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# Mathematical Modeling of the Transmission Dynamics of Fowl Pox in Poultry

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**Abstract:** In this study, researchers present two models that examine the transmission dynamics of fowl pox among birds based on the mode of transmission of the disease in poultry. Using methods from dynamical systems theory equilibrium analysis of the first model showed that the disease free equilibrium is stable if $\alpha N < (d_1+\mu+r_1)$, $\beta<\gamma$. The endemic equilibrium is asymptotically stable if $\beta-\gamma<\alpha (d_1+\mu+r_1)/k$. That is, fowl pox will not invade the poultry if the rate at which the susceptible birds ($\beta$) are introduced into the poultry is greater than the rate at which the susceptible birds become exposed to infection ($\gamma$). It was also established that $R_0<1$ if $S_0>Sc$ where $S_c = (d_1+\mu+r_1)/\alpha$ and $R_0 = \gamma S_0/(d_1+\mu+r_1)$. The second model is stable if the rate at which the infected birds recover and the rate at which mosquitoes die are high. Also if the growth rate of mosquito is less than the death rate of mosquito.

**Key words:** Fowl pox, poultry, birds, mosquitoes, exposed birds, Nigeria

## INTRODUCTION

Fowl pox, pox or avian pox is a relatively slow-spreading viral disease characterized by skin lesions or plagues in the pharynx. It is prevalent among chickens, turkey, pigeons, canaries, etc., worldwide. Morbidity is 10-95% and mortality usually 0-50%. Infection occurs through the skin abrasions and bites or by the respiratory route. The virus persists in the environment for months. The duration of the disease is about 14 days on individual bird bases.

The infected birds display some of the following symptoms: Warty spreading eruption, scabs on comb and wattles gaseous deposits in mouth throat and sometimes trachea, depression, poor growth and poor egg production. Because of its slow-spreading nature, it is possible to vaccinate to stop an out break. Flocks and individuals still unaffected may be vaccinated usually with chicken strain by wing web vaccinating method. If there is evidence of secondary bacterial infection, broad-spectrum, antibiotics may be of some benefit.

Fowl pox or avian pox is transmitted by direct contact between infected birds and susceptible birds or by contact between infected mosquitoes and susceptible birds. Virus-containing scabs also can be slough from infected birds and serve as a source of infection. The virus can enter the blood stream through the eye, skin wound or respiratory tracts. Mosquitoes become infected by feeding on birds with fowl pox in their blood stream. There is some evidence that the mosquitoes remain infected for life. Mosquitoes are the primary reservoir and spreaders of fowl pox on poultry ranges. Several species of mosquito can transmit fowl pox. Often mosquitoes winter-over in poultry houses so outbreak can occur during winter and early spring (Perry, 1992). Poultry is a general name for birds of several species such as chickens or domestic fowls, turkeys, ducks, geese, guinea fowls, pen fowls, ostriches, pheasant and other game birds. Poultry industry is regarded as one of the most successful industries in Nigeria's economy today. Individuals can afford to start the business whether on small scale or large scale. One of the challenges facing the management of poultry is epidemic outbreak. If the rate of epidemic outbreak is controlled or minimized, it will reduce the risk in the industry thereby encouraging more investment in this area. This can be done if the dynamics of the effective rate of population growth and the transmission dynamics of the infection are well understood. A lot of research has been carried on in this area.

Inyama (2009a) shown a mathematical for the transmission dynamics of bird flu among birds and humans. The model assumes that there is no migration of birds in the susceptible bird population immediately the disease starts. The analysis of the steady state and stability showed that the system will be stable if there is a bound on the growth (birth) rate of birds in the community. Okuonghae and Okuonghae (2006) shows a mathematical model for the dynamics of Lassa fever. The research showed that contributions from regular contact

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with the species of rats that carry the virus which cause Lassa fever and infectious contact with those suffering from the disease is seen as significant in the spread of the disease. Steady states of the model were examined for disease free and endemic situations. A second model that incorporates the effect of vaccination on the set of the target population was proposed, although at the moment there is no vaccine against the disease. However, the model showed that in the interim, control of the rodents carrying the virus and some isolation policy for infected individuals are the best strategies against the spread of the disease.

