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Star Coloring on Double Star Graph Families

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Abstract: The purpose of this study is to find the star chromatic number for the central graph, middle graph, total graph and line graph of double star graph $K_{1,n,n}$ denoted by C ($K_{1,n,n}$), M ($K_{1,n,n}$), T ($K_{1,n,n}$) and L ($K_{1,n,n}$), respectively. We discuss the relationship between star chromatic number with other type of chromatic number such as equitable chromatic number.

Key words: Central graph, middle graph, total graph, line graph, equitable coloring, star coloring

INTRODUCTION

For a given graph G = (V, E), we do an operation on G by subdividing each edge exactly once and joining all the non adjacent vertices of G. The graph obtained by this process is called central graph (Vernold *et al*., 2009a, b) of G denoted by C (G). Let G be a graph with vertex set V (G) and edge set E (G). The middle graph (Michalak, 1981) of G denoted by M (G) is defined as follows. The vertex set of M (G) is V (G)∪E (G). About 2 vertices x, y in the vertex set of M (G) are adjacent in M (G) in case one the following holds:

- x, y are in E (G) and x, y are adjacent in G
- x is in V (G), y is in E (G) and x, y are incident in G

Let G be a graph with vertex set V (G) and edge set E (G). The total graph (Michalak, 1981; Harary, 1969) of G denoted by T (G) is defined as follows. The vertex set of T (G) is V (G)∪E (G). About 2 vertices x, y in the vertex set of T (G) are adjacent in T (G) in case one the following holds:

- x, y are in V (G) and x is adjacent to y in G
- x, y are in E (G) and x, y are adjacent in G
- x is in V (G), y is in E (G) and x, y are incident in G

The line graph (Harary, 1969) of G denoted by L (G) is the graph with vertices are the edges of G with 2 vertices of L (G) adjacent whenever, the corresponding edges of G are adjacent. Double star K ($_{1,n,n}$) is a tree obtained from the star $K_{1,n}$ by adding a new pendant edge of the existing n pendant vertices. It has 2n+1 vertices and 2n edges. The notion of star chromatic number was introduced by Grunbaum (1973). A star coloring (Albertson *et al*., 2004; Grunbaum, 1973; Fertin *et al*., 2004) of a graph G is a proper vertex coloring in which every path on four vertices uses at least 3 distinct colors. Equivalently, in a star coloring, the induced subgraphs formed by the vertices of any 2 colors has connected components that are star graphs. The star chromatic number X_s (G) of G is the least number of colors needed to star color G. The notion of equitable coloring was introduced by Meyer (1973).

If the set of vertices of a graph G can be partitioned into k classes $V_1, V_2, ..., V_k$ such that each V_i is an independent set and the condition $||V_i| - |V_j|| \leq 1$ holds for every pair (i, j) then G is said to be equitably k-colorable. The smallest integer k for which G is equitable k-colorable is known as the equitable chromatic number (Meyer, 1973) of G and denoted by X = (G).

STAR COLORING ON CENTRAL GRAPH OF DOUBLE STAR GRAPH

Algorithm 1: Input; the number n of K ($_{1,n,n}$). Output; assigning star coloring for the vertices in C ($K_{1,n,n}$).

```
begin:
for i = 1 to n
{
V1 = {ui};
```

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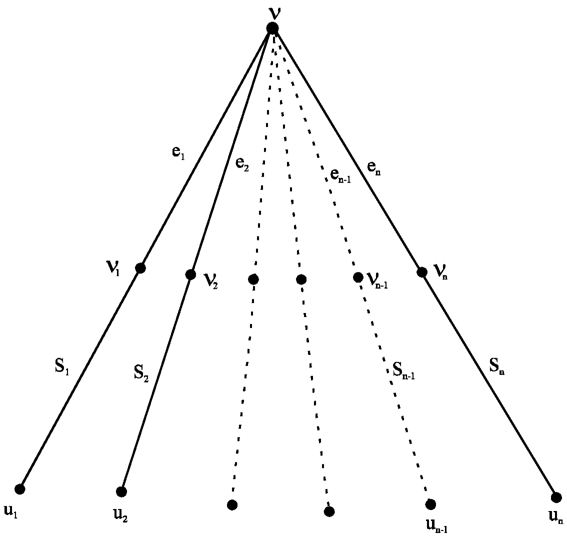

