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Heat Transfer to Poiseuille Flow of a Reacting Pressure and Temperature Dependent Viscosity in a Channel

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Abstract: In this research, we studied the flow of a reacting pressure/temperature dependent viscous fluid, when air or oxygen is introduced into a channel containing hydrocarbon, oxidation or combustion is induced, we study the heat transfer in the channel. The model leads to system of momentum and energy equation and the resulting non-linear differential equation is solved numerically, we showed that the momentum equation just like the energy equation has multiple solutions.

Key words: Pioseuille, reacting pressure, temperature, viscosity, channel

INTRODUCTION

Fluid flows through porous media are not only important to both plant and animals for continuity of life it is also important to man due to the recovery of crude oil from the pores of the reservoir rocks.

Marchesin and Schecter (2003) considered the oxidation heat pulses in two phase expansive flow in porous media, following them, low temperature oxidation is a method of oil recovery that uses a chemical reaction to cause a temperature increase, thereby reducing oil viscosity and allowing the oil to flow more readily. It has been successfully used in various oil fields for example, total oil's Horse greek project, North Dakota U.S.A Germain and Geyelin (1997) and has been the subject of a number of papers in the petroleum engineering literature (Baibakov and Garushev, 1989; Cram and Redford 1977; Dabbous and Fulton, 1994; Fassihi *et al*., 1990; Grabowski *et al*., 1979; Prats, 1986) and the mathematics literature (Barkve, 1989; Brvining and Duijn, 2000; Da-Mota *et al*., 1999; Da-Mota, 1992). The mathematical theory of low temperature oxidation is an aspect of the theory of combustion in multiple flows in porous medium.

Bear (1972) considered the effect of pressure and temperature on viscosity and concluded that most fluid shows a pronounced variation with temperature but are relatively insensitive to pressure until high pressures have been attained he also reported that for gases at twice the critical temperature variations of viscosity with temperature are quite small until pressures of the order of the critical pressure have been reached. For gases, at high pressure an increase in temperature causes a decrease in viscosity while a decrease in temperature causes the viscosity of a gas at low density to decrease. Also Adesanya and Ayeni (2007) studied the couette flow of a reacting pressure/temperature dependent viscous flow and we gave conditions for which viscosity can depend on pressure and temperature exponentially.

The need to investigate heat transfer to a viscous incompressible fluid arises because it has been proven useful for the description of polymer melts, metal melts, blood flow as well as many other industrial flows Okoya (2006) and Makinde and Mhone (2005) investigated the heat transfer to MHD oscillatory flow in a channel filled with porous media, they assumed a constant viscosity. when the thermal conductivity (k) of the fluid is constant and its viscosity a function of temperature, analytical solutions have been presented by Frank-Kameneskii (1987), Gainutdinov (2001) and Adesanya *et al*. (2006).

In this study, we shall investigate the heat transfer in the channel with respect to the boundary conditions with the following assumptions;

- Temperature and velocity have reached the steady state.
- Viscosity is a function of pressure and temperature.
- Heat transfer through radiation is not negligible.
- Fluid is reacting and satisfies the Arrhenius temperature dependence law.
- The walls of the channel are chemically inert.
- The rmal conductivity (k) is a constant.

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MATHEMATICAL MODEL

Let us consider the flow of fluid between 2 parallel plates with gap h between them in the y-direction. The plates and the fluid between them in the fractured reservoir are assumed to extend very far in the ±x-direction the upper plate has the velocity U, the lower plate is stationary also the fluid is reacting. These assumptions lead to the steady equation: Take a Cartesian coordinate system (x, y), then, the equation governing the motion are given by

Momentum equation

$$0=\frac{d}{dy}\left(\mu\frac{du}{dy}\right)-\frac{dp}{dx}+\rho g\beta\left(T-T_0\right) \tag{1}$$

Energy equation

$$0=\frac{d}{dy}\left(k\frac{dT}{dy}\right)+Qe^{\frac{-E}{RT}} \tag{2}$$

with boundary conditions

$$u(-h)=0 \tag{3}$$

$$u(h)=0 \tag{4}$$

$$T(-h)=T_0 \tag{5}$$

$$T(h)=T_0 \tag{6}$$

We assume that,

$$-\frac{dp}{dx}=f\left(y\right)=Ky\left(1-\frac{y}{h}\right) \tag{7}$$

$$\mu=\mu_0 e^{\delta\frac{dp}{dx}+\sigma(T-T_0)} \tag{8}$$

We now introduce the following dimensionless parameters

$$\phi=\frac{u}{U},\qquad \theta=\frac{1}{RT_0^2}E\left(T-T_0\right),\qquad \bar{y}=\frac{y}{h}$$

