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A Modified Super Convergent Line Series Algorithm for Solving Unconstrained Optimization Problems

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Abstract: This study presents a new algorithm for solving unconstrained optimization problems known as Modified Super Convergent Line Series (MSCLSu). The development of the new algorithm makes use of the principles of optimal experimental design. Illustrative example shows that the optimizer could be reached in just one move, as compared to some other known algorithms say the Gradient method which requires >1 iteration to reach the optimizer. Thus, the new algorithm compares favourably with the Gradient method say.

Key words: Optimal experimental design, MSCLS, support points, partitioning of design matrices, unconstrained optimization, Nigeria

INTRODUCTION

The Modified Super Convergent Line Series (MSCLS) algorithm for solving linearly constrained optimization problems, say Linear Programming Problems (LPP) has already been developed by Etukudo and Umoren (2008). The object of this study is to show that this algorithm could be modified and extended to handle unconstrained optimization problems. The procedure in the new method for solving unconstrained optimization problems which makes use of the principles optimal experimental design is essentially the same as for constrained, except that in the new method, the step length is obtained by taking the derivative of the objective function after appropriate substitutions. The modification is further made on Super Convergent Line Series (SCLS) (Onukogu, 2002) in which the entire experimental space is partitioned into k* segments from which the support points that make up the initial design matrices are obtained for each segment. In the modified super convergent line series algorithm for solving unconstrained optimization problems, the support points that make up the initial design matrix are obtained from the entire experimental space and the initial design matrix partitioned into the desired k* groups.

Thus, rather than representing each segment by a first order model as by Onukogu (2002), the MSCLSu simply represents the entire experimental space or response surface by a single response function.

As by Etukudo and Umoren (2008), the MSCLSu provides a well defined method to handle interactive effects in the case of quadratic surfaces. Also, the MSCLSu provides a well defined method of determining the direction of search d*, the optimal starting point $\bar{x}^*$ and a mathematically tractable stopping rule, stop if:

$$\left| f(x_2^*) - f(\bar{x}_2^*) \right| < \varepsilon$$

which is adaptable to computer programs. This is not the case with Onukogu (2002) which simply says stop if:

$$\bar{x}_2^* = x_f^*$$

which is not recognizable in computer operations, $\bar{x}_f^*$ is the value of the x^*at the current iteration.

Modified Super Convergent Line Series algorithm for solving unconstrained optimization problems (MSCLSu): The sequential steps involved in MSCLSu are given as follows:

Step 1: Let the response surface be:

$$y = c_0 + c_1x_1 + c_2x_2 + q_1x_1^2 + q_2x_1x_2 + q_3x_2^2$$
$$x_1, x_2 \in G_i,\ i = 1, 2, \ldots, k^*$$

Select N support points such that:

$$3k^* \le N \le 4k^*$$

where, $2 \le k^* \le 3$ is the number of partitioned groups desired. By arbitrarily choosing the support points, make up the initial design matrix:

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$$X = \begin{bmatrix} 1 & x_{11} & x_{21} \\ 1 & x_{12} & x_{22} \\ \vdots & \vdots & \vdots \\ 1 & x_{1N} & x_{2N} \end{bmatrix}$$

Step 2: Partition X into k* groups with equal number of support points and obtain the design matrix, X_i, i = 1, 2, …, k* for each group. Obtain the information matrices:

$M_i = x'_i x_i$, i = 1, 2, …, k* and their inverses, M^{-1}, i = 1, 2, …, k* such that:

$$M_i^{-1} = \begin{bmatrix} v_{i11} & v_{i21} & v_{i31} \\ v_{i12} & v_{i22} & v_{i32} \\ v_{i13} & v_{i23} & v_{i33} \end{bmatrix}$$