Yusuf (2006) shows a mathematical model to examine the population dynamics with respect to the bird flu and its transmission. The appropriate systems of ordinary differential equations were formulated and solved numerically. The analysis of the system showed that spread of the virus will continue as long as the infected birds are in circulation and the tendency of human infection sooner or later. Joshua showed a simple SIR mathematical model of the mortality rate of broiler chickens. This is to help farmers initiate management programmes that will reduce mortality rate and its overall effect on poultry production. The analysis of the model showed that good welfare condition must be observed to avoid the extreme case of effective mortality rate being large.

## MATERIALS AND METHODS

**Assumptions:**

- The chicks are vaccinated few days after being hatched
- Infected birds are treated
- The exposed (latent) birds are not treated
- Treated birds can join the exposed or treated population class
- Individuals die only by infection
- The exposed birds are not treated as such they do not join the susceptible population class

The population of birds is divided into Susceptible (S), Exposed (E), Infected (I) and Removed (R) (Fig. 1).

**Parameters/symbols:**

S = Susceptible population of birds at time t
E = Exposed (Latent) population of birds at time t
I = Infected population of birds at time t
R = Removed population of birds at time t
$\beta$ = Recruitment term-new birds that enter into the susceptible population
$\alpha$ = Transmission rate of infection

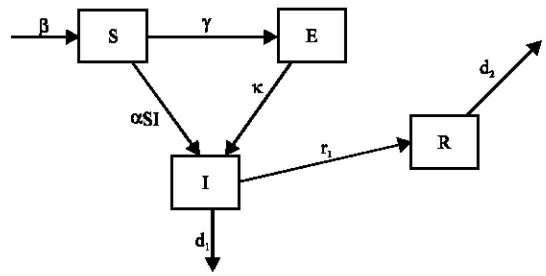


Fig. 1: Division of population of birds

$\gamma$ = The rate at which the susceptible join the exposed population class
$\mu$ = The rate at which the treated infective join the susceptible population class
$r_1$ = The rate at which the infected birds that do not respond to treatment are removed
k = Rate at which the exposed moves to infected population
$d_1$ = Death rate due to infection
$d_2$ = Death rate of the removed population
r = The rate at which the susceptible birds are bought in the poultry
$\delta$ = The rate at which susceptible birds are born into the poultry

**The model:** Using the above symbols and assumptions, researchers develop the model as follows:

$$\frac{dS}{dt} = \beta S - \alpha SI + \mu I - \gamma S \quad (1)$$

$$\frac{dE}{dt} = \gamma S - \kappa E \quad (2)$$

$$\frac{dI}{dt} = \alpha SI + \kappa E - d_1 I - \mu I - r_1 I \quad (3)$$

$$\frac{dR}{dt} = r_1 I - d_2 R \quad (4)$$

$$\beta = r + \delta$$

The equations describe the dynamics of the susceptible, exposed infected and removed population of birds. Equation 1 describes the dynamics of the susceptible.

The first term of Eq. 1 is the recruitment term-new birds that enter into the susceptible population in the second term the susceptible become infected at the rate as a result of interaction with the infected. In the third term, the treated infected joins the susceptible population at the rate. In the last term, the susceptible birds join the exposed population at the rate $\gamma$.

Equation 2 describes the dynamics of the exposed birds in the poultry. That is those that do not manifest the symptoms of fowl pox infection in the poultry. The first term shows the growth of the exposed population of birds as a result of movement from the susceptible to the exposed population of birds while the second term indicates the movement from the exposed to the infected population (full blown fowl pox).

Equation 3 describes the dynamics of the infected birds (those suffering from fowl pox infection). The first and second terms show that the infected population increases as the susceptible birds become infected and the exposed birds move to the infected population. Decrease in the equation is as a result of infected birds, those treated and those that do not respond to treatment are moved away. Equation 4 describes the population of the removed birds. The first term is the movement from the infected that do not respond to treatment while the second term is the death of the removed.

**Equilibrium analysis:** When modeling infectious diseases, the aim is to find out whether or not the infection will invade the community. The researchers therefore carry out equilibrium and stability analysis to have a better understanding of the dynamics of the disease. To determine the behaviour of the population near the equilibrium solution, researchers need to compute the linearization of the system which is obtained from the jacobian matrix of the system. From the system of Eq. 4, the Jacobian matrix is follows:

$$J = \begin{pmatrix} \beta - \alpha I^0 - \gamma & 0 & \mu - \alpha S^0 & 0 \\ \gamma & -k & 0 & 0 \\ \alpha I^0 & k & \alpha S^0 - (d_1 + \mu + r_1) & 0 \\ 0 & 0 & r_1 & -d_2 \end{pmatrix}$$