Fig. 1: Double star graph K $(_{1,n,n})$

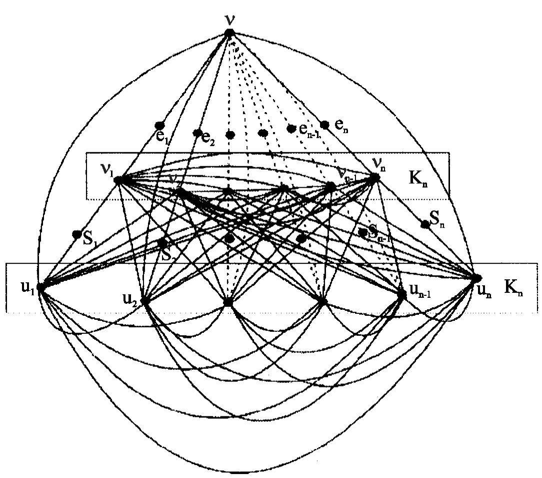

Fig. 2: Central graph of double star graph C $(K_{1,n})$

```
C (ui) = i;
V2 = {si};
C (si) = n+1;
}
V3 = {v};
C (v) = n+1;
for i = 1 to n
{
V4 = {vi};
C (vi) = n+1+i;
}
for i = 3 to n
V5 = {ei};
C (ei) = i-2;
}
C (e1) = n-1;
C (e2) = n;
V = V1 ∪ V2 ∪ V3 ∪ V4 ∪ V5;
end
```

Theorem 1: For any double star graph K $(_{1,n,n})$ (Fig. 1 and 2) the star chromatic number is:

$$X_s\,[C\,(K_{1,n,n})] = 2n+1$$

Proof: Let v, $v_1, v_2, ..., v_n$ and $u_1, u_2, .., u_n$ be the vertices in K $(_{1,n,n})$, the vertex v be adjacent to v_i $(1 \le i \le n)$. The vertices v_i $(1 \le i \le n)$ be adjacent to u_i $(1 \le i \le n)$. Let the edge vv_i and uu_i $(1 \le i \le n)$ be subdivided by the vertices e_i $(1 \le i \le n)$ and s_i $(1 \le i \le n)$ in C $(K_{1,n,n})$. Clearly V [C $(K_{1,n,n})$] = $\{v\} \cup \{v_i/1 \le i \le n\} \cup \{u_i/1 \le i \le n\} \cup \{e_i/1 \le i \le n\} \cup \{s_i/1 \le i \le n\}$. The vertices v_i $(1 \le i \le n)$ induce a clique of order n (say K_n) and the vertices v, u_i $(1 \le i \le n)$ induce a clique of order n+1 (say K_{n+1}) in C $(K_{1,n,n})$, respectively also each v_i $(1 \le i \le n)$ adjacent to u_j $(1 \le j \le n)$, $\forall i \ne j$. Thus by proper star coloring, we have, X_s [C $(K_{1,n,n})$] $\ge$ 2n+1.

Now consider the vertex set V [C $(K_{1,n,n})$] and the color classes $C_1 = \{c_1, c_2, c_3, ..., c_n\}$ and $C_2 = \{c_1, c_2, c_3, ..., c_n, c_{n+1}\}$, assign the proper star coloring to C $(K_{1,n,n})$ by Algorithm 1. Therefore, X_s[C $(K_{1,n,n})$] $\le$ 2n+1. Hence, X_s[C $(K_{1,n,n})$] = 2n+1.