Step 3: Compute the matrices of the interaction effect of the variables for the groups. That is compute:

$$X_{iI} = \begin{bmatrix} x_{i11}^2 & x_{i11}x_{i21} & x_{i21}^2 \\ x_{i12}^2 & x_{i12}x_{i22} & x_{i22}^2 \\ \vdots & \vdots & \vdots \\ x_{i1N}^2 & x_{i1N}x_{i2N} & x_{i2N}^2 \end{bmatrix}$$

where, i = 1, 2, …, k* and the vector of the interaction parameters obtained from f(x) is given by:

$$g = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

The interaction vectors for the groups are given by:

$$I_i = M_i^{-1} X_i' X_{iI} \mathbf{g}$$

and the matrices of mean square error for the groups are given by:

$$\overline{M}_i = M_i^{-1} + I_i I_i' = \begin{bmatrix} \bar{v}_{i11} & \bar{v}_{i21} & \bar{v}_{i31} \\ \bar{v}_{i12} & \bar{v}_{i22} & \bar{v}_{i32} \\ \bar{v}_{i13} & \bar{v}_{i23} & \bar{v}_{i33} \end{bmatrix}$$

Step 4: The matrices of coefficient of convex combinations of the matrices of mean square error are:

$$H_i = \text{diag}\left\{\frac{\bar{v}_{i11}}{\sum \bar{v}_{i11}}, \frac{\bar{v}_{i22}}{\sum \bar{v}_{i22}}, \frac{\bar{v}_{i33}}{\sum \bar{v}_{i33}}\right\}$$

$$= \text{diag}\ \{h_{i1}, h_{i2}, h_{i3}\},\ i = 1, 2, \ldots, k^*$$

By normalizing H_i such that $\sum H_i^* H_i^{*1} = I$, we have:

$$H_i^* = \text{diag}\left\{\frac{h_{i1}}{\sqrt{\sum h_{i1}^2}}, \frac{h_{i2}}{\sqrt{\sum h_{i2}^2}}, \frac{h_{i3}}{\sqrt{\sum h_{i3}^2}}\right\}$$

The average information matrix is given by:

$$M(\xi_N) = \sum_{i=1}^{k} H_i^* M_i H_i^{*'}$$

$$= \begin{bmatrix} \overline{m}_{11} & \overline{m}_{21} & \overline{m}_{31} \\ \overline{m}_{12} & \overline{m}_{22} & \overline{m}_{32} \\ \overline{m}_{13} & \overline{m}_{23} & \overline{m}_{33} \end{bmatrix}$$

Step 5: From f(x), obtain the response vector:

$$\mathbf{z} = \begin{bmatrix} z_0 \\ z_1 \\ z_2 \end{bmatrix}$$

Where:

$$z_0 = f(\overline{m}_{12}, \overline{m}_{13})$$
$$z_1 = f(\overline{m}_{22}, \overline{m}_{23})$$
$$z_2 = f(\overline{m}_{32}, \overline{m}_{33})$$

Hence, we define the direction vector:

$$d = \begin{bmatrix} d_0 \\ d_1 \\ d_2 \end{bmatrix} = M^{-1}(\xi_N) z$$

and by normalizing d such that $d^{*1} d^* = 1$, we have:

$$d^* = \begin{bmatrix} d_1^* \\ d_2^* \end{bmatrix} = \begin{bmatrix} \frac{d_1}{\sqrt{d_1^2 + d_2^2}} \\ \frac{d_2}{\sqrt{d_1^2 + d_2^2}} \end{bmatrix}$$

Step 6: Compute the optimal starting point, $\bar{x}_1^*$ from:

$$\bar{x}_1^* = \sum_{m=1}^{N} u_m^* x_m \qquad u_m^* > 0;$$

$$\sum_{m=1}^{N} u_m^* = 1$$

$$u_m^* = \frac{a_m^{-1}}{\sum_{m=1}^{N} a_m^{-1}} \qquad m = 1, 2, \ldots, N$$

$$a_m = x_m^{'} x_m, \qquad i = 1, 2, \ldots, N$$

Step 7: Obtain the step length, ρ_1 from:

$$\frac{df(x_2^*)}{d\rho_1} = 0$$

Step 8: Make a move to the point:

$$x_2^* = \bar{x}_1^* - \rho_1 d^*$$

for a minimization problem or:

$$x_2^* = \bar{x}_1^* + \rho_1 d^*$$

for a maximization problem.

