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Mathematical Models for the Dynamics of Typhoid Fever in Kassena-Nankana District of Upper East Region of Ghana

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Abstract: This study deals with formulation of a Mathematical Model of Typhoid Fever in the Kassena-Nankana District in Upper East Region of Ghana. The equilibrium points of the model system are presented and their stability is investigated. Threshold conditions for the disease free equilibrium are presented. Both numerical and qualitative analyses of the model were analyzed. The results of the analysis show that there exist permanent immunity equilibrium points and were found to be globally asymptotically stable.

Key words: Salmonella paratyphi, typhoid fever, compartmental model, stability, simulation

INTRODUCTION

Typhoid Fever is one of the most dangerous human infections diseases caused by a bacterium called *Salmonella typhi.* It belongs to the family *Enterobacteriaceae;* members of this family are: *Salmonella paratyphi* A, *Salmonella paratyphi* B, *Salmonella choleraesurs* and *Salmonella typhi.* (www.mcw.edu/article/955158962.html-17k).

These bacteria are primarily pathogens of humans and animals. They have been insolated from almost every animal source known, example poultry, livestock and other bird. *Salmonella typhi* is the one that is responsible for typhoid fever. *Salmonella typhi* is associated with the ability of the organism to multiply within mononuclear phagocytes. After ingesting organism, it penetrates intestinal mucosa with minimal epithelial damage. They are transported through cells in vacuoles through lymphatics to the intestinal lymph nodes and then carried to the blood stream. They are disseminated to the liver spleen and bone marrow. The bacterium then multiplies in the cells of the organs and re-enters the blood bladder, biliary system and the lymphatic tissue of the bowel, then multiply in high numbers. The incubation period for typhoid fever is eight to fourteen days and the duration of the illness is about four to 6 weeks. The patient or the infected humans show the following signs and symptoms: mild fever, slight abdominal pains, constipation, headache, loss of appetite, nausea, vomiting and appearance of a rash (rose rash) on the abdomen. Intensity of fever and abdominal pain increases and diarrhea follows. (www.medicinenet.com/typhoin _fever/article.htm-43k).

The French physician Pierre Charles Alexander Louis was the first man and that proposed the name "Typhoid Fever". Between 1822 and 1827, he studied a total of 138 patients with typhoid fever, 50 of whom died. The post mortem findings were compared to post-mortem results from 83 people who died from other causes. Dr. William Budd (1808-1882) investigated an outbreak of typhoid fever in the small Welsh border town of Cambridge in 1853. He noticed that in the outbreak locality, there was a local well close to septic pit of an inn, suggesting water contamination. He also notice that a patient who was recovery from typhoid fever had left the inn 2 days before and then all the people who became ill has been given lemonade, prepared with water from the contaminated well. He reinforced his theories in 1866, when he and a colleague Dr. Grace Traced similar outbreak in several farm cottages. (www.who.int/vaccines-document/).

In recent years, data indicating that typhoid fever is a major cause of morbidity and mortality among the urban and peri-urban population. In several community-based studies from South Asia, the incident rate seems to be especially high among young children, with rates exceeding 500-1000 cases per 100,000 populations (WHO, 2003).

A study was conducted between October 2006 and July 2007 to investigate the dynamics of typhoid fever in Kassena-Nankana district in the Upper East of Ghana. In this study, we have, therefore, developed a model for typhoid fever in Kassena-Nankana district of the Upper East Region in Ghana. Kassena-Nankana district is one of the 6 districts in the Upper East Region with Navrongo been its district Capital. The district is geographically

located at the extreme Northern part of Ghana .It shares international boarder with Burkina Faso to the North, Bongo and Bolgatanga district to the East, Builsa and Sisala to the West and Mamprusi West district to the South. The district lies between Latitudes $10^0 30^1$ and $11^0 00^1$ north of the Equator and between Longitudes $1^0 00^1$ and $1^0 30^1$ west of the zero Meridian and covers an area of 1,675 square kilometers along the Ghana Burkina Faso boarder.89% of the houses in the district are mud huts with thatched roofs.The rest, which are built with cement blocks, are mostly found in the Urban area. Almost 2/3rd of (65%) of the roofs are constructed with Straw. Zinc Sheets are used for the remaining 35%. The main sources of water supply in the district are wells and bore holes. In few urban houses, however, pipelines have been installed to provide treated water. Similarlly, only 7% of the compounds have access to properly constructed toilet facilities, suggesting that as high as 93% of the houses use the bushes in their immediate surroundings. Since this district shares common boarder with the neighbouring Country Burkina Faso, this make the Social life of the people to be high in term of chilling. Eating of Meat (Cow, Goat, Pig, Sheep, Dog meat) that had been contaminated or water. Patients with acute illness can contaminate the surrounding water supply through the stool, which contains a high concentration of the bacteria. Contamination of the water supply can in turn, taint the food supply. There is high level of disease incidence, hence there is that need to look into the health condition of the people living in Kassena-Nankana.

