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Facility Location Selection: An Interval Valued Intuitionistic Fuzzy TOPSIS Approach

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Abstract: The aim of this study is to select the best location for multi-criteria decision making facility location with interval valued intuitionistic fuzzy information in which the information about attribute weights is completely known and the attribute values take the form of interval valued intuitionistic fuzzy numbers. The weighted Euclidean distances between every facility location alternative with positive ideal solution and negative ideal solution are calculated. Then according to the weighted Euclidean distances, the relative closeness degree to the positive ideal solution is calculated to rank all location alternatives. Finally, an illustrative example about facility location selection is considered to verify the developed approach.

Key words: Facility location selection, fuzzy logic, multi-criteria decision making, intuitionistic fuzzy topsis

INTRODUCTION

The facility location decision involves organizations seeking to locate, relocate or expand their operations for finding the lowest cost plan for distributing stocks of goods or supplies from multiple origins to multiple destinations that demand the goods. The facility location decision process encompasses the identification, analysis, evaluation and selection among alternatives (Yang and Huei, 1997). Selecting a facility location is a very important decision for firms because they are costly and difficult to reverse and they entail a long term commitment and also location decisions have an impact on operating costs and revenues. For an instance, a poor choice of location might result in excessive transportation costs, a shortage of qualified labor, lost of competitive advantage, inadequate supplies of raw materials or some similar condition that would be detrimental to operations.

There are many criteria that influence the location decisions of firms (Stevenson, 1993), however some criteria are so important that they tend to dominate the decision. In the study, we take 5 criteria into consideration; these are favorable labor climate, proximity to markets, community considerations, quality of life, proximity to suppliers and resources (Ertugrul and Karakasglu, 2008). Schilling *et al.* (1993) provide a detailed review of the covering models in facility site. An integrated approach to warehouse site selection process where both quantitative and qualitative aspects were considered by Korpela and Tuominen (1996). The conventional approaches like locational cost volume analysis, factor rating and center of gravity method (Stevenson, 1993) for facility location problems tend to be less effective in dealing with the imprecise or vague nature of the linguistic assessment (Kahraman *et al.*, 2003). In real life, the evaluation data of plant location suitability for various subjective criteria and the weights of the criteria are usually expressed in linguistic terms and also to efficiently resolve the ambiguity frequently arising in available information and do more justice to the essential fuzziness in human judgment and preference, the fuzzy set theory has been used to establish an ill defined multiple criteria decision making problems (Liang, 1999). In order to deal with vagueness of human thought (Zadeh, 1965) first introduced the fuzzy set theory. Atanassov (1986, 1989) introduced the concept of Intuitionistic Fuzzy Set (IFS) which is a generalization of the concept of fuzzy set (Zadeh, 1965).

It has received more and more attention since its appearance. Atanassov and Gargov (1989) and Atanassov (1994) further introduced the Interval-valued Intuitionistic Fuzzy Set (IVIFS) which is a generalization of the IFS.

The fundamental characteristic of the IVIFS is that the values of its membership function and non-membership function are intervals rather than exact numbers. TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) developed by Hwang and Yoon (1981) views a MADM problem. Since then, it is one of the useful MADM techniques to manage real world problems.

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According to this technique, the best alternative would be ideal solution and farthest from the negative ideal solution (Benitez *et al*., 2007). In short, the positive ideal solution is composed of all best values attainable of criteria whereas the negative ideal solution consists of all worst values attainable of criteria (Wang, 2008). TOPSIS defines an index called similarity (or relative closeness) to the positive ideal solution and the remoteness from the negative ideal solution.

Then the method chooses an alternative with the maximum similarity to the positive ideal solution. A fuzzy TOPSIS model for solving the facility location selection problem have been presented by Chu (2002a, b) and Yong (2006). Liang and Wang (1991), Kahraman *et al.* (2003), Chou *et al.* (2008) and Ishii *et al.* (2007) have considered other fuzzy multicriteria decision making methods for facility location selection.

Chen (2000) extend the concept of TOPSIS to develop a methodology for solving multi-person MADM problems in fuzzy environment.

Jahanshahloo *et al.* (2006) extend the concept of TOPSIS to develop a methodology for solving MADM problems with interval data. Xu (2007) has applied interval-valued intuitionistic fuzzy sets to pattern recognitions.

