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An Extension of Rayleigh Distribution

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Abstract: In this study, we propose a model which is an extension of Rayleigh distribution. This model includes Rayleigh distribution, exponential and some new models as a special case.

Key words: Probability density function, random variable, statistical model, Rayleigh distribution, special case, India

INTRODUCTION

In the literature (Johnson and Kotz, 1970; Hogg and Craig, 1970; Bain and Engelhardt, 1991; Lawless, 2003), there exists a large number of continuous type distributions.

Since, a distribution stands on some stipulated assumptions and any variation in these assumptions leads to a different situation. It is natural to study a modification and a revision of a distribution depending upon the nature of change in the situation or violation of assumption which gives rise to a new class of distributions. In this study, we propose a new model and as a special case obtain Rayleigh distribution from it. A random variable X of continuous type is said to have a Rayleigh distribution if its probability density function with parameter θ (>0) is of the form:

$$f(x) = \frac{2x}{\theta^2}\exp\left[-\left\{\frac{x}{\theta}\right\}^2\right];\ x > 0 \quad (1)$$
$$= 0 \text{ elsewhere}$$

The noncentral Chisquare distribution regarded as a generalized Rayleigh distribution or Rayleigh rice or Rice distribution (Miller *et al*., 1958; Park, 1961) can be used in Mathematical Physics and especially in communication theory (Helstroem, 1960).

MATERIALS AND METHODS

Proposed statistical model: A random variable X of continuous type has a probability density function given by:

$$f(x) = \frac{\alpha\beta}{\theta}\left(\frac{\alpha x - \lambda}{\theta}\right)^{\beta-1}\exp\left[-\left\{\frac{\alpha x-\lambda}{\theta}\right\}^{\beta}\right];\ x > 0$$
$$\frac{\lambda}{\alpha} < x < \infty,\ \beta, \theta > 0 \text{ and } \alpha \neq 0$$
$$= 0 \text{ elsewhere} \quad (2)$$

where, α, β, θ and λ the parameters of the Eq. 2.

Deduction of some well known distributions from the proposed model: In fact a number of distributions can be deduced from the proposed Eq. 2 by choosing the parameters suitably. Here, we mention only a few. Taking $\lambda = 0$ and $\alpha = 1$ in Eq. 2, we have:

$$f(x) = \frac{\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1}\exp\left[-\left\{\frac{x}{\theta}\right\}^{\beta}\right];\ x > 0$$
$$= 0 \text{ elsewhere}$$

Which is a pdf of Weibull distribution with two parameters β (>0) and θ (>0). Taking $\lambda = 0$, $\beta = 1$ and $\alpha = 1$ in Eq. 2, we have:

$$f(x) = \frac{1}{\theta}\exp\left[-\left\{\frac{x}{\theta}\right\}\right];\ x > 0$$
$$= 0 \text{ elsewhere}$$

Which is a pdf of well-known Exponential distribution with parameter θ>0. Taking $\alpha = 1$ in Eq. 2, we have:

$$f(x) = \frac{\beta}{\theta}\left(\frac{x-\lambda}{\theta}\right)^{\beta-1}\exp\left[-\left\{\frac{x-\lambda}{\theta}\right\}^{\beta}\right];\ x > 0, \lambda < x$$
$$= 0 \text{ elsewhere}$$

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Which is a pdf of a continuous random variable X with 3 parameters β (>0), λ (>0) and θ (>0). Taking β = 1 and α = 1 in Eq. 2, we have:

$$f(x) = \frac{1}{\theta}\exp\left[-\left\{\frac{x-\lambda}{\theta}\right\}\right]; \ x > 0$$
$$= 0 \text{ elsewhere}$$

Which is a pdf of well-known two parameter Exponential distribution with parameters $\theta>0$ and $\lambda>0$ ($\lambda<x$). Taking $\beta = 2$, $\alpha = 1$ and $\lambda = 0$ in Eq. 2, we have:

$$f(x) = \frac{2x}{\theta^2}\exp\left[-\left\{\frac{x}{\theta}\right\}^2\right]; \ x > 0$$
$$= 0 \text{ elsewhere}$$

Which is a pdf of well-known Rayleigh distribution with parameter $\theta>0$.

RESULTS AND DISCUSSION

We have proposed a new model which is an extension of Rayleigh distribution. We have also derived some well known distributions fron the proposed model. Rayleigh distribution is an important distribution which is frequently used to model wave heights in oceanography and in communication theory to describe hourly median and instantaneous peak power of received radio signals. It has been used to model the frequency of different wind speeds over a year at wind turbine sites.

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