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Mathematical Model of the Impact of Vaccination on the Transmission Dynamics of Fowl pox in Poultry

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Abstract: In this study, the researchers present the mathematical model of the impact of vaccination on the transmission dynamics of fowl pox in poultry. The model resulted in a system of 1st order ordinary differential equation. Analyzing the system using methods from dynamical system theory together with Routh-Harwitz theorem, it was established that the disease-free equilibrium is locally stable if the effective reproductive ratio $R_\rho = (1 - \rho)\ \alpha\beta/d_1+r_1+\mu$ in the presence of vaccination is <1 and unstable if it is >1. Using the condition for control, the critical proportion that needs to be vaccinated to achieve herd immunity for fowl pox is established as $\rho_c = \alpha\beta - (d_1+r_1+\mu)/\alpha\beta$. From this research, researchers discover that fowl pox can be eradicated from the poultry through vaccination provided the critical proportion ρ_c is achieved.

Key words: Fowl pox, vaccination, herd immunity, critical proportion, reproductive ratio, Nigeria

INTRODUCTION

In the past 15 years, outbreak of fowl pox has cause severe mortality losses in both vaccinated and unvaccinated flocks. Fowl pox alone can cause high mortality of up to 50-60% in unvaccinated birds. Fowl pox is a viral disease of various avian species that causes skin lesion (dry pox) seen around the comb, wattle, ear lobes and eyes. The wet pox is associated with oral cavity and upper respiratory tract. The wet pox is more serious, the cause of the present industrial problem in poultry. There are many types of avian pox viruses and they tend to be specific to particular species of birds. All age group are at risk and the distribution of this disease is worldwide.

The design of strategies for the control of an infectious disease in a host population; in this case, avian pox or fowl pox is based on quantitative information of the transmission dynamics under different condition. In principle, such knowledge can only be obtained by analyzing the data from natural outbreaks and animal experiments; mathematical modeling is of great importance here. Mathematical modeling is used whenever there is need to extrapolate from current or previous conditions (De Jong and Hagenaars, 2009).

Avian pox or fowl pox has a slow-spreading nature; it is possible to vaccinate to intercept an outbreak. Flocks and birds still unaffected may be vaccinated usually with chicken strain by Wing Web Vaccinating Method. The fundamental characteristics of vaccination is its ability to reduce the incidence of the disease in those immunized, the susceptible. Also, vaccination protects indirectly non-vaccinated susceptible against infection by producing herd immunity.

Finding threshold conditions that determine whether an infectious disease will spread or die out in a population remains one of the fundamental questions of epidemiological modeling. For this purpose there remain a key epidemiological quantity R_0 the basic reproduction ratio. R_0 is the secondary cases that result from single infectious individuals in an entirely susceptible population. The current usage of R_0 is the following: if $R_0<1$, the modeled disease die out and if $R_0>1$, the disease spreads in the population. Reproductive ratios turn out to be an important factor for determining target for vaccination coverage. In mathematical models the reproductive number R_0 is determined by the dominant eigenvalue of the Jacobian matrix at the infection-free equilibrium for models in a finite dimensional space (Tessa, 2006).

If the proportion of the population that is immune exceeds the herd immunity level for the disease then the can no longer persist in the population. Thus if this level can be exceeded by vaccination the disease can be eliminated (Savill *et al*., 2006). Fowl pox in chicken does not present much problem to most poultry farmers since, satisfactory vaccine is available for prevention. Serious manifestation of the infection may occur when not protected become infected with fowl pox virus.

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Herd immunity theory proposes that in contagious disease that is transmitted from individuals to individuals, chain of infection is likely to be disrupted where large number of individuals is vaccinated. Herd immunity is a public health concept developed by epidemiologist with the intension of protecting indirectly individual that are not immune.

Lee and Suarez (2007) on their research on Avian influenza. Prospects for prevention and control by vaccination reported that although, vaccination does not always prevent infection of Avian Influenza (AI) virus, the clear benefit of vaccination is it ability to prevent and reduce the amount of virus in circulation. Thus judicious use of vaccination can be an important component of avian influenza control.

Da Silva *et al.* (2009) in her research on Fowl pox: Identification and adoption of prophylactic measures in backyard chicken in Bahia, Brazil, recommended the control and vaccination of non-infected birds as an intervention strategy to control fowl pox. According to the research, when the impact of the intervention strategy was evaluated, it was discovered that for a number of 700 chickens that were vaccinated only 9% of new cases were reported.