We obtain the dimensionless equations (after dropping of bar)

$$0=\frac{d}{dy}\left(be^{\alpha y(1-y)+\beta\theta}\frac{d\phi}{dy}\right)+gy\left(1-y\right)+a\theta \tag{9}$$

$$\phi\left(-1\right)=0=\phi\left(1\right) \tag{10}$$

$$0=d\frac{d^2\theta}{dy^2}+fe^{\frac{\theta}{1+\epsilon\theta}} \tag{11}$$

$$\theta\left(-1\right)=0=\theta\left(1\right) \tag{12}$$

Where:

$$a=\frac{g\beta\rho RT_0^2}{E},\ b=\frac{U\mu_0}{h^2},\ g=Kh,\ \alpha=\delta Kh,$$
$$\beta=\frac{\sigma RT_0^2}{E},d=\frac{KRT_0^2}{Eh^2},\ f=Qe^{\frac{-E}{RT_0^2}} \tag{13}$$

We now proceed to solve the Eq. 9 and 11 under the boundary conditions (10) and (12).

METHOD OF SOLUTION

Let a = 1,b = 1, ε = 0, d = 1, g = 1, f = 0.5, β = 0.5, α = 0.5

$$\frac{d}{dy}\left(e^{0.5y(1-y)+\frac{\theta}{2}}\frac{d\phi}{dy}\right)+y\left(1-y\right)+\theta=0 \tag{14}$$

$\phi(-1) = 0 = \phi(1)$

$$\frac{d^2\theta}{dy^2}+0.5e^{\theta}=0 \tag{15}$$

$\theta(-1) = \theta(1) = 0$

The 2 solutions of (11) (Adesanya *et al*., 2006) are:

$$e^{\frac{\theta}{2}}=e^{0.156}\,\text{sech}\ \ 0.58y \tag{16}$$

and

$$e^{\frac{\theta}{2}}=e^{1.438}\,\text{sech}\ \ 2.11y \tag{17}$$

We now find the solution of Eq. 10,

$$\frac{d}{dy}\left(e^{0.5y(1-y)+\frac{\theta}{2}}\frac{d\phi}{dy}\right)+y(1-y)+\theta=0 \tag{18}$$

with $\phi(-1) = 0$, $\phi(1) = 0$

When

$$e^{\frac{\theta}{2}}=e^{0.156}\,\text{sech}\ 0.58y$$

$$\frac{d}{dy}\left(e^{0.156}\,\text{sech}0.58ye^{0.5y(1-y)}\frac{d\phi}{dy}\right)+$$
$$y\left(1-y\right)+2\ln e^{0.156}\,\text{sech}\ 0.58y=0 \tag{19}$$

$$\frac{d}{dy}\left(e^{0.5y(1-y)}e^{0.156}\,\text{sech}\ 0.58y\frac{d\phi}{dy}\right)+$$
$$y(1-y)+0.312-2\ln\left(\frac{e^{0.58y}+e^{-0.58y}}{2}\right)=0$$

We obtains

$$\frac{d}{dy}\left(e^{0.5y(1-y)}e^{0.156}\operatorname{sech}0.58y\frac{d\phi}{dy}\right)+y(1-y)+3.0844=0$$

Integrating

$$\frac{d\phi}{dy}+\frac{y^2\left(\frac{1}{2}-\frac{y}{3}\right)+3.0844y+q}{e^{0.5y(1-y)}e^{0.156}\operatorname{sech}0.58y}=0 \quad (20)$$

Where q is the integration constant
And
When

$$e^{\frac{\theta}{2}}=e^{1.438}\operatorname{sech}\ 2.11y$$

Repeating the procedure for Eq. 12 in 13
We have

$$\frac{d}{dy}\left(e^{0.5y(1-y)}e^{1.438}\operatorname{sech}2.11y\frac{d\phi}{dy}\right)+ y(1-y)+2\ln e^{1.438}\operatorname{sech}2.11y=0 \quad (21)$$

This becomes

$$\frac{d}{dy}\left(e^{0.5y(1-y)}e^{0.156}\operatorname{sech}0.58y\frac{d\phi}{dy}\right)+ y(1-y)+2.876-2\ln\left(\frac{e^{2.11y}+e^{-2.11y}}{2}\right)=0 \quad (22)$$

We finally obtains

$$\frac{d}{dy}\left(e^{0.5y(1-y)}e^{1.438}\operatorname{sech}2.11y\frac{d\phi}{dy}\right) + y(1-y)+5.6484=0 \quad (23)$$

Integrating

$$\frac{d\phi}{dy}+\frac{y^2\left(\frac{1}{2}-\frac{y}{3}\right)+5.6484y+q}{e^{0.5y(1-y)}e^{1.438}\operatorname{sech}2.11y}=0 \quad (24)$$

Equation 20 and 24 has no analytic solution, we therefore solve numerically using maple soft ware which clearly satisfy the 2 boundary conditions as seen in the Table 1.

The rate of heat transfer is given as the Nussel number (NU).