THE MATHEMATICAL MODEL

Variables and parameters: According to May (1975), Mazumda (1999) and Jay and Smith (1996) We use the following notations and/or definitions for the human populations.

Susceptibles S(t): The number of individuals who can be infected but have not yet contracted the *Salmonella typhi* but may contract it if exposed to any mode of its transmission.

Infectives I(t): The number of individuals who have contracted the Salmonella typhi and are actively or capable of transmitting it.

Carriers C(t): The number of individuals who, although apparently healthy themselves, harbour infection which can be transmitted to others.

Recoveries R(t): The number of individuals who are recovered after treatment and are immune to the disease.

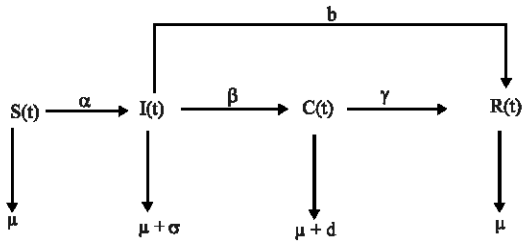

Fig. 1: Compartmental model

Assuming in the give population at time say t, S(t), I(t), C(t) and R(t) denote susceptible, infective carriers and recoveries, respectively, we define the following parameters as follows:

μ : The per capital natural mortality rate.
σ : The typhoid fever-indicated mortality rate.
α : The rate of infection.
β : The rate of progression from infective to carrier.
γ : The rate of recovery from carrier stage.
δ : The carrier-induced mortality rate.
b : The rate of recovery from the infectious stage.

The dynamics of a one strain/stage model of typhoid fever can be described as the compartmental model Fig. 1.

Asumptions of the model: The following assumptions are taken into consideration:

- Susceptible ingest solid or liquid food, a proportion of which are infective. Assume that this proportion is equal to the prevalence of infective in the population
- All susceptible individuals are equally susceptible, all infected individuals are equally infectious and all carriers are equally infectious too.
- The disease is transmitted by direct transmission or by ingestion of contaminated food or liquid by susceptible individuals.
- There is no latent period for the disease; hence the disease is transmitted instantaneous on contact.
- The population under consideration is fixed in size, that is no migration or birth occur and all deaths are taken into account.

Hence N is the population size, this implies that $N = S(t) + I(t) + C(t) + R(t)$.

Equations of the model: Applying the assumption and the inter-relations between the variables and parameters as described in the compartmental model in Fig. 1, the model of the typhoid fever is described by the following equations.

$$\frac{ds}{dt} = -\alpha SI - \mu S \quad (1)$$

$$\frac{dI}{dt} = \alpha SI - (\mu + \sigma)I - \beta I - bI \quad (2)$$

$$\frac{dc}{dt} = \beta I - (\mu + \delta)c - \gamma c \quad (3)$$

$$\frac{dR}{dt} = bI + \gamma c - \mu R \quad (4)$$

Where, N = S(t) +I(t) + C(t) + R(t) is the total population size.

ANALYSIS OF THE MODEL

Existence of steady states: Let $E(S^*, I^*, C^*, R^*)$ be the equilibrium points of the system described by the Eq. 1-4.
At an equilibrium point, we have

$$\frac{dS}{dt} = \frac{dI}{dt} = \frac{dC}{dt} = \frac{dR}{dt} = 0$$

Equilibrium of the model: The governing system of equations of the model (Eq. 1-4) has two equilibrium points namely;

- $E_0\left(\frac{\mu+\beta}{\alpha}, \frac{-\mu}{\alpha}, \frac{-\beta}{\alpha}, 0\right)$ at permanent immunity (Here no body has salmonella typhi); hence $b = \sigma = \delta = \gamma = 0$
- $E^*, (S^*, I^*, C^*, R^*)$ the endemic equilibrium.

Stability of the equilibra: We now need to determine the stability of E_0 and E^*, the following matrices are computed corresponding to equilibrium points E_0 and E^*.
From matrix M_0, let

$$K = \begin{bmatrix} (-\mu + \sigma + \beta + b) - \lambda & 0 \\ \beta & -(\mu + \delta + \gamma) - \lambda \end{bmatrix}$$

Where λ is the eigenvalues of M_0. the system is stable.