Fowl pox prevention and control in birds is accomplished by vaccination using the web win method with commercially available fowl pox vaccines. All the birds should be vaccinated at the same time. Vaccinated birds should be examined for takes about 7-10 days following inoculation, takes consist of the swelling of the skin or a scab at the site where the vaccination was applied. A high percentage of the birds showing takes indicate a satisfactory vaccination.

Precaution should be taken when a administering pox vaccine as it is a type of live vaccine. Only healthy birds should be vaccinated. Mosquito control should be part of the prevention programme (Butcher and Rossi, 2003). Vaccination should be done prior to expected exposure of the disease virus. Areas that have mosquito throughout the year often use two vaccinations, one early and one later for permanent protection. Chicks can be vaccinated as early as 1 day of age. In this study, researchers present a mathematical modeling that incorporates vaccination of those entering the susceptible population.

MATERIALS AND METHODS

Model formulation

Assumptions:

- The susceptible birds are vaccinated, both those that are bought and those that are born within the system, 1 day of age
- The infected individuals (birds) are not vaccinated
- Birds that recover and join the susceptible class are also vaccinated
- Birds that die as result of infection are eliminated from the environment immediately
- Recovered birds do not remain carriers, one application of fowl pox result in permanent immunity
- During this period, the total population N is fixed that is S+E+I+R = N

Symbols/parameters:

S = Susceptible population of birds at time t
E = Exposed (Latent) population of birds at time t
I = Infected population of birds at time t
R = Removed population of birds at time t
β = Recruitment term-new birds that enter into the susceptible population
α = Transmission rate of infection
γ = The rate at which the susceptible join the exposed population class
μ = The rate of recovery of infected birds
ρ = Proportion of those successfully vaccinated
k = Rate at which the exposed moves to infected population
d_1 = Death rate due to infection
d_2 = Death rate of the removed population
r = The rate at which the susceptible birds are brought into the poultry
δ = The rate at which susceptible birds are born into the poultry $\delta + r = \beta$

The model:

$$\begin{aligned} \frac{dS}{dt} &= (1-\rho)\beta S - \alpha SI + \mu I - \gamma S \\ \frac{dE}{dt} &= \gamma S - \kappa E \\ \frac{dI}{dt} &= \alpha SI + \kappa E - d_1 I - \mu I - r_1 I \\ \frac{dR}{dt} &= \rho\beta S + r_1 I - d_2 R \\ \beta &= r + \delta \end{aligned} \qquad (1)$$

Stability of the disease-free state: The Jacobian of Eq.1 at the equilibrium point (S^*, E^*, I^*, R^*) is:

$$J = \begin{bmatrix} (1-\rho)\beta - \alpha I^* - \gamma & 0 & \mu - \alpha S^* & 0 \\ \gamma & -k & 0 & 0 \\ \alpha I^* & k & \alpha S^* - (d_1 + \mu + r_1) & 0 \\ \beta\rho & 0 & r_1 & d_2 \end{bmatrix}$$

From the recommendation of other researchers (Tessa, 2006), researchers assume all the susceptible are vaccinated. In the absence of infection $E^* = I^* = 0$, the Jacobian of Eq. 1 at the disease-free J_0 (ρ, 0, 0 N-ρ) is:

$$J_0 = \begin{bmatrix} (1-\rho)\beta-\gamma & 0 & \mu-\alpha\rho & 0 \\ \gamma & -k & 0 & 0 \\ \alpha I^* & k & \alpha\rho-(d_1+\mu+r_1) & 0 \\ \beta\rho & 0 & r_1 & d_2 \end{bmatrix}$$

Its eigenvalues are $X_1 = -d_2$ and the roots of:

$$X^3 + (\gamma + k + \rho\beta - \beta - d_1 - r_1 - \mu)X^2 + \begin{bmatrix} (\beta\rho + \gamma + d_1 + r_1 + \mu)k + (\beta\rho + \gamma) \\ (d_1 + r_1 + \mu) - (k + d_1 + r_1 + \mu)\beta \end{bmatrix} X + (1-\rho)(\beta-\gamma)(d_1 + r_1 + \mu)k + \gamma k(\alpha\rho - \mu) = 0$$

Theorem 1: The disease-free equilibrium is locally stable if $R_p<1$ and unstable if $R_p>1$ where, $R_p = (1 - \rho)\ \alpha\beta/d_1+r_1+\mu$.