STAR COLORING ON MIDDLE AND TOTAL GRAPH OF DOUBLE STAR GRAPH

Algorithm 2: Input; the number n of K $(_{1,n,n})$. Output; assigning star coloring for vertices in M $(K_{1,n,n})$ and T $(K_{1,n,n})$.

```
begin:
for i = 1 to n
{
V1 = {ei};
C (ei) = i;
}
V2 = {v};
C (v) = n+1;
for i = 2 to n
{
V3 = {vi};
C (vi) = i-1;
}
C (v1) = n;
for i = 3 to n
{
V4 = {si};
C (si) = i-2;
}
C (s1) = n-1;
C (s1) = n;
for i = 1 to n
{
V5 = {ui};
C (ui) = n+1;
}
V = V1∪V2∪V3∪V4∪V5;
end
```

Theorem 2: For any double star graph K $(_{1,n,n})$ (Fig. 3), the star chromatic number is:

$$X_s\,[M\,(K_{1,n,n})] = n + 1$$

Proof: Let V $(K_{1,n,n}) = \{v\}\cup\{v_i/1\le i\le n\}\cup\{u_i/1\le i\le n\}$. By definition of middle graph, each edge vv_i and v_iu_i $(1\le i\le n)$ in K $(_{1,n,n})$ are subdivided by the vertices u_i and s_i in M $(K_{1,n,n})$, i.e., V [M $(K_{1,n,n})$] = $\{v\}\cup\{v_i/1\le i\le n\}\cup\{u_i/1\le i\le n\}\cup\{e_i/1\le i\le n\}\cup\{s_i/1\le i\le n\}$, the vertices v, e_1, e_2, ..., e_n induce a clique of order n+1(say K_{n+1}) in M $(K_{1,n,n})$. Therefore by proper star coloring, X_s[M $(K_{1,n,n})$]$\ge$n + 1. Now consider the vertex set V [M $(K_{1,n,n})$] and colour class C = $\{c_1, c_2, c_3,.., c_n, c_{n+1}\}$, assign the proper star coloring to M $(K_{1,n,n})$ by Algorithm 2. Thus, we have X_s [M $(K_{1,n,n})$]$\le$n+1. Hence, X_s [M $(K_{1,n,n})$] = n+1.

Theorem 3: For any double star graph K $(_{1,n,n})$, the star chromatic number is X_s [T $(K_{1,n,n})$] = n+1.

Proof: Let V $(K_{1,n,n}) = \{v, v_1, v_2, .., v_n\}\cup\{u_1, u_2, .., u_n\}$ and E $(K_{1,n,n}) = \{e_1, e_2, ..., e_n\}\cup\{s_1, s_2, s_3,..., s_n\}$. By the definition of total graph, we have V [T($K_{1,n,n}$)] = $\{v\}\cup\{v_i/1\le i\le n\}\cup\{u_i/1\le i\le n\}\cup\{e_i/1\le i\le n\}\cup\{s_i/1\le i\le n\}$ in which the vertices v, e_1, e_2,...,e_n induce a clique of order n+1 (say K_{n+1}).

Therefore, by proper star coloring, X_s [T $(K_{1,n,n})$] $\ge$n+1. Now consider the vertex set V [T $(K_{1,n,n})$] and colour class C = $\{c_1, c_2, c_3, ...c_n, c_{n+1}\}$, assign the proper coloring to T $(K_{1,n,n})$ by: Algorithm 2 (Fig. 4). Thus, we have X_s [$(K_{1,n,n})$]$\le$n+1. Hence, X_s [T $(K_{1,n,n})$] = n+1.

