, water and gas is high and consequently, the recovery rate is boosted. This is of great economic importance. The same is true of parameter v which is the flame speed.

In this reserch, we proposed suitable expression for the permeability. From the findings of this research, it is seen that non-constant permeability affects the *in situ* combustion processes. In addition, we report that the parameters involved in the *in situ* combustion models have considerable effects on the phenomena of the flows in the medium.

CONCLUSION

We expect the results of this reserch to be useful for field and production engineers in the oil fields (especially in the layered reservoirs which have variable permeability distribution) during processes of oil recovery as well as other engineering field.

We also expect the results to form basis for further mathematical modelling and analysis of *in situ* combustion processes.

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