Step 9: Compute $f(\bar{x}_2^*)$ and $f(\bar{x}_1^*)$ is $\left|f(\bar{x}_2^*) - f(\bar{x}_1^*)\right| < \varepsilon$ where $\varepsilon = 0.0001$, then stop.

The current solution is optimal, otherwise, replace $f(\bar{x}_1^*)$ by $f(\bar{x}_2^*)$ and return to step 7. If the new step length, p_2 is negligibly small or zero then an optimizer had been located at the first move.

Computational procedure

Determination of the average information matrix: This shall be devoted to obtaining the average information matrix of the design when the response surface is quadratic. Here, we first obtain the support points from the entire experimental space and then partition the supports into two groups. Let the design matrix of the response function be:

$$X = \begin{bmatrix} 1 & x_{11} & x_{21} \\ 1 & x_{12} & x_{22} \\ \vdots & \vdots & \vdots \\ 1 & x_{1N} & x_{2N} \end{bmatrix}$$

then the information matrices of the groups are $M_1 = x_1' x_1$ and $M_2 = x_2' x_2$ and their inverses are respectively:

$$M_1^{-1} = \begin{bmatrix} v_{111} & v_{121} & v_{131} \\ v_{112} & v_{122} & v_{132} \\ v_{113} & v_{123} & v_{133} \end{bmatrix}$$

and;

$$M_2^{-1} = \begin{bmatrix} v_{211} & v_{221} & v_{231} \\ v_{212} & v_{222} & v_{232} \\ v_{213} & v_{223} & v_{233} \end{bmatrix}$$

Since, there exists interaction between and within variables, the matrices of the interaction effect of the variables for the two groups are respectively:

$$X_{1I} = \begin{bmatrix} x_{111}^2 & x_{111}x_{121} & x_{121}^2 \\ x_{112}^2 & x_{112}x_{122} & x_{122}^2 \\ \vdots & \vdots & \vdots \\ x_{11N}^2 & x_{11N}x_{12N} & x_{12N}^2 \end{bmatrix}$$

and;

$$X_{2I} = \begin{bmatrix} x_{211}^2 & x_{211}x_{221} & x_{221}^2 \\ x_{212}^2 & x_{212}x_{222} & x_{222}^2 \\ \vdots & \vdots & \vdots \\ x_{21N}^2 & x_{21N}x_{22N} & x_{22N}^2 \end{bmatrix}$$

and the vector of the interaction parameters obtained from f(x) is given by:

$$g = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

The interaction vectors for the groups are given by:

$$I_1 = M_1^{-1} X_1^{'} X_{1I} g \quad \text{and} \quad I_2 = M_2^{-1} X_2^{'} X_{2I} g$$

and the matrices of mean square error for the groups are respectively:

$$\overline{M}_1 = M_1^{-1} + I_1 I_1^{'} = \begin{bmatrix} \bar{v}_{111} & \bar{v}_{121} & \bar{v}_{131} \\ \bar{v}_{112} & \bar{v}_{122} & \bar{v}_{132} \\ \bar{v}_{113} & \bar{v}_{123} & \bar{v}_{133} \end{bmatrix}$$

and;

$$\overline{M}_2 = M_2^{-1} + I_2 I_2^{'} = \begin{bmatrix} \bar{v}_{211} & \bar{v}_{221} & \bar{v}_{231} \\ \bar{v}_{212} & \bar{v}_{222} & \bar{v}_{232} \\ \bar{v}_{213} & \bar{v}_{223} & \bar{v}_{233} \end{bmatrix}$$

The matrices of coefficient of convex combinations of the matrices of mean square error are:

$$H_1 = \text{diag}\left\{\frac{\bar{v}_{111}}{\bar{v}_{111} + \bar{v}_{211}}, \frac{\bar{v}_{122}}{\bar{v}_{122} + \bar{v}_{222}}, \frac{\bar{v}_{133}}{\bar{v}_{133} + \bar{v}_{233}}\right\} = \text{diag}\{h_{11}, h_{12}, h_{13}\}$$

$$H_2 = I - H_1 = \text{diag}\{1 - h_{11}, 1 - h_{12}, 1 - h_{13}\} = \text{diag}\{h_{21}, h_{22}, h_{23}\}$$