**Disease Free Equilibrium (DFE):** The disease free equilibrium point is given as $(S^0, E^0, I^0, R^0) = (N, 0, 0, 0)$. The aim is to find out what happens to the poultry if a small number of fowl pox infected birds are introduced into the poultry. Can the disease free state be achieved? Researchers therefore, perform a stability analysis for the steady state. For the point $(S^0, E^0, I^0, R^0) = (N, 0, 0, 0)$ the Jacobian of the system is obtained as follows:

$$J_0 = \begin{pmatrix} \beta - \gamma & 0 & \mu - \alpha N & 0 \\ \gamma & -k & 0 & 0 \\ 0 & k & \alpha N - (d_1 + \mu + r_1) & 0 \\ 0 & 0 & r_1 & -d_2 \end{pmatrix}$$

The eigenvalues of the Jacobian are:

$$\lambda_1 = \beta - \gamma,\ \lambda_2 = -k,\ \lambda_3 = \alpha N - (d_1+\mu+r_1),\ \lambda_4 = -d_2$$

For the disease free state to be stable, the real part all the eigenvalues of the Jacobian matrix must be negative. This is possible if the:

$$\alpha N - (d_1 + \mu + r_1) < 0$$

$$\Rightarrow \alpha N < (d_1 + \mu + r_1) \Rightarrow N < \frac{d_1 + \mu + r_1}{\alpha} \quad (5)$$

$$\beta - \gamma < 0 \Rightarrow \beta < \gamma \quad (6)$$

If Eq. 4 and 5 are satisfied then the infection dies out of the population and there is no invasion of the poultry by the disease. That is this disease may easily be eradicated from the poultry after sometimes. The condition for this to happen is that there is a bound on the recruitment of susceptible birds into the population which is equal to ratio of the sum of death due to infection ($d_1$), rate of treatment of infection ($\mu$) and the rate of removal of those that do not respond to treatment ($r_1$) to that of the infection rate ($\alpha$). From Eq. 5 researchers obtain the threshold condition for the equation researchers see that:

$$N<d_1+\mu+r_1/\alpha$$

Hence, the critical susceptible pool ($S_c$) is:

$$S_c = \frac{d_1 + \mu + r_1}{\alpha} \quad (7)$$

Let $S_0$ be the initial susceptible birds. If $S_0$>Sc then the infection will spread and there will be epidemic otherwise the infection will die out. Using the method of new generation matrix, the basic reproduction number ($R_0$) of infectious process is obtained as:

$$R_0 = \frac{\gamma S_0}{d_1 + \mu + r_1}$$

If $R_0$<1 then there will be no sufficient transmission potential to sustain the chain of infection that is expected number of secondary infection will reduce and gradually vanishes out from the system and the disease will die out (Inyama, 2009b).

**Endemic fowl pox:** At the endemic fowl pox, all the birds is assumed to be exposed or infected. That is there is no susceptible birds. Again setting the derivatives in Eq. 4 to zero and solving algebraically, researchers obtain the equilibrium points thus:

$$(S^0, E^0, I^0, R^0) = (0, \frac{d_1+\mu+r_1}{k}, N - \frac{d_1+\mu+r_1}{k}, 0)$$

For;

$$(S^0, E^0, I^0, R^0) = (0, \frac{d_1+\mu+r_1}{k}, N - \frac{d_1+\mu+r_1}{k}, 0)$$

the Jacobian matrix is:

$$J_E = \begin{pmatrix} \beta-\gamma-\frac{\alpha(d_1+\mu+r_1)}{k} & 0 & \mu & 0 \\ \gamma & -k & 0 & 0 \\ \frac{\alpha(d_1+\mu+r_1}{k} & k & 1-(d_1+\mu+r_1) & 0 \\ 0 & 0 & r_1 & -d_2 \end{pmatrix}$$

The eigen values of the Jacobian matrix $J_E$ are:

$$\lambda_1 = \beta-\gamma-\frac{\alpha(d_1+\mu+r_1)}{k},\ \lambda_2 = -k,$$
$$\lambda_3 = -(d_1+\mu+r_1),\ \lambda_4 = -d_2$$

For the endemic equilibrium to be stable, all the eigenvalues of the jacobian matrix $J_E$ must be negative. This is possible if:

$$\beta-\gamma-\frac{\alpha(d_1+\mu+r_1)}{k} < 0 \quad (8)$$

Hence, the endemic equilibrium is stable if the rate at which the susceptible birds ($\beta$) is introduced into the poultry is greater than the rate at which the susceptible birds become exposed to infection ($\gamma$). $S^0$, $E^0$, $I^0$ and $R^0$ are the equilibrium points of various epidemiological states.