Table 1: The 2 boundary condition

y	Φ1θ1	Φ2θ2
-1	0	0
-0.9	0.45867455	0.806082412
-0.8	0.817455649	1.315031122
-0.7	1.094920945	1.632869323
-0.6	1.305837235	1.827795545
-0.5	1.461850338	1.943627106
-0.4	1.572064876	2.008389465
-0.3	1.643517502	2.039807443
-0.2	1.681554006	2.048806488
-0.1	1.690123566	2.041725973
0	1.672002256	2.021684729
0.1	1.628955959	1.989371541
0.2	1.561851956	1.943431068
0.3	1.470725191	1.880525055
0.4	1.354801306	1.795109572
0.5	1.212480295	1.678913408
0.6	1.041277056	1.520057986
0.7	0.837718037	1.301705004
0.8	0.591851497	1.00004621
0.9	0.313698147	0.581347364
1	-0.023794776	-0.002389601

Table 2: The rate of heat transfer for θ1, θ2

y	Nu1	Nu2
-1	-0.60629	-4.09774
-0.9	-0.55592	-4.03496
-0.8	-0.50267	-3.94102
-0.7	-0.44668	-3.80182
-0.6	-0.38814	-3.59844
-0.5	-0.32728	-3.30739
-0.4	-0.26439	-2.90303
-0.3	-0.19983	-2.36368
-0.2	-0.13396	-1.68215
-0.1	-0.0672	-0.87744
0	0	0
0.1	0.0672	0.87744
0.2	0.13396	1.68215
0.3	0.19983	2.36368
0.4	0.26439	2.90303
0.5	0.32728	3.30739
0.6	0.38814	3.59844
0.7	0.44668	3.80182
0.8	0.50267	3.94102
0.9	0.55592	4.03496
1	0.60629	4.09774

For $\theta 1$,
we have

$$Nu=-\frac{d\theta}{dy}=-1.16\tanh 0.58y \quad (25)$$

And for $\theta 2$
We have,

$$Nu=-\frac{d\theta}{dy}=-4.22\tanh 2.11y \quad (26)$$

The result (25) and (26) are given in Table 2.

DISCUSSION

Table 1 shows the result given in (20) and (24) while Table 2 present the result obtained in (25) and (26). While

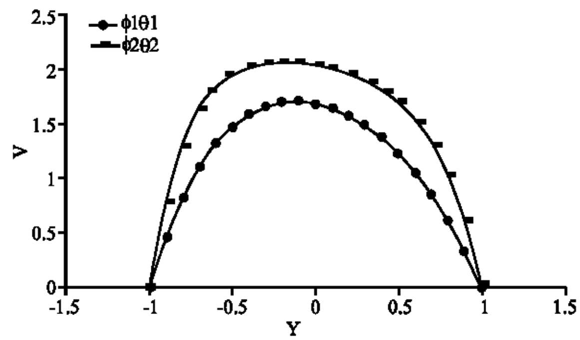

Fig. 1: Combine velocity profile

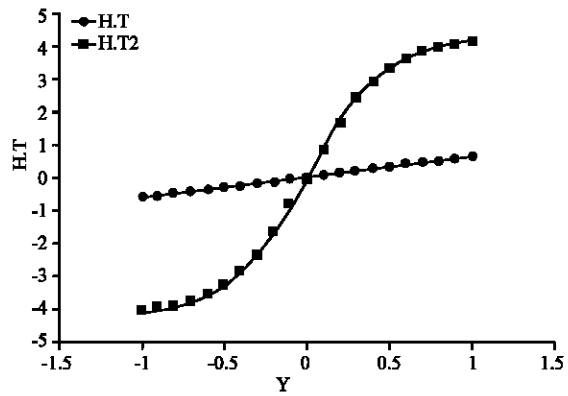

Fig. 2: Heat transfer rate

Fig. 1 shows the relationship between the two results presented in Table 1 and Fig. 2 shows the rate of heat transfer.

CONCLUSION

We have studied the heat transfer during combustion reaction in a channel, the possible application of this result is in fractured reservoir, when air or oxygen is introduced into oil reservoir and oxidation or combustion is induced since the viscosity of crude oil depends on pressure and temperature, the heat reduces the viscosity thus allowing the oil to flow more readily during oil recovery. We shall study the effect of a uniform transverse magnetic field on the flow in our next research.

Nomenclature:

U	:	Axial velocity.
T	:	Fluid temperature.
p	:	Pressure.
g	:	Gravitational force.
β	:	Coefficient of volume due to expansion.
k	:	Thermal conductivity.
μ	:	Fluid dynamic viscosity.
ρ	:	Fluid density.
Q	:	Heat per unit mass during reaction.
R	:	Universal gas constant.
E	:	Activation energy.
U_0	:	The flow mean velocity.
x	:	The co-ordinate in flow direction.
y	:	The co-ordinate across flow.
h	:	The distance between plate.
T_1 and T_0	:	Are wall temperature.

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