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Hydromagnetic Flow of a Radiating Gas with Temperature Dependent Thermal Conductivity

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Abstract: We studied the effect of radiation on a hydromagnetic flow of a radiating gas where the thermal conductivity depends linearly on the temperature of the system. The energy equation in an optically thin limit case was considered. The problem is reduced to a non-linear partial differential equation. Successive approximation method is employed to study the problem. It was observed that an increase in radiation parameter causes a decrease in temperature.

Key words: Transient, non-grey, thermal conductivity, optically thin, radiation parameter

INTRODUCTION

We know that the radiation effect is important under many non-isothermal situations. If the entire system involving the polymer extrusion process is placed in a thermally controlled environment then radiation could become important. The knowledge of radiation heat transfer in the system can perhaps lead to a desired product with sought characteristics. Recently, the effects of radiation on the flow and heat transfer of a micro polar fluid past a continuously moving plate have been studied by many authors (Kum and Fedorov, 2003; Raptis, 1998). Olajuwon (2003) investigated unsteady temperature field of a power-law fluid flow with variable thermal conductivity. The heat dissipation is essential while thermal conductivity varies with time. Kum *et al.* (2003) presented an integral approach to estimate temperature dependent thermal conductivity in a transient non-linear heat conducting medium. It is assumed that the thermal conductivity is a monotonic function of the temperature and it can be expressed as a piece wise linear function. The flow and heat transfer of an electrically conducting micro polar fluid on a continuously moving plate embedded in a non-Darcian porous medium in the present of a uniform magnetic field and radiation was studied by Mostafa *et al.* (2006) Lacey and Wake (1982) proposed that thermal conductivity various exponentially with temperature. They showed that critical behavior disappears when the thermal conductivity increases with temperature faster than the rate of heating.

For simplicity, the Cogley *et al.* (1968) approximation for an optically thin non-grey is invoked. Gupta and Gupta (1974), both employed the approximation of (Cogley *et al.*, 1968) so that close form analytic solutions are possible. Goody (1956) initiator of the problem of radiative transfer in a vertical surface considered a neutral fluid. Cess (1966), however, considered absorbing-emitting gray fluids with black vertical plate.

The present work considers the case when the thermal conductivity depends linearly on temperature and also effect of radiation parameter was considered (Adesanya, 1995).

MATHEMATICAL FORMULATION

The physical model consists of a long porous channel of width l. Fluid is sucked with velocity v_0 from the wall at $y^1=0$, while it is blown with the same velocity at the wall $y^1= 1$ in the presence of a transversely applied magnetic field. The wall at $y^1 = 0$ is maintained at a constant temperature. T_0 while that at $y^1 = 1$ is at temperature $T_1\{1+\epsilon^1 H(t^1-t^1_0)\}$. H(x) is the Heaviside step function. Mathematically a steady state situation exists for $t^1 \le t^1_0$, thereafter a transient component is super imposed. In this analysis we shall assume that the difference between T_1 and T_0 is small, so that the approximation in Cogley *et al.* (1968).

$$\text{i.e} \quad \frac{\partial q^1_y}{\partial y} = 4(T - T_w)\int_0^\infty \left(\alpha_\lambda \frac{\partial B_\lambda}{\partial T}\right)_w d\lambda \tag{1}$$

may be applicable

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The energy transfer of a radiating gas of the system is governed by:

$$\rho_\infty \; c_p \left(\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) - \frac{\partial q^1_y}{\partial y} \qquad (2)$$

with the boundary conditions
$T = T_0$ on $y^1 = 0$

$$T = T_1\left\{1+ \in^1 H(t-t_0)\right\} \text{ on } y^1=1 \qquad (3)$$

In Eq. 2, the value of the derivative of the radiative flux is given by Eq. 1 and in Eq. 2 $k = k_0\,(\alpha\theta+1)$ where k_0 and α are constants.

q^1_y : The components of radiative flux
α_λ : The absorption coefficient
B_λ : The Planck's function.

Subscript w refers to condition at the wall.