By normalizing H_1 and H_2 such that:

$$H_1^* H_1^{*'} + H_2^* H_2^{*'} = I$$

we have:

$$H_1^* = \text{diag}\left\{\frac{h_{11}}{\sqrt{h_{11}^2 + h_{21}^2}}, \frac{h_{12}}{\sqrt{h_{12}^2 + h_{22}^2}}, \frac{h_{13}}{\sqrt{h_{13}^2 + h_{23}^2}}\right\}$$

$$H_2^* = \text{diag}\left\{\frac{h_{21}}{\sqrt{h_{11}^2 + h_{21}^2}}, \frac{h_{22}}{\sqrt{h_{12}^2 + h_{22}^2}}, \frac{h_{23}}{\sqrt{h_{13}^2 + h_{23}^2}}\right\}$$

The average information matrix is given by:

$$M(\xi_N) = H_1^* X_1' X_1 H_1^{*'} + H_2^* X_2' X_2 H_2^{*'} = H_1^* M_1 H_1^{*'} + H_2^* M_2 H_2^{*'} = \begin{bmatrix} \bar{m}_{11} & \bar{m}_{21} & \bar{m}_{31} \\ \bar{m}_{12} & \bar{m}_{22} & \bar{m}_{32} \\ \bar{m}_{13} & \bar{m}_{23} & \bar{m}_{33} \end{bmatrix}$$

Determination of the direction vector of the response function: If f(x) is the response function or the objective function then the response vector, z is given by:

$$\mathbf{z} = \begin{bmatrix} z_0 \\ z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix}$$

$$z_0 = f(\bar{m}_{12}, \bar{m}_{13}, \ldots\ldots\ldots\ldots, \bar{m}_{1,n+1})$$

$$z_1 = f(\bar{m}_{22}, \bar{m}_{23}, \ldots\ldots\ldots, \bar{m}_{2,n+1})$$

$$z_n = f(\bar{m}_{n+1,2}, \bar{m}_{n+1,3}, \ldots\ldots, \bar{m}_{n+1,n+1})$$

Hence, we define the direction vactor:

$$d = \begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix} = M^{-1}(\xi_N)\mathbf{z}$$

where;

$$M^{-1}(\xi_N)\mathbf{z}$$

is the inverse of the average information matrix obtained in information matrix obtained and by normalizing d such that $d^{*'} d^* = 1$, we have:

$$d^* = \begin{bmatrix} d_1^* \\ d_2^* \\ \vdots \\ d_n^* \end{bmatrix} = \begin{bmatrix} \frac{d_1}{\sqrt{d_1^2 + d_2^2 + \cdots + d_n^2}} \\ \frac{d_2}{\sqrt{d_1^2 + d_2^2 + \cdots + d_n^2}} \\ \vdots \\ \frac{d_n}{\sqrt{d_1^2 + d_2^2 + \cdots + d_n^2}} \end{bmatrix}$$

where, $d_0 = 1$ is ignored.

Determination of optimal starting point: In order to locate the optimizer efficiently, it is important to pay particular attention to the determination of optimal starting point obtained from the initial design matrix of the entire response surface.

We illustrate that even though the problem is unconstrained, the independent variables are in fact constrained as they can only take on values within a finite dimensional region which therefore constrain the acceptable values of the response function. Thus, it is better to have the support points that make up the initial design matrix from which the optimal starting point is obtained from the boundary of the experimental region (Onukogu, 1997).

