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Solution of Fredholm Integral-Differential Equations Systems by Adomian Decomposition Method

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Abstract: In this study, Adomian decomposition method, as a well-known method for solving functional equations, has been employed to solve systems of linear and nonlinear Fredholm Integral-differential equations. Theoretical considerations are discussed and convergence of the method for these systems is addressed. Some examples are presented to show the ability of the method for such systems.

Key words: Adomian decomposition method, systems of Ferdholm integral-differential equations, numerical examples

INTRODUCTION

Adomian decomposition method has been known as a powerful device for solving many functional equations as algebraic equations, ordinary and partial differential equations, integral equations and so on (Adomian, 1998; Delves, 1985).

In Biazar (2003) used the Adomian decomposition method for solving the system voltera Integral-differential equations. Later in that year Davari also applied the some method for solving linear Fredholm Integral-differential equation.

Consider the following system of integro-differential equation:

$$\begin{cases} u_1^{(n1)}(x) = f_1(x) + \int_0^1 k_1\left(x, t, u_1(t), u_2(t), \dots, u_p(t)\right) dt, \\ u_2^{(n2)}(x) = f_2(x) + \int_0^1 k_2\left(x, t, u_1(t), u_2(t), \dots, u_p(t)\right) dt, \\ \vdots \\ u_p^{(np)}(x) = f_p(x) + \int_0^1 k_p\left(x, t, u_1(t), u_2(t), \dots, u_p(t)\right) dt. \end{cases} \tag{1}$$

with initial conditions:

$$u_i^{(j)}\left(x_0\right) = u_{ij},\ i = 1, \dots, p,\ j = 0, \dots, n_i - 1$$

This system of integral-differential equation can be converted to a system of Fredholm integral equations of the second kind.

We suppose be the operator $L_x = d/dx$ and its inverse operator. Now if we act n_i times the operator L_x^{-1} on each equation of the above system, we obtain

$$\begin{cases} u_1(x) = y_1(x) + \underbrace{L_x^{-1} \cdots L_x^{-1}}_{n_1 \text{times}} \\ \left(f_1(x) + \int_0^1 k_1\left(x, t, u_1(t), u_2(t), \dots, u_p(t)\right) dt\right) \\ u_2(x) = y_2(x) + \underbrace{L_x^{-1} \cdots L_x^{-1}}_{n_2 \text{times}} \\ \left(f_2(x) + \int_0^1 k_2\left(x, t, u_1(t), u_2(t), \dots, u_p(t)\right) dt\right) \\ \vdots \\ u_p(x) = y_p(x) + \underbrace{L_x^{-1} \cdots L_x^{-1}}_{n_p \text{times}} \\ \left(f_p(x) + \int_0^1 k_p\left(x, t, u_1(t), u_2(t), \dots, u_p(t)\right) dt\right) \end{cases} \tag{2}$$

Let us write Adomian series of as the following

$$u_i\left(x\right) = \sum_0^\infty u_{ij}\left(x\right),\ i = 1, 2, \dots \tag{3}$$

Now if we substitute from Eq. 3 in 2, we set

$$\begin{cases} u_{10}(x) = y_1(x) + \underbrace{L_x^{-1} \cdots L_x^{-1}}_{n_1 \text{times}}\left(f_1(x)\right), \\ u_{20}(x) = y_2(x) + \underbrace{L_x^{-1} \cdots L_x^{-1}}_{n_2 \text{times}}\left(f_2(x)\right), \\ \vdots \\ u_{p0}(x) = y_p(x) + \underbrace{L_x^{-1} \cdots L_x^{-1}}_{n_p \text{times}}\left(f_p(x)\right). \end{cases}$$

and

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$$\begin{cases} u_{11}(x) = \underbrace{L_x^{-1} \dots L_x^{-1}}_{n_1 \text{times}} \left(\int_0^1 k_1 \left(x, t, u_{10}(t), u_{20}(t), \dots, u_{p0}(t) \right) dt \right) \\ u_{21}(x) = \underbrace{L_x^{-1} \dots L_x^{-1}}_{n_2 \text{times}} \left(\int_0^1 k_2 \left(x, t, u_{10}(t), u_{20}(t), \dots, u_{p0}(t) \right) dt \right) \\ \vdots \\ u_{p1}(x) = \underbrace{L_x^{-1} \dots L_x^{-1}}_{n_p \text{times}} \left(\int_0^1 k_p \left(x, t, u_{10}(t), u_{20}(t), \dots, u_{p0}(t) \right) dt \right) \end{cases} \tag{5}$$

or generally we have

$$\begin{cases} u_{i0}(x) = y_i(x) + \underbrace{L_x^{-1} \dots L_x^{-1}}_{n_i \text{times}} \left(f_i(x) \right) \\ u_{i,s+1}(x) = \underbrace{L_x^{-1} \dots L_x^{-1}}_{n_i \text{times}} \left(\int_0^1 k_i \left(x, t, u_{2,s}(t), u_{2,s}(t), \dots, u_{p,s}(t) \right) dt \right) \\ i = 1, 2, \dots, p,\ s = 0, 1, 2, \dots \end{cases} \tag{6}$$