K : The thermal conductivity
δ_c : The electrical conductivity
ρ : The density

C_p is the specific heat at constant pressure subscript ∞ is for conditions in the static flux, N is the radiation parameter.

$$N = \frac{4l^2}{k^1}\int_0^\infty \left(\alpha_\lambda \frac{\partial B_\lambda}{\partial T}\right)_0 d\lambda$$

Subscript 0 on the integrand in N indicates that it is evaluated at $T=T_0$

V : The velocity vector.
Re : The Reynolds number
Rm : The magnetic Reynolds number.

METHOD OF SOLUTION

Equation (2) above will be reduced to the form that can be easily handled.

It is convenient to introduce the non-dimensional flow variables

$$y = {y^1}/{l}\;,\; t = \frac{v_0 t^1}{l}\;,\; \theta = \frac{T-T_0}{T_0-T_\infty}\;,$$
$$m = \frac{T_1-T_0}{T_0-T_\infty}\;,\; \in = \frac{\in^1 T_0}{T_0-T_\infty}\;,\; Re = \frac{V_0 l}{\nu} \qquad (4)$$

$Rm = v_0 1\, \delta_c$

Using (4) in Eq. 2 and 3 to obtain

$$Re\,Pr\left(\frac{\partial\theta}{\partial t} - \frac{\partial\theta}{\partial y}\right) = \alpha\left(\frac{\partial\theta}{\partial y}\right)^2 + (\alpha\theta+1)\frac{\partial^2\theta}{\partial y^2} - N\theta \qquad (5)$$

subject to the boundary conditions

$$\begin{aligned}&\theta = 0 \text{ on } y=0\\ &\theta = m + \in H(t-t_0) \text{ on } y=1\end{aligned} \qquad (6)$$

The primary concern of this study is the discussions of the transient Eq. 5:

$$\text{i.e.}\quad Re\,Pr\left(\frac{\partial\theta}{\partial t} - \frac{\partial\theta}{\partial y}\right) = \alpha\left(\frac{\partial\theta}{\partial y}\right)^2 + (\alpha\theta+1)\frac{\partial^2\theta}{\partial y^2} - N\theta \qquad (7)$$

Subject to the boundary conditions

$$\begin{aligned}&\theta = 0 \text{ on } y=0\\ &\qquad\qquad 0 \le y \le 1\\ &\theta(1)=1 \text{ on } y=1\end{aligned}$$

Apply method of successive approximation to Eq. 7 by dropping the non-linear terms we have

$$\frac{\partial^2\theta}{\partial y^2} + Re\,Pr\frac{\partial\theta}{\partial y} - N\theta = Re\,Pr\frac{\partial\theta}{\partial t} \qquad (8)$$

Apply separation of variables to Eq. 8 to obtain

$$\theta_0(y,t) = \lambda e^{at}\left(e^{\phi_1 y} - e^{\phi_2 y}\right) \qquad (9)$$

Where,

$$\lambda = \frac{e^{-a}}{e^{\phi^1} - e^{\phi_2}}$$

$$a = \frac{-k^2}{Re\,Pr}$$

$$\phi_1 = -\frac{\alpha_1 + \sqrt{\alpha_1^2 - 4(k^2-N)}}{2}$$

$$\phi_2 = -\frac{\alpha_1 - \sqrt{\alpha_1^2 - 4(k^2-N)}}{2}$$

where, k^2 is the separation constant. Replacing the non-linear terms in Eq. 7 with 9 to obtain

$$\alpha_1\frac{\partial\theta}{\partial t} - \alpha_1\frac{\partial\theta}{\partial y} = \alpha\left(\frac{\partial\theta_0}{\partial y}\right)^2 + \alpha\theta_0\frac{\partial^2\theta_0}{\partial y^2} + \frac{\partial^2\theta}{\partial y^2} - N\theta \qquad (10)$$

Where, α_1 = Re Pr and α =1 = constant
From Eq. 10 we have

$$\frac{\partial^2\theta}{\partial y^2} + \alpha_1 \frac{\partial\theta}{\partial y} - \alpha_1 \frac{\partial\theta}{\partial t} - N\theta = -\left[\alpha\left(\frac{\partial\theta_0}{\partial y}\right)^2 + \alpha\theta_0 \frac{\partial^2\theta_0}{\partial y^2}\right]$$