By arbitrarily choosing the support points, we make up the design matrix:

$$X = \begin{bmatrix} 1 & x_{11} & x_{21} & \cdots & x_{n1} \\ 1 & x_{12} & x_{22} & \cdots & x_{n2} \\ 1 & x_{13} & x_{23} & \cdots & x_{n3} \\ \vdots & & & & \vdots \\ 1 & x_{1N} & x_{2N} & \cdots & x_{nN} \end{bmatrix}$$

and compute the optimal starting point, $\bar{x}_1^*$ from:

$$\bar{x}_1^* = \sum_{m=1}^{N} u_m^* x_m$$

where, $u_{111}^{*}>0$ is the weight of the design measure and:

$$\sum_{m=1}^{N} u_m^* = 1$$

$$u_m^* = \frac{a_m^{-1}}{\sum_{m=1}^{N} a_m^{-1}} \quad m = 1, 2, ..., N$$

$$a_m = x_m' x_m, \quad i = 1, 2, ..., N$$

Determination of optimal step length: The optimal step length, p_1 is obtained by solving the differential equation:

$$\frac{df(x_2^*)}{d\rho_1} = 0$$

Numerical illustration: We now illustrate the working of the Modified Super Convergent Line Series algorithm (MSCLSu) by solving an unconstrained optimization problem. The example below provides the desired illustration:

$$\text{Maximize}\, f(x) = 2x_1x_2 + 2x_2 - x_1^2 - 2x_2^2$$

Solution

Step 1: Select N support points such that:

$$3\,k^* \le N \le 4\,k^*$$

where, $2 \le k^* \le 3$ is the number of partitioned groups desired. By choosing $k^* = 2$, we have $6 \le N \le 8$. Hence, by arbitrarily choosing 8 support points, the initial design matrix is:

$$X = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0.5 \\ 1 & -1 & 0.5 \\ 1 & -1 & -0.5 \\ 1 & 1 & -0.5 \\ 1 & -1 & -1 \\ 1 & 1 & -1 \end{bmatrix}$$

Step 2: Partition X into 2 groups such that:

$$G1 = \{x_1, x_2;\ x_1 = -1,\ -1 \le x_2 \le 1\}$$
$$G2 = \{x_1, x_2;\ -1 \le x_1 \le 1,\ 0.5 \le x_2 \le 1\}$$

and the design matrices for the two groups are:

$$X_1 = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0.5 \\ 1 & -1 & 0.5 \end{bmatrix}$$

$$X_2 = \begin{bmatrix} 1 & -1 & -0.5 \\ 1 & 1 & -0.5 \\ 1 & -1 & -1 \\ 1 & 1 & -1 \end{bmatrix}$$

The respective information matrices are:

$$M_1 = X_1'X_1 = \begin{bmatrix} 4.0000 & 0 & 3.0000 \\ 0 & 4.0000 & 0 \\ 3.0000 & 0 & 2.5000 \end{bmatrix}$$

and:

$$M_2 = X_2'X_2 = \begin{bmatrix} 4.0000 & 0 & -3.0000 \\ 0 & 4.0000 & 0 \\ -3.0000 & 0 & 2.5000 \end{bmatrix}$$

Step 3: The matrices of the interaction effect of the variables are:

$$X_{1I} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 0.5 & 0.25 \\ 1 & -0.5 & 0.25 \end{bmatrix}$$

and:

$$X_{2I} = \begin{bmatrix} 1 & 0.5 & 0.25 \\ 1 & -0.5 & 0.25 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

and the vector of the interaction parameters obtained from f(x) is given by:

$$g = \begin{bmatrix} -1 \\ 2 \\ -2 \end{bmatrix}$$

The interaction vectors for the groups are:

$$I_1 = M_1^{-1}X_1'X_{11}g = \begin{bmatrix} 0 \\ 1.5 \\ -3 \end{bmatrix}$$

and;

$$I_2 = M_2^{-1}X_2'X_{21}g = \begin{bmatrix} 0 \\ -1.5 \\ 3 \end{bmatrix}$$

The matrices of mean square error for the groups are respectively:

$$\overline{M}_1 = M_1^{-1} + I_1I_1' = \begin{bmatrix} 2.5000 & 0 & -3.0000 \\ 0 & 2.5000 & -4.5000 \\ -3.0000 & -4.5000 & 13.0000 \end{bmatrix}$$

and;

$$\overline{M}_2 = M_2^{-1} + I_2I_2' = \begin{bmatrix} 2.5000 & 0 & 3.0000 \\ 0 & 2.5000 & -4.5000 \\ 3.0000 & -4.5000 & 13.0000 \end{bmatrix}$$