**Interaction with mosquito:** As discussed before that mosquitoes are the primary reservoir and spreaders of fowl pox on poultry ranges. Fowl pox can also be transmitted by direct contact between the susceptible birds and infected mosquitoes. Researchers shall present a mathematical model of the interaction between infected mosquito and the susceptible birds. Here, researchers regard the mosquito as the vector. Researchers assume that the mosquitoes remain infected for life. The researchers assume that the treated birds can recover and join the susceptible population. Researchers, also assume that the vector (mosquito) has logistic growth:

S = Susceptible birds population at time t
E = Exposed birds population at time t
V = Mosquito population at the time t
$\alpha$ = Transmission rate between the susceptible birds and infected birds
$\mu$ = Transmission rate between infected mosquito and infected birds
P = Transmission rate between the exposed and infected mosquito
$r_1$ = The rate of treatment of infected birds
$\varphi$ = Natural death rate
$r_2$ = Growth rate of mosquito
K = Carrying capacity
$k_1$ = The rate of recovery of the exposed birds
$d_1$ = Rate of death due to infection
$\gamma$ = The rate at which the susceptible birds join the exposed birds population

Using the above assumption and symbols, the model is developed as follows:

$$\begin{aligned} \frac{dS}{dt} &= \beta S - \alpha SI - \mu SV + r_1 I - \gamma S + k_1 E - \varphi S \\ \frac{dE}{dt} &= \gamma S - k_1 E - pVE - \varphi E \\ \frac{dI}{dt} &= \alpha SI + pVE + \mu SV - r_1 I - d_2 I \\ \frac{dV}{dt} &= r_2 V\left(1-\frac{V}{K}\right) - \varphi V \end{aligned} \quad (9)$$

**Equilibrium analysis:** The different epidemiological states are represented by S, E, I, V. Researchers should consider the cases:

**Case 1:** In case 1; (S, 0, 0, 0) = ($S^*$, 0, 0, 0). This implies that no infective, no mosquito, no exposed and total population is susceptible.

**Case 2:** (S, E, 0, 0) = ($S^*$, N – $S^*$, 0, 0). This implies that no infective no mosquito, the total population is made up of the susceptible and the exposed (Inyama, 2009a). Linearizing (Yusaf, 2006), obtain the following Jacoian matrix:

$$J = \begin{pmatrix} -\alpha I^* - \mu V^* - (\gamma+\varphi) & k_1 & r_1 - \alpha S^* & -\mu S^* \\ \gamma & -(k_1+\varphi) - \rho V^* & 0 & -\rho E^* \\ \alpha I^* + \mu V & \rho V^* & \alpha S^* - (d_1+r_1) & \rho E^* + \mu S^* \\ 0 & 0 & 0 & r_2(1-\frac{V^*}{K}-\frac{r_2 V^*}{K}-\varphi \end{pmatrix}$$

Taking case 1 (S, 0, 0, 0) = ($S^*$, 0, 0, 0):

$$J_1 = \begin{pmatrix} -(\gamma+\varphi) & k_1 & r_1-\alpha S^* & -\mu S^* \\ \gamma & -(k_1+\varphi) & 0 & 0 \\ 0 & 0 & \alpha S^*-(d_1+r_1) & \mu S^* \\ 0 & 0 & 0 & r_2-\varphi \end{pmatrix}$$

$$|J_1-\lambda I| = \begin{vmatrix} -(\gamma+\varphi)-\lambda & k_1 & r_1-\alpha S^* & -\mu S^* \\ \gamma & -(k_1+\varphi)-\lambda & 0 & 0 \\ 0 & 0 & \alpha S^*-(d_1+r_1)-\lambda & \mu S^* \\ 0 & 0 & 0 & r_2-\varphi-\lambda \end{vmatrix}$$