Proof: As X_1 is negative, it remains to prove X_2, X_3 and X_3 the roots of the cubic part of the characteristic polynomial of J_0 are both negative. This is done using the Routh-Hurwitz theorem. This is true when $a_0>0$, $a_1>0$, $a_2>0$ and $a_3>0$ where, a_0, $a_{1\text{-}3}$ are the coefficients of the cubic part of the characteristic polynomial of J_0:

$$a_3 = 1 > 0$$
$$a_2 = \gamma + k + \rho\beta - \beta - d_1 - r_1 - \mu > 0$$
$$a_1 = (\beta\rho + \gamma + d_1 + r_1 + \mu)k + (\beta\rho + \gamma)$$
$$(d_1 + r_1 + \mu) - (k + d_1 + r_1 + \mu)\beta > 0$$
$$a_0 = (1-\rho)\beta - \gamma)(d_1 + r_1 + \mu)k + \gamma k(\alpha\rho - \mu) > 0$$

Remark: $R_p = (1 - \rho)\ \alpha\beta/d_1 + r_1 + \mu$ is the effective reproductive number in the presence of vaccination. If $\rho = 0$, $R_0 = \alpha\beta/d_1+r_1+\mu$ is the basic reproductive number in the absence of vaccination.

Optimal vaccination strategies: Herd immunity describes a form of immunity that occurs when the vaccination of a significant portion of a population (or herd) provides a measure of protection for individuals who have not developed immunity (John and Samuel, 2000). Herd immunity theory proposes that in contagious diseases that are transmitted from individual to individual, chains of infection are likely to be disrupted when large numbers of a population are immune or less susceptible to the disease. The greater the proportion of individuals who are resistant, the smaller the probability that a susceptible individual will come into contact with an infectious individual (History of Epidemiology of Global Smallpox Eradication).

Vaccination acts as a sort of firewall in the spread of the disease, slowing or preventing further transmission of the disease to others (Fine, 1993). Unvaccinated individuals are indirectly protected by vaccinated individuals as the latter will not contract and transmit the disease between infected and susceptible individuals (History of Epidemiology of Global Smallpox Eradication). Hence, a public health policy of herd immunity may be used to reduce spread of an illness and provide a level of protection to a vulnerable, unvaccinated subgroup. Since, only a small fraction of the population (or herd) can be left unvaccinated for this method to be effective, it is considered best left for those who cannot safely receive vaccines.

The condition for control: Let ρ be the proportion immune after vaccination. To reach the critical proportion ρ_c that is the proportion that needs to be vaccinated to achieve herd immunity. The condition; $R_0\ (1 - \rho_c)<1$ must be satisfied:

$$R_0(1-\rho_c)=1 \Rightarrow \rho_c = 1 - \frac{1}{R_0} \Rightarrow \rho_c = \frac{\alpha\beta - (d_1 + r_1 + \mu)}{\alpha\beta}$$

Under this condition, the minimal coverage to control or prevent fowl pox is such that every bird does not need to be immuned through vaccination. We need the critical proportion. $\rho_c = \alpha\beta - (d_1+r_1+\mu)/\alpha\beta$ to achieve herd immunity necessary to control or prevent the infection.

RESULTS AND DISCUSSION

Finding threshold conditions that determine whether fowl pox infections will spread or die out in a population remains one of the fundamental questions of epidemiological modeling that this research help to answer. In this research, researshers present the mathematical modeling of the impact of vaccination on the transmission dynamics of fowl pox in poultry. The model resulted in a system of 1st order ordinary differential equation. Analyzing the system using methods from dynamical system theory together with Routh-Harwitz theorem, it was established that the disease-free equilibrium is locally stable if the effective reproductive ratio.

$R_p = (1 - \rho)\ \alpha\beta/d_1+r_1+\mu$ in the presence of vaccination is <1 and unstable if it is >1. The researchers have also studied the local stability of the endemic equilibrium by linearization, Jacobian matrix and Routh-Harwitz theorem. Using the condition for control, the critical proportion that needs to be vaccinated to achieve herd immunity for fowl pox is established as $\rho_c = \alpha\beta - (d_1+r_1+\mu)/\alpha\beta$.

CONCLUSION

From this research, researchers discover that fowl pox can be eradicated from the poultry through vaccination provided the critical proportion ρ_c is achieved.

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