CONVERGENCE OF THE METHOD

Since, after the n_i, i = 1, 2, 3, ..., p step of the mentioned procedure we derive the system (4), which is a system of Fredholm integral equations of the second kind, for the convergence of the method, we refer the reader to Babolian *et al*. (2004) and Cherruault and Saccomandi (1992) in which the problem of convergence has been discussed briefly.

NUMERICAL EXAMPLES

In this study, we present two example for testing the accuracy of our proposed method. In all of our experiments, we use the package Maple 9.5 for numerical computations.

Example 1: In this example the following linear system of integral-differential equations is solved,

$$\begin{cases} u''_1(x) = \dfrac{8}{9} + \displaystyle\int_0^1 x\left(\frac{u_1(t)}{3} + \frac{u_2(t)}{4}\right) dt \\ u''_2(x) = 6x - \dfrac{x^2}{18} + \displaystyle\int_0^1 \left(\frac{u_1(t)}{6} - \frac{u_2(t)}{3}\right) dt \end{cases}$$

with initial conditions: $u_1(0) = 0$, $u_2(0) = 0$, $u'_1(0) = 1/3$, $u'_2(0) = -1/2$ and the exact solution

$$u_1(x) = \frac{x^2}{2} + \frac{x}{3},\ u_2(x) = x^3 - \frac{x}{2}.$$

Using inverse operation, we have

$$\begin{cases} u_1(x) = \dfrac{x}{3} + \dfrac{4}{9}x^2 + \dfrac{1}{2}x^2 \displaystyle\int_0^1 \left(\frac{u_1(t)}{3} + \frac{u_2(t)}{4}\right) dt \\ u_2(x) = \dfrac{-1}{2}x + x^3 - \dfrac{x^4}{126} + \displaystyle\int_0^1 \frac{x^4}{12}(u_1(t) - u_2(t))\, dt \end{cases}$$

Using Adomian method, we have

$$\begin{cases} u_{1,0}(x) = \dfrac{x}{3} + \dfrac{4}{9}x^2 \\ u_{2,0}(x) = \dfrac{-1}{2}x + x^3 - \dfrac{x^4}{126} \end{cases}$$

Solutions with three terms are shown in Table 1:

Approximated solutions for some values of x and the corresponding absolute errors are presented in Table 1.

Example 2: Consider the following system of two nonlinear Fredholm integral-differential equations, with initial values

$$\begin{cases} u'_1(x) = 1 - 3x + \displaystyle\int_0^1 x(u_1(t)^2 + 2u_2(t))\, dt \\ u''_2(x) = 2 - \dfrac{25}{12}x + \displaystyle\int_0^1 (u_1(t) + u_1(t)u_2(t))\, dt \end{cases}$$

with initial conditions: $u_1(0) = 1$, $u_2(0) = 0$, $u'_2(0) = 0$ and the exact solution.

$$u_1(x) = x + 1,\ u_2(x) = x^2$$

Using inverse operation, we have

$$\begin{cases} u_1(x) = 1 + x - \dfrac{3}{9}x^2 + \dfrac{1}{2}x^2 \displaystyle\int_0^1 (u_1(t)^2 + 2u_2(t))dt \\ u_2(x) = x^2 - \dfrac{25}{72}x^3 + \displaystyle\int_0^1 \frac{x^3}{6}(u_1(t) + u_1(t)u_2(t))dt \end{cases}$$

Using Adomian method, we have

$$\begin{cases} u_{1,0}(x) = 1 + x - \dfrac{3}{2}x^2, \\ u_{2,0}(x) = x^2 - \dfrac{25}{72}x, \end{cases}$$

Table 1: Error of the Adomian decomposition method with 5 terms

x	$u_1(x)$	$\varphi_{1,5}(x)$	$e(\varphi_{1,5}(x))$	$u_2(x)$	$\varphi_{2,5}(x)$	$e(\varphi_{2,5}(x))$
0	0	0	0	0	0	0
0.1	0.038333	0.038332	0.0087E-004	-0.049000	-0.049000	0.0000003
0.2	0.086666	0.086663	0.0350E-004	-0.092000	-0.092005	0.000005
0.3	0.145000	0.144992	0.0788E-004	-0.123000	-0.123026	0.000026
0.4	0.213333	0.213319	0.1402E-004	-0.136000	-0.136084	0.000084
0.5	0.291666	0.291644	0.2191E-004	-0.125000	-0.125205	0.000205
0.6	0.380000	0.379968	0.3155E-004	-0.084000	-0.084426	0.000426
0.7	0.478333	0.478290	0.4295E-004	-0.007000	-0.007789	0.000789
0.8	0.586666	0.586610	0.5610E-004	0.112000	0.110652	0.001347
0.9	0.705000	0.704928	0.7100E-004	0.279000	0.276842	0.002157
1	08333333	0.833245	0.8766E-004	0.500000	0.496710	0.003289