In this study, by considering the fact that in some cases, determining precisely the exact value of the values are considered as interval valued intuitionistic fuzzy information, therefore we extended the concept of TOPSIS to develop a methodology for solving MADM problems to deal with the facility location selection with interval-valued intuitionistic fuzzy information in which the information about attribute weights is completely known and the attribute values take the form of interval valued intuitionistic fuzzy numbers.

Preliminaries: In the following, some basic concepts related to intuitionistic fuzzy sets and interval-valued intuitionistic fuzzy sets have been discussed.

Definition 1: Let X is a universe of discourse then a fuzzy set is defined as:

$$A = \{\langle x, \mu_A(x) | x \in X\rangle\}$$

which is characterized by a membership function,

$$\mu_A(x): X \to [0, 1]$$

Where μ_A (x)is the degree of membership of the element x to the set A (Zadeh, 1965).

Atanassov (1994) extended the fuzzy set to the IFS shown as follows:

Definition 2: An IFS A in X is given by:

$$A = \{\langle x, \mu_A(x), v_A(x) | x \in X\rangle\}$$

Where $\mu_A(x)$: X→ [0, 1] and $v_A(x)$: X→ [0, 1] with the condition $0 \le \mu_A(x) + v_A(x) \le 1$. The numbers $\mu_A(x)$ and $v_A(x)$ represent, respectively, the membership degree and nonmembership degree of the element x to the set A (Atanassov, 1986, 1989)

Definition 3: For each IFS A in X if $\pi_A(x) = 1 - \mu_A(x) - v_A(x)$, $\forall x \in X$.

Then $\pi_A(x)$ is called the degree of indeterminacy of x to A (Atanassov, 1986, 1989).

Definition 4: Let X be a universe of discourse, an IVIFS $\tilde{A}$ over X is an object having the form (Atanassov and Gargov, 1989; Atanassov, 1994):

$$\tilde{A} = \{\langle x, \tilde{\mu}_A(x), \tilde{v}_A(x) | x \in X\rangle\}$$

Where, $\tilde{\mu}_A(x) \subset [0, 1]$ and $\tilde{v}_A(x) \subset [0, 1]$ are interval numbers and $0 \le \sup(\tilde{\mu}_A(x)) + \mathrm{sum}(\tilde{v}_A(x)) \le 1,\ \forall x \in X$. For convenience, let:

$$\tilde{\mu}_A(x) = [a, b],\ \tilde{v}_A(x) = [c, d]$$
$$\tilde{A} = ([a, b], [c, d])$$

Definition 5: Let,

$$\tilde{a}_j^1 = ([a_j^1, b_j^1], [c_j^1, d_j^1])\ (j = 1, 2, \ldots, n)$$

and

$$\tilde{a}_j^2 = ([a_j^2, b_j^2], [c_j^2, d_j^2])\ (j = 1, 2, \ldots, n)$$

be two collections of interval-valued intuitionistic fuzzy values, then the weighted Euclidean distance between $\tilde{a}_j^1$ $(j = 1, 2, \ldots, n)$ and $\tilde{a}_j^2$ $(j = 1, 2, \ldots, n)$ is defined as follows:

$$d(a_j^1, a_j^2) = \frac{1}{4}\sum_{j=1}^{n} w_j$$
$$\sqrt{(a_j^1 - a_j^2)^2 + (b_j^1 - b_j^2)^2 + (c_j^1 - c_j^2)^2 + (d_j^1 - d_j^2)^2},$$
$$j = 1, 2, \ldots, n$$

Where, $w = \{w_1, w_2, \ldots, w_n\}$ is weight vector of $\tilde{a}_j$ (j = 1, 2,...,n).

Interval valued intuitionistic fuzzy TOPSIS approach: The following are the steps to select the best facility location using inter valued intuitionistic fuzzy TOPSIS. The suitable facility location selection has become one of the most important issues for a company success. The facility location selection is a Multiple Criteria Decision Making (MCDM) problem that includes both qualitative and quantitative attributes such as favourable labour climate, proximity to markets, community considerations, quality of life, proximity to suppliers and resources etc. A MADM problem to deal with the facility location selection can be concisely expressed in matrix format as:

$$A = \begin{array}{c|cccc} & G_1 & G_2 & & G_n \\ \hline A_1 & a_{11} & a_{12} & \dots & a_{1n} \\ A_2 & a_{21} & a_{22} & \dots & a_{2n} \\ A_m & a_{m1} & a_{m2} & \dots & a_{mn} \end{array}$$

$$w = \left[w_1 \, w_2 \dots w_n\right]$$

Where:

$A = \{A_1, A_2, \dots, A_n\}$ = It is a discrete set of facility location alternatives

$G = \{G_1, G_2, \dots, G_n\}$ = The set of attributes

$w = \{w_1, w_2, \dots, w_n\}$ = The weighting vector of the attribute G_j

$$j = 1, 2, \dots, n$$

Where:

$$w_j \in [0,1], \sum_{j=1}^{n} w_j = 1$$

Suppose that;

$$\tilde{A} = \left[\tilde{a}_{ij}\right]_{m\times n} = \left(\left[a_{ij}, b_{ij}\right], \left[c_{ij}, d_{ij}\right]\right)_{m\times n}$$

is the inter valued intuitionistic fuzzy decision matrix where $[a_{ij}, b_{ij}]$ indicates the degree that the facility alternatives A_i satisfies the attribute G_j given by the decision maker, $[c_{ij}, d_{ij}]$ indicates the degree that the facility alternatives; A_i does not satisfies the attribute G_j given by the decision maker;

$$[a_{ij}, b_{ij}] \subset [0, 1] \; [c_{ij}, d_{ij}] \subset [0, 1]$$
$$b_{ij} + d_{ij} \le 1, i = 1, 2, \dots, m; j = 1, 2, \dots, n$$

Step 1: Determine the positive ideal and negative ideal location alternatives based on interval-valued intuitionistic fuzzy information:

$$\tilde{r}^{+} = \begin{pmatrix} \left([a_1^{+}, b_1^{+}], [c_1^{+}, d_1^{+}]\right), \left([a_2^{+}, b_2^{+}], [c_2^{+}, d_2^{+}]\right), \\ \dots, \left([a_n^{+}, b_n^{+}], [c_n^{+}, d_n^{+}]\right) \end{pmatrix}$$

$$\tilde{r}^{-} = \begin{pmatrix} \left([a_1^{-}, b_1^{-}], [c_1^{-}, d_1^{-}]\right), \left([a_2^{-}, b_2^{-}], [c_2^{-}, d_2^{-}]\right), \\ \dots, \left([a_n^{-}, b_n^{-}], [c_n^{-}, d_n^{-}]\right) \end{pmatrix}$$

Where:

$$([a_j^{+}, b_j^{+}], [c_j^{+}, d_j^{+}]) = \left([\max_i a_{ij}, \max_i b_{ij}], [\min_i c_{ij}, \min_i d_{ij}]\right)$$
$$([a_j^{-}, b_j^{-}], [c_j^{-}, d_j^{-}]) = \left([\min_i a_{ij}, \min_i b_{ij}], [\max_i c_{ij}, \max_i d_{ij}]\right)$$

Step 2: Calculate the weighted Euclidean distance. The weighted Euclidean distances of each location alternative from the ideal location alternative is given as:

$$d(\tilde{r}_i, \tilde{r}^{+}) = \frac{1}{4}\sum_{j=1}^{n} w_j$$
$$\sqrt{(a_{ij} - a_j^{+})^2 + (b_{ij} - b_j^{+})^2 + (c_{ij} - c_j^{+})^2 + (d_{ij} - d_j^{+})^2},$$
$$i = 1, 2, \dots, m$$

Similarly, the weighted hamming distances of each location alternative from the negative ideal location alternative is given as:

$$d(\tilde{r}_i, \tilde{r}^{-}) = \frac{1}{4}\sum_{j=1}^{n} w_j$$
$$\sqrt{(a_{ij} - a_j^{-})^2 + (b_{ij} - b_j^{-})^2 + (c_{ij} - c_j^{-})^2 + (d_{ij} - d_j^{-})^2},$$
$$i = 1, 2, \dots, m$$

Step 3: Calculate the relative closeness to the ideal location alternative. The relative closeness of the location alternative with respect to is defined as:

$$c(\tilde{r}_i, \tilde{r}^{+}) = \frac{d(\tilde{r}_i, \tilde{r}^{-})}{d(\tilde{r}_i, \tilde{r}^{+}) + d(\tilde{r}_i, \tilde{r}^{-})} \quad i = 1, 2, \dots, m$$