Step 4: Obtain the matrices of coefficients of convex combinations from $\overline{M}_1$ and $\overline{M}_2$ as follows:

$$H_1 = \text{diag}\left\{\frac{2.5}{2.5+2.5}, \frac{2.5}{2.5+2.5}, \frac{13}{13+13}\right\}$$

$$= \text{diag}\ \{0.5000, 0.5000, 0.5000\}$$

$$H_2 = I - H_1 = \text{diag}\ \{0.5000, 0.5000, 0.5000\}$$

and by normalizing H_1 and H_2 such that:

$$H_1^*H_1^{*'} + H_2^*H_2^{*'} = I$$

we have:

$$H_1^* = \text{diag}\left\{\frac{0.5}{\sqrt{0.5^2+0.5^2}}, \frac{0.5}{\sqrt{0.5^2+0.5^2}}, \frac{0.5}{\sqrt{0.5^2+0.5^2}}\right\}$$

$$= \text{diag}\ \{0.7071, 0.7071, 0.7071\}$$

$$H_2^* = \text{diag}\left\{\frac{0.5}{\sqrt{0.5^2+0.5^2}}, \frac{0.5}{\sqrt{0.5^2+0.5^2}}, \frac{0.5}{\sqrt{0.5^2+0.5^2}}\right\}$$

$$= \text{diag}\ \{0.7071, 0.7071, 0.7071\}$$

The average information matrix is given by:

$$M(\xi_N) = H_1^*X_1'X_1H_1^{*'} + H_2^*X_2'X_2H_2^{*'}$$
$$= \begin{bmatrix} 3.9999 & 0 & 0 \\ 0 & 3.9999 & 0 \\ 0 & 0 & 2.5000 \end{bmatrix}$$

Step 5: From $f(x_1, x_2)$, obtain the response vector:

$$z = \begin{bmatrix} z_0 \\ z_1 \\ z_2 \end{bmatrix}$$

That is:

$z_0 = f(0, 0) = 0$

$z_1 = f(3.9999, 0) = -15.9992$

$z_2 = f(0, 2.5) = -7.5$

Therefore:

$$\mathbf{z} = \begin{bmatrix} 0 \\ -15.9992 \\ -7.5 \end{bmatrix}$$

Here, we define the direction vector:

$$d = \begin{bmatrix} d_0 \\ d_1 \\ d_2 \end{bmatrix} = M^{-1}(\xi_N)\, z = \begin{bmatrix} \underline{0.0000} \\ -3.9999 \\ -3.0001 \end{bmatrix}$$

and by normalizing d such that $d^{*1}\, d^* = 1$, we have:

$$d^* = \begin{bmatrix} d_1^* \\ d_2^* \end{bmatrix} = \begin{bmatrix} -0.8000 \\ -0.6000 \end{bmatrix}$$

Step 6: Obtain the optimal starting point:

$$\overline{x}_1^* = \sum_{m=1}^{N} u_m^* x_m \quad u_m^* > 0;$$

$$\sum_{m=1}^{N} u_m^* = 1$$

$$u_m^* = \frac{a_m^{-1}}{\sum_{m=1}^{N} a_m^{-1}} \quad m = 1, 2, \ldots\ldots, N$$

$$a_m = x_m' x_m \quad m = 1, 2, \ldots\ldots, N$$

$$a_1 = x_1' x_1 = \begin{bmatrix} 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = 3 \quad a_1^{-1} = 0.3333$$

By similar calculation, we obtain:

$$a_2^{-1} = 0.3333,\ a_3^{-1} = 0.4444,\ a_4^{-1} = 0.4444$$
$$a_5^{-1} = 0.4444,\ a_6^{-1} = 0.4444,\ a_7^{-1} = 0.3333$$
$$a_8^{-1} = 0.3333$$
$$\sum_{m=1}^{8} a_m^{-1} = 3.1108$$