Table 2: Error of the Adomian decomposition method with 15 terms

x	$u_1(x)$	$\varphi_{1,12}(x)$	$e(\varphi_{1,12}(x))$	$u_2(x)$	$\varphi_{2,12}(x)$	$e(\varphi_{2,12}(x))$
0	1	1	0	0	0	0
0.1	1.1	1.099790	0.000209	0.01	0.009997	0.000002
0.2	1.2	1.199161	0.000838	0.04	0.039977	0.000022
0.3	1.3	1.298114	0.001885	0.09	0.089925	0.000074
0.4	1.4	1.396647	0.0033523	0.16	0.159822	0.000177
0.5	1.5	1.494761	0.005238	0.25	0.249653	0.000346
0.6	1.6	1.592457	0.007542	0.36	0.359401	0.000598
0.7	1.7	1.689733	0.001026	0.49	0.489048	0.000951
0.8	1.8	1.786590	0.013409	0.64	0.638580	0.000141
0.9	1.9	1.883028	0.01697	0.81	0.807978	0.002021
1	2.0	1.979047	0.020952	1	0.997227	0.002772

And

$$\begin{cases} u_{1,n+1}(x) = A_{1,n}(u_{1,0},\ \ldots,\ u_{1,0},\ u_{2,0},\ \ldots,\ u_{2,0}), \\ u_{2,n+1}(x) = A_{2,n}(u_{1,0},\ \ldots,\ u_{1,0},\ u_{2,0},\ \ldots,\ u_{2,0}),\ n = 0,\ 1,\ 2,\ \ldots \end{cases}$$

For the first iteration, we have:

$$\begin{cases} u_{1,1}(x) = A_{1,0}(u_{1,0},\ u_{2,0}) = \\ \int_0^1 \frac{x^2}{2}\left(u_{1,0}(t)^2 + 2u_{2,0}(t)\right) dt = 0.7631944445t^2, \\ u_{2,1}(x) = A_{2,0}(u_{1,0},\ u_{2,0}) = \\ \int_0^1 \frac{x^3}{6}\left(u_{1,0}(t) + u_{1,0}(t)u_{2,0}(t)\right) dt = 0.2023148148t^2 \end{cases}$$

For the second iteration, we have:

$$\begin{cases} u_{1,2}(x) = A_{1,1}(u_{1,0},\ u_{1,1},\ ,\ u_{2,0},\ u_{2,1}) = \\ \int_0^1 \frac{x^2}{2}\left(2u_{1,0}(t)u_{1,1}(t) + 2u_{2,1}(t)\right) dt = 0.2668171296t^2, \\ u_{2,1}(x) = A_{2,0}(u_{1,0},\ u_{1,1},\ u_{2,0},\ u_{2,1}) = \\ \int_0^1 \frac{x^3}{6}\left(u_{1,1}(t) + u_{1,1}(t)u_{2,0}(t) + u_{1,0}(t)u_{2,1}(t)\right) \\ \quad dt = 0.06722227582t^2 \end{cases}$$

Solutions with three terms are:

$$\begin{cases} \varphi_{1,3}(x) = u_{1,0}(x) + u_{1,1}(x) + u_{1,3}(x) = 1 + x - 0.4699884255x^2, \\ \varphi_{2,3}(x) = u_{2,0}(x) + u_{2,1}(x) + u_{2,3}(x) = x^2 - 0.0776851316x^3 \end{cases}$$

The solutions after eleven iteration and for the first 12 terms are given as:

$$\begin{cases} \varphi_{1,12}(x) = u_{1,0}(x) + u_{1,1}(x) + \ldots + u_{1,11}(x) = 1 + x - 0.0390016769x^2, \\ \varphi_{2,11}(x) = u_{2,0}(x) + u_{2,1}(x) + \ldots + u_{2,11}(x) = x^2 - 0.0052469604x^3 \end{cases}$$

Approximated solutions for some values of x and the corresponding absolute errors are presented in Table 2.

CONCLUSION

In this study, we used Adomian decomposition method for solving systems of integral-differential equations. As it was shown, this method has the ability of solving systems of both linear and nonlinear integral-differential equations.

Our results shows that this method is very accurate for solving the linear integral-differential equations but also for better performance for nonlinear integral-differential equations, we must use high iterations.

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