Step 4: Rank all the locations alternatives and select the best one(s) in accordance with:

$$c(\tilde{r}_i, \tilde{r}^{+}), \quad i = 1, 2, \dots, m$$

Illustrative example: In this, an example adapted from Ertugrul and Karakasglu (2008) for a multicriteria decision making facility location selection is used as a demonstration of the application of the proposed fuzzy decision making method in a realistic scenario. There is a panel with three possible alternatives to facility location considered in the comparison are A-C. The five attributes include labor climate (G_1) proximity to markets (G_2), community considerations (G_3), quality of life (G_4) and proximity to suppliers and resources (G_5), respectively.

The three possible facility location are to be evaluated using the interval-valued intuitionistic fuzzy information by the decision maker under the above five attributes in the matrix given as:

$$\tilde{A}=\begin{bmatrix} ([0.4, 0.5], [0.3, 0.4])([0.4, 0.6], [0.2, 0.4])([0.1, 0.3],[0.5, 0.6]) \\ ([0.3, 0.4], [0.4, 0.6])([0.3, 0.4], [0.5, 0.7]) \\ ([0.6, 0.7], [0.2, 0.3])([0.6, 0.7], [0.2, 0.3])([0.4, 0.7], [0.1, 0.2]) \\ ([0.2, 0.3], [0.5, 0.7])([0.4, 0.6], [0.5, 0.7]) \\ ([0.3, 0.6], [0.3, 0.4])([0.5, 0.6], [0.3, 0.4])([0.5, 0.6],[0.1, 0.3]) \\ ([0.1, 0.4], [0.3, 0.5])([0.2, 0.4], [0.5, 0.6]) \end{bmatrix}_{3\times 5}$$

And the weighting vector of criteria labor climate (G_1), proximity to markets (G_2), community considerations (G_3), quality of life (G_4) and proximity to suppliers and resources (G_5) are:

$$w = (0.30, 0.25, 0.13, 0.12, 0.20)$$

Step1: Determine the positive ideal and negative ideal solution:

$$\tilde{r}^{+} = \begin{bmatrix} ([0.6, 0.7], [0.2, 0.3])([0.6, 0.7], [0.2, 0.3]) \\ ([0.5, 0.7], [0.1, 0.2])([0.3, 0.4], [0.3, 0.5]) \\ ([0.4, 0.6][0.5, 0.6]) \end{bmatrix}$$

$$\tilde{r}^{-} = \begin{bmatrix} ([0.3, 0.5], [0.3, 0.4])([0.4, 0.6], [0.3, 0.4]) \\ ([0.1, 0.3], [0.5, 0.6])([0.1, 0.3], [0.5, 0.7]) \\ ([0.2, 0.4][0.5, 0.7]) \end{bmatrix}$$

Step 2: Calculate the weighted Euclidean distances of each location alternative from the ideal solution and negative ideal solution:

$$d(\tilde{r}_1, \tilde{r}^{+}) = 0.33, d(r_2, \tilde{r}^{+}) = 0.071, d(\tilde{r}_3, \tilde{r}^{+}) = 0.253$$

$$d\,(\tilde{r}_1, \tilde{r}^{-}) = 0.11\, d(r_2, \tilde{r}^{-}) = 0.35 \;\; d(\tilde{r}_3, \tilde{r}^{-}) = 0.203$$

Step 3: Calculate the relative closeness to the ideal solution:

$$c(\tilde{r}_1, \tilde{r}^{+}) = 0.25, c(r_2, \tilde{r}^{+}) = 0.83, c(\tilde{r}_3, \tilde{r}^{+}) = 0.44$$

Step 4: Rank all the facility location alternatives A-C and select the best one(s) in accordance with $c(\tilde{r}_i, \tilde{r}^{+}),\ i = 1,2,3,4$. The most suitable facility location alternative is B.

CONCLUSION

In this study, we utilize the interval valued intuitionistic fuzzy TOPSIS to select the best location for multicriteria decision making facility location in which the information about attribute weights is completely known and the attribute values take the form of interval valued intuitionistic fuzzy numbers. Finally, an illustrative example is proved to illustrate the developed approach. In further research, other multicriteria decision making methods in fuzzy environment can be used to handle facility location problems.

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