Since:

$$u_m^* = \frac{a_m^{-1}}{\sum_{m=1}^{8} a_m^{-1}} \quad m = 1, 2, \ldots\ldots, 8$$

then;

$$u_1^* = \frac{0.3333}{3.1108} = 0.1071$$

Similarly:

$$u_2^* = 0.1071,\ u_3^* = 0.1429,\ u_4^* = 0.1429$$
$$u_5^* = 0.1429,\ u_6^* = 0.1429,\ u_7^* = 0.1071$$
$$u_8^* = 0.1071$$

Hence, the optimal starting point is:

$$\bar{x}_1^* = \sum_{m=1}^{8} u_m^* x_m = 0.1071 \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + 0.1071 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + 0.1429 \begin{bmatrix} 1 \\ 1 \\ 0.5 \end{bmatrix} +$$
$$0.1429 \begin{bmatrix} 1 \\ -1 \\ 0.5 \end{bmatrix} + 0.1429 \begin{bmatrix} 1 \\ -1 \\ -0.5 \end{bmatrix} + 0.1429 \begin{bmatrix} 1 \\ 1 \\ -0.5 \end{bmatrix} +$$
$$0.1071 \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} + 0.1071 \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1.0000 \\ 0.0000 \\ 0.0000 \end{bmatrix}$$

Step 7: Obtain the step length, ρ_1 from:

$$x_2^* = \bar{x}_1^* + \rho_1 d^* = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \rho_1 \begin{bmatrix} -0.8000 \\ -0.6000 \end{bmatrix} = (-0.8\rho_1, -0.6\,\rho_1)$$

That is:

$$f(x_2^*) = f(-0.8\,\rho_1, -0.6\rho_1) = -1.2\,\rho_1 - 0.4\,\rho_1$$
$$\frac{d f(x_2^*)}{d\rho_1} = -1.2 - 0.8\,\rho_1^2 = 0$$

Hence:

$$\rho_1 = -1.5$$

Step 8: Make a move to the point:

$$x_2^* = \bar{x}_1^* + \rho_1 d^* = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - 1.5 = \begin{bmatrix} -0.8000 \\ -0.6000 \end{bmatrix} \begin{bmatrix} 1.2 \\ 0.9 \end{bmatrix}$$

Step 9: By calculation, $f(x_2^*) = 0.9$ and $f(xx_1^*) = 0$

Since $|f(x^*_2) - f(\bar{x}^*_1)| = 0.9$ is large, we make a second move by replacing:

$$\bar{x}_1^* = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ by } x_2^* = \begin{bmatrix} 1.2 \\ 0.9 \end{bmatrix}$$

The new step length is obtained as follows:

$$f(x^*_3) = f(1.2 - 0.8\rho_2,\ 0.9 - 0.6\,\rho_2) = 0.9 - 0.4\,\rho^2_2$$

$$\frac{d f(x_3^*)}{d\rho_2} = -0.8\rho_2 = 0$$

Hence, $\rho_2 = 0$. Since the new step length is zero, the optimal solution was obtained at the first move and hence:

$$x_2^* = \begin{bmatrix} 1.2 \\ 0.9 \end{bmatrix}$$

and;

$$f(x_2^*) = 0.9$$

CONCLUSION

This study has successfully executed the primary objective of this study, namely the development of the Modified Super Convergent Line Series (MSCLSu) algorithm for solving unconstrained optimization problems. The illustrative example showing a sequential procedure of obtaining optimal solution of the unconstrained optimization problem gives the value of the optimizer as:

$$x_2^* = \begin{bmatrix} 1.2 \\ 0.9 \end{bmatrix}$$

and the value of the objective function as $f(x^*_2) = 0.9$. This result compares favourably with:

$$x^* = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

and f(x*) = 1 obtained by the Gradient method (Hillier and Lieberman 1995, 2001).

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