Let,

$$f(y,t) = \alpha\left(\frac{\partial\theta_0}{\partial y}\right)^2 + \alpha\theta_0 \frac{\partial^2\theta_0}{\partial y^2}$$

Where,

$$\theta_0\,(y,t) = \lambda\, e^{at}\left(e^{\phi_1 y} - e^{\phi_2 y}\right)$$

Thus we have

$$\frac{\partial^2\theta}{\partial y^2} + \alpha_1 \frac{\partial\theta}{\partial y} - \alpha_1 \frac{\partial\theta}{\partial t} = N\theta = f(y,t) \qquad (11)$$

$$\theta_p\,(y,t) = c_1\, e^{2at+2\phi_1 y} + c_2\, e^{2at+(\phi_1+\phi_2)y} + c_3\, e^{2at+2\phi_2 y} \qquad (12)$$

Putting (12) into the LHS of (11) and comparing coefficients with the RHS of (11)

$$\text{i.e}\quad \frac{\partial^2\theta_p}{\partial y} + \alpha_1 \frac{\partial\theta_p}{\partial y} - \alpha_1 \frac{\partial\theta_p}{\partial t} - N\theta_p = -\ f(y,t)$$

$$\text{Thus}\ C_1 = \frac{2\alpha\lambda^2\phi_1^2}{4\phi_1^2 + 2\alpha_1\phi_1 - 2\alpha_1 a - N}$$

$$C_2 = \frac{-\alpha\lambda^2\left(2\phi_1\phi_2 + \phi_1^2 + \phi_2^2\right)}{(\phi_1+\phi_2)^2 + \alpha_1(\phi_1+\phi_2) - 2\alpha_1 a - N}$$

$$C_3 = \frac{2\alpha\lambda^2\phi_2^2}{4\phi_2^2 + 2\alpha_1\phi_2 - 2\alpha_1 a - N}$$

$$\therefore\ \theta(y,t) = \theta_c\,(y,t) + \theta_p\,(y,t)$$

Hence,

$$\theta(y,t) = Q\left(e^{\phi_2 y+at} - e^{\phi_1 + at}\right) - \theta_p\,(0,t)\,e^{\phi_1 y} + \theta_p\,(y,t)$$

$$\text{Where,}\quad Q = \frac{1+\theta_p(0,t)\,e^{\phi_1} - \theta_p\,(1,t)}{e^{\phi_2+at} - e^{\phi_1+at}}$$

θ_c and θ_p is the complementary and particular solution to the problem.

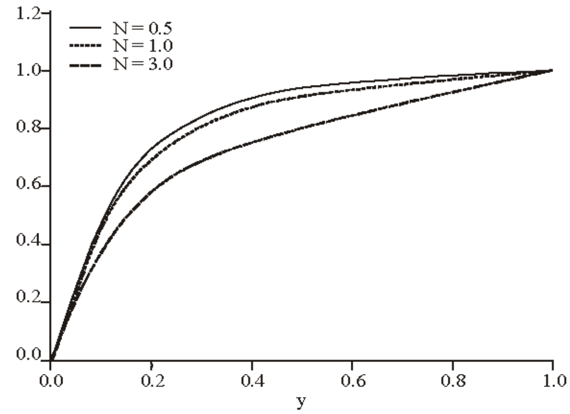

Fig. 1: Graph of temperature against position with various values of radiation parameter

DISCUSSION

In the numerical analysis we have taken Pr = 0.71 which corresponds to the Prandtl number for air. Also we take Re = 10.0, Rm = 1.0. The other quantities are varied to simulate physically realistic situations. In free convection flow involving radiation, the parameter N has an overriding effect.

In Fig. 1 below, the transient temperature distribution is depicted. An increase in radiation parameter N causes a decrease in the temperature field.

CONCLUSION

Unsteady hydromagnetic flow of a radiating gas with temperature dependent thermal conductivity in a vertical porous channel within the optically thin limit approximation was investigated. It was observed that an increase in radiation parameter causes a decrease in temperature profile. With the variation in thermophysical parameter, the effects are more pronounced than when the thermophysical parameter is a constant.

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