Taking case 2 (S, E, 0, 0) = ($S^*$, N – $S^*$, 0, 0). Let = $p_2$, = $p_1$:

$$J_2 = \begin{pmatrix} -p_1 & k_1 & r_1-\alpha S^* & -\mu S^* \\ \gamma & -p_2 & 0 & -\rho(N-S^*) \\ 0 & 0 & \alpha S^*-(d_1+r_1) & \rho(N-S^*)+\mu S^* \\ 0 & 0 & 0 & r_2-\varphi \end{pmatrix}$$

$$|J_2-\lambda I| = \begin{vmatrix} -p_1-\lambda & k_1 & r_1-\alpha S^* & -\mu S^* \\ \gamma & -p_2-\lambda & 0 & -\rho(N-S^*) \\ 0 & 0 & \alpha S^*-(d_1+r_1)-\lambda & \rho(N-S^*)+\mu S^* \\ 0 & 0 & 0 & r_2-\varphi-\lambda \end{vmatrix}$$

A careful analysis of case 1 and 2 shows that they have the same eigenvalues:

$$\Rightarrow r_2-\varphi-\lambda = 0 \quad (10)$$

$$\alpha S^*-(d_1+r_1)-\lambda = 0 \quad (11)$$

$$(-p_2-\lambda)(-p_1-\lambda)-k_1\gamma = 0 \quad (12)$$

From Eq. 10 and 11 researchers have:

$$\lambda_1 = r_2-\varphi$$
$$\lambda_2 = \alpha S^*-(d_1+r_1)$$

From Eq. 12:

$$\lambda = \frac{-\delta_1 \pm \sqrt{\delta_1^2-4\delta_2}}{2}$$

$$\Rightarrow \lambda_3 = \frac{-\delta_1 + \sqrt{\delta_1^2-4\delta_2}}{2}$$

$$\Rightarrow \lambda_4 = \frac{-\delta_1 - \sqrt{\delta_1^2-4\delta_2}}{2}$$

Where, $P_1+P_2 = \boldsymbol{\delta}_1$ and $P_1P_2 - k_1\,\boldsymbol{\gamma} = \boldsymbol{\delta}_2$. Let:

$$e = \frac{\sqrt{\delta_1^2-4\delta_2}}{2}$$

Then:

$$\lambda_3 = \frac{-\delta_1}{2}+e, \quad \lambda_4 = \frac{-\delta_1}{2}-e$$

For the disease free state to be stable, all the real part of the eigen values must be negative. This possible if:

$$\varphi-\lambda < 0$$
$$\alpha S^*-(d_1+r_1) < 0$$
$$\frac{-\delta_1}{2}+e < 0$$
$$\frac{-\delta_1}{2}-e < 0$$

## RESULTS AND DISCUSSION

In this study, researchers showed two models that examine the transmission dynamics of fowl pox among birds. This is based on the mode transmission of the disease in poultry. As discussed before, mosquitoes are the primary reservoir and spreaders of fowl pox on poultry ranges. Fowl pox or avian pox is transmitted by direct contact between infected birds and susceptible birds or by contact between infected mosquitoes and susceptible birds. The first model completely pictures and examines the transmission dynamics between infected birds and susceptible birds. One of the reasons for making models of infectious disease is to design a policy aimed at eradicating or at least controlling them. The most significant policy is to reduce the $R_0$, the basic reproductive ratio <1. Using methods from dynamical systems theory, equilibrium analysis of the system showed that the Disease Free equilibrium is stable if:

$$\alpha N < (d_1+\mu+r_1)$$
$$\beta < \gamma$$

That is fowl pox will not invade the poultry if the rate at which the susceptible birds (β) are introduced into the poultry is greater than the rate at which the susceptible birds become exposed to infection (γ). It was also established that $R_0<1$ if $S_0>S_c$ where $S_c = d_1+\mu+r_1/\alpha$; $S_0$ is the initial susceptible birds and $S_c$ is the critical susceptible pools. The endemic equilibrium is asymptotically stable if:

$$\beta-\gamma < \frac{\alpha(d_1+\mu+r_1)}{k}$$

The second model is the extension of the first model that incorporates the interaction between the susceptible birds and the infected mosquitoes. From the analysis of the model, it was discovered that the disease free state is stable if:

$$r_2 < \varphi, \alpha S^* < (d_1 + r_1), \frac{\delta_1}{2} > e$$

That is, it is stable if the rate at which the infected birds recover and the rate at which mosquitoes die are high. Also, if the growth rate of mosquito is less than the death rate of mosquito.

## CONCLUSION

In this study, reseirachers shall adopt a simple SEIR and SEIV Model to formulate the mathematical model of the transmission dynamics of fowl pox (avian pox) and population growth of birds in poultry.

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