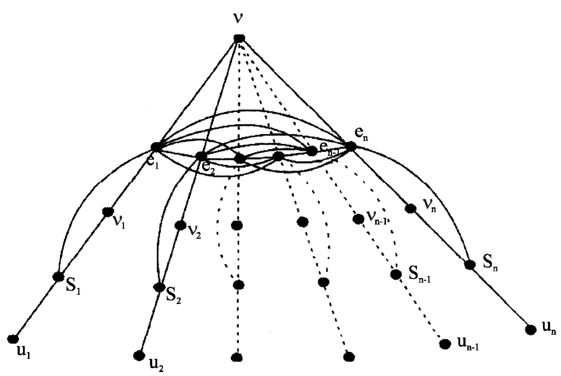

Fig. 3: Middle graph of double star graph M $(K_{1,n,n})$

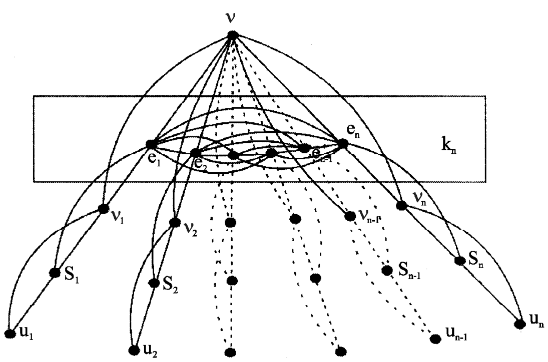

Fig. 4: Total graph of double star graph T $(K_{1,n,n})$

STAR COLORING ON LINE GRAPH OF DOUBLE STAR GRAPH

Algorithm 3: Input; the number n of K $(_{1,n,n})$. Output; assigning star coloring for vertices in L $(K_{1,n,n})$.

```
begin:
for i = 1 to n
{
V1 = {ei};
C (ei) = i;
}
for i = 2 to n
V2 = {si};
C (si) = i - 1;
}
C (s1) = n;
V = V1∪V2;
end
```

Theorem 4: For any double star graph K $(_{1,n,n})$, the star chromatic number is:

$$X_s [L (K_{1,n,n})] = n$$

Proof: Let V $(K_{1,n,n}) = \{v\}\cup\{v_i/1\le i\le n\}\cup\{u_i/1\le i\le n\}$ and E $(K_{1,n,n}) = \{e_1, e_2, ...e_n\}\cup\{s_1, s_2, s_3, ..., s_n\}$. By the definition of line graph, each edge of K $(_{1,n,n})$ taken to be as vertex in L $(K_{1,n,n})$.

The vertices e_1, e_2, ..., e_n induce a clique of order n in L $(K_{1,n,n})$, i.e., V [L $(K_{1,n,n})$] = $\{e_i/1\le i\le n\}\cup\{s_i/1\le I\le n\}$. Therefore by proper star coloring, X_s [L $(K_{1,n,n})$]$\ge$n. Now consider the vertex set V [L $(K_{1,n,n})$] and a color class C = $\{c_1, c_2, c_3,...c_n\}$, assign the proper star coloring to L $(K_{1,n,n})$ by Algorithm 3 (Fig. 5). Thus we have, X_s [L $(K_{1,n,n})$]$\le$n. Hence:

$$X_s [L (K_{1,n,n})] = n$$

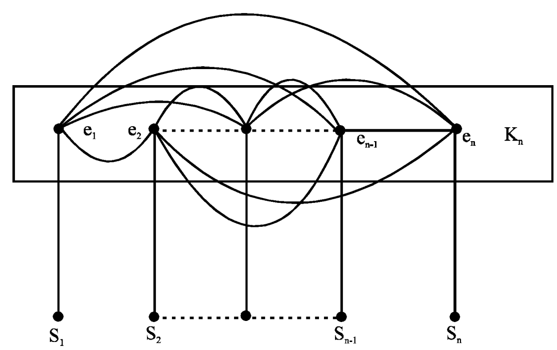

Fig. 5: Line graph of double star graph L $(K_{1,n,n})$

MAIN THEOREM

Theorem 5: For any double star graph K $(_{1,n,n})$, the equitable chromatic number, X = (C $(K_{1,n,n})$) = X = (M $(K_{1,n,n})$) =X = (T $(K_{1,n,n})$) = n+1.

Now, we characterize the graph for which the star chromatic number and equitable chromatic number are the same. The proof of the main theorem follows from theorem 2, 3, 5 (Vernold and Venkatachalam, 2010).

Theorem 6: For any double star graph K $(_{1,n,n})$, the star chromatic number and equitable chromatic number, X = (C $(K_{1,n,n})$) = X = (M $(K_{1,n,n})$) = X= (T $(K_{1,n,n})$) = X_s [M $(K_{1,n,n})$] = X_s [T $[K_{1,n,n})$] = n+1.

CONCLUSION

In this present study, we have proved for the star chromatic number and the equitable chromatic number are equal for some double star graph families. As a motivation from this study can be extended by classifying the different families of graphs for which these two chromatic numbers are equal.

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