It is clear tat $\det(K) = (\mu + \sigma + \beta + b)(\mu + \delta + \gamma) > 0$ hence, there is permanent immunity, which implies that the system is stable.

Also for M^*, using the basic reproductive number, R_0 in analysing the stability of the epidemic equilibrium. We discovered that if $R_0<1$, the system will have a unique endemic equilibrium which will be globally asymptotically stable.

We need to note that if $R_0<1$, the permanent immunity equilibrium point is globally asymptotically stable while if $R_0>1$, the permanent immunity is unstable. It is clear that if $R_0 < S_0$, the initial value of S(t), dI/dt remains negative, indicating that the epidemic does not build up. However, if $R_0 < S_0$, the epidemic will build up and in the case all the susceptibles do not get infected. Alternatively, if the density of the susceptible is low, the epidemic will disappear.

NUMERICAL ANALYSIS

In the study, we present results of the numerical simulation of the model. Below are some of the parameters values used.

- Natural Mortality rate of individuals (μ): The time unit is set at year and the constant natural mortality rate, is assumed to be inversely proportional or related to the life expectancy of birth which is approximately 56 years in Ghana. Therefore $\mu = 1/56 = 0.018$ year^{-1}.
- Per capital recovery rate of infective, (b): This is taken as the inverse of time between typhoid fever infection and recovery by treatment. This is within 4 and 6 weeks. (clinical and pathogenic Microbiology by Barbara J. Howard, John Klass J. Howard, Sally Jo Rubin, Alice S. Weissfeld and Richard C. Tilton)$b < 1$ per year.
- Taking the treatment to be 5 weeks (35 days) we obtain recovery rate for infection to be $b = 1/0.096\ 10.4 \approx 10$ individuals per year. Per capital recovery rate for carrier (γ). This is also taken to be the inverse of time between carrier state and recovery by continuous treatment. According to clinical and laboratory presentation of Typhoid fever 2002 by Ahmed Yaramis and others, it is 6 weeks and beyond.
- Per capita Recovery rate for carrier, (γ): This is also taken to be the inverse of time between carrier state and recovery by continues treatment. According to Clinical and Laboratory presentation of Typhoid Fever-2002 by Ahmed Yaramis and others, it is 6 weeks and beyond, implying that $\gamma < 1$ per year. Taken the treatment to be 42 days, we obtain the recovery rate for carriers to be;

$\gamma = 1/0.115 = 8.7 \approx 9$ individuals per year.

- Per capita disease-induced mortality rate, (σ): This varies from country to country. In the developed countries, it as low as 0.06. In the endemic prone countries like Ghana, it 0.9 per year (CHIM/PPME-GHS, 2003).
- Per capita carrier-induced mortality rate, (δ): This also varies from country to country depending on the attitudes towards medication (illiteracy rate). It is therefore assumed to be the inverse of the illiteracy rate (74.8% in Ghana)

$\delta = 1/74.8 = 0.013$ per year.

- Per capita Progression rate from Infective to Carrier, (β): This range from months to decades. Most infective never progress towards the carrier state. However, average progression is short and becoming shorter due to the availability of vaccines now. It is about three to five percent (3-5%) of typhoid fever patients become carriers.
- Per capita rate of Infection, (α): Is set to match the expected number of infections produced in units of time.

$\alpha = 1044/144754 = 0.0072$, where 1044 is the annual infection rate for the district and the 144,754 is the total population for the district.

NUMERICAL SIMULATION

A computer program (Visual BASIC) was used to solve the system of equations in this study 2.3 (Eq. 1-4).

Below are the diagrams that resulted from the numerical simulation.

From the Fig. 2, we realized that as the constant of progression from the infective compartment to the carrier compartment increases, the carrier population will also increase. This means that more susceptibles will be infected with the disease to decrease the susceptible population.

As the Fig. 3 suggests, the higher the rate of recovery from the carrier compartment, the lower the carrier population will be and the higher the recovered population will also be.

The recovered population will also increase as the rate of recovery from carrier compartment to the recovered compartment (Fig. 4).

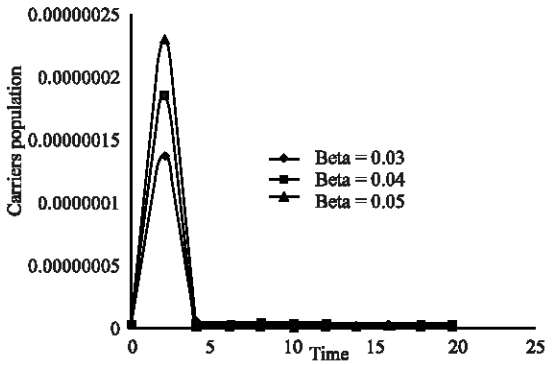

Fig. 2: Carrier population for varying rate of progression from infectives to carriers

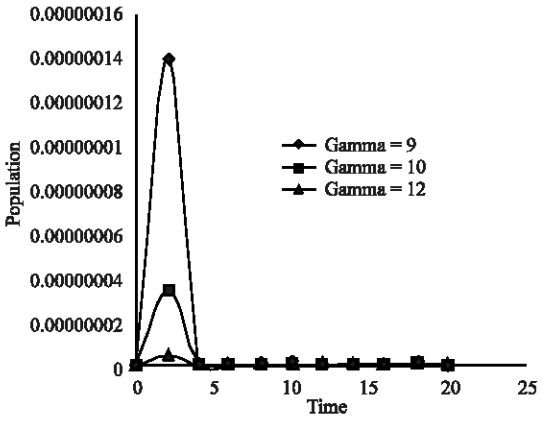

Fig. 3: Carrier population for varying carrier recovery rate

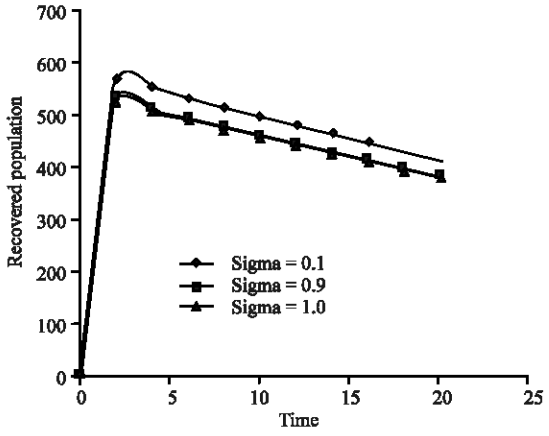

Fig. 4: Recovered patients population for varying recovery rate

The Fig. 5 also suggests that as the rate of recovery from the infective compartment to the recovered compartment increase, the recovered population also increase also increase with respect to time.

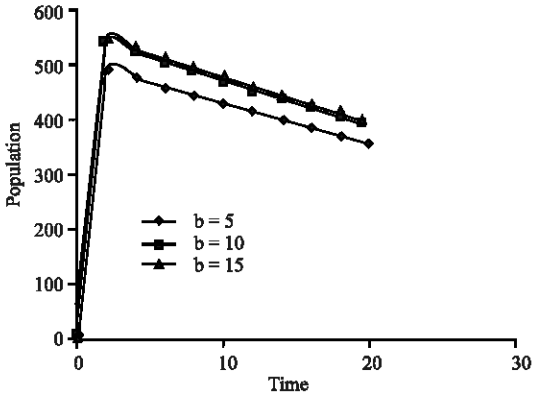

Fig. 5: Recovered population for varying rate of infectives

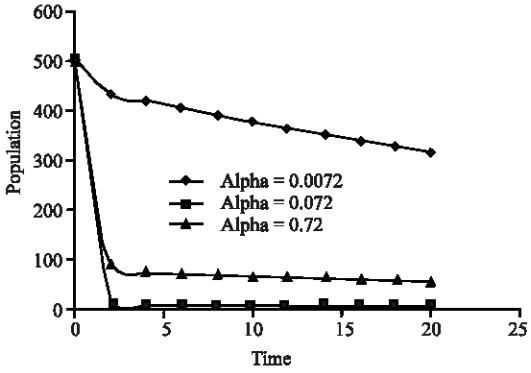

Fig. 6: Susceptible population for varying infection rate

An increase in the rate of infection with time decreases the susceptible population (Fig. 6).

CONCLUSION

In the qualitative analysis of the model, the existence of steady states and their stabilities were analyzed. The analysis showed that a permanent immunity equilibrium point existed and was found to be globally asymptotically stable provided the dI/dt remains negative.

The basic reproductive number, Rî, is one of the tools used in determining disease trends. The necessary condition for Ro being less than unity, in which case there would be no more typhoid epidemics, is the same as the requirement for the stability of the permanent immunity equilibrium point. The size of the initial value of S (t) plays a vital role in the determination of the basic reproductive number in this case of ours. The necessary condition to have Ro < 1 clearly indicates the role of population density in determining the disease trends in a district. The higher the density, the greater the risk of instability of the permanent immunity equilibrium, this implies that, there is a possibility of an epidemic in the district (population).

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