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An Accurate Continuous Hybrid Block Method for the Direct Solution of General Second Initial Value Problem

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Abstract: In this research, a new hybrid block method of order 5 for solving second order Initial Value Problems (IVPs) is determined. One step with 3 off step points is used to generate the coefficients of the new method. This method approximates the solutions of the second order IVPS at x_n+1/6, x_n+2/6, x_n+3/6, x_{n+1}, concurrently by using interpolation and collocation approach with fixed step size. The developed method don't need to reduce the IVPs to its equivalent system of first order IVPs. The numerical results obtained show that the new method outperforms the existing methods in terms of error.

Key words: Hybrid method, second order differential equation, power series, three off step points, block, IVPs

INTRODUCTION

In this study attention is being focused on finding a direct approximate solution to the second order initial value problem of the form:

$$y'' = f(x, y, y'), y(a) = \lambda_0, y'(a) = \lambda_1, x \in [a, b] \quad (1)$$

In some cases, most of the analytical solution of these problems is not available or more than one solution exists. Hence, the use of numerical methods is advocated. These methods have been introduced by many researchers (Abdelrahim and Omar, 2015; Abdelrahim *et al.*, 2016; Omar and Abdelrahim, 2016) and others. In order to proffer solution to the zero stability barrier in numerical multistep method which states that the highest order for zero stability linear multistep method is k+2 when steplength k is even and k+1 when k is odd (Lambert, 1973), the introduction of a hybrid method which includes using the off step points is considered.

Therefore, in bringing improvement to numerical methods, this study attempts to develop one step self starting hybrid block method with three off step points for solving second initial value problems directly.

Derivation of the method: In this part, the derivation of one step hybrid block method with three off-step points $x_{n+1/6}$, $x_{n+2/6}$ and $x_{n+3/6}$ for solving Eq. 1 is examined. Taking the power series of the form:

$$y(x) = \sum_{i=0}^{v+m-1} a_i \left(\frac{x-x_n}{h}\right)^i, x \in [x_n, x_{n+1}] \quad (2)$$

to be an approximate solution to Eq. 1 where n = 0, 1, 2, ..., N-1, h = x_n-x_{n-1} is fixed step size of partition of interval (a, b) which is given by a = x_0<x_1<, ..., <x_{N-1}<x_N = b, v indicates to the number of interpolation points and m represents the number of collocation points. Equation 2 is differentiated twice to give:

$$y''(x) = f(x, y, y') = \sum_{i=2}^{v+m-1} \frac{i(i-1)}{h^2} a_i \left(\frac{x-x_n}{h}\right)^{i-2} \quad (3)$$

Where:
v = 2
m = 5

Interpolation is made on Eq. 2 at the two off step points before the last point while Eq. 3 is collocated at all points in the selected interval. This gives the following equations which can be reformed in matrix form as:

$$\begin{pmatrix} 1 & \frac{1}{3} & \frac{1}{9} & \frac{1}{27} & \frac{1}{81} & \frac{1}{243} & \frac{1}{729} \\ 1 & \frac{1}{2} & \frac{1}{4} & \frac{1}{8} & \frac{1}{16} & \frac{1}{32} & \frac{1}{64} \\ 0 & 0 & \frac{2}{h^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{h^2} & \frac{1}{h^2} & \frac{1}{(3h^2)} & \frac{5}{(54h^2)} & \frac{5}{(216h^2)} \\ 0 & 0 & \frac{2}{h^2} & \frac{2}{h^2} & \frac{4}{(3h^2)} & \frac{20}{(27h^2)} & \frac{10}{27h^2} \\ 0 & 0 & \frac{2}{h^2} & \frac{3}{h^2} & \frac{3}{h^2} & \frac{5}{(2h^2)} & \frac{15}{8h^2} \\ 0 & 0 & \frac{2}{h^2} & \frac{6}{h^2} & \frac{12}{h^2} & \frac{20}{h^2} & \frac{30}{h^2} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{pmatrix} = \begin{pmatrix} y_{n+\frac{2}{6}} \\ y_{n+\frac{3}{6}} \\ f_n \\ f_{n+\frac{1}{6}} \\ f_{n+\frac{2}{6}} \\ f_{n+\frac{3}{6}} \\ f_{n+1} \end{pmatrix} \quad (4)$$

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The MATLAB Software code is employed in finding the coefficients of ai for i = 0 (1) 6. Then, these values of ai are substituted into Eq. 2 and this yields a continuous implicit scheme:

$$y\,(x) = \sum_{i=2}^{3}\alpha_{\frac{1}{5}} y_{n+\frac{i}{6}} + \sum_{i=0}^{1}\beta_i f_{n+i} + \sum_{i=1}^{3}\beta_{\frac{i}{6}} f_{n+\frac{i}{6}} \qquad (5)$$

The first derivative of Eq. 5 is:

$$y'(x) = \sum_{i=2}^{3}\frac{\partial}{\partial x}\alpha_{\frac{i}{6}}(x) y_{n+\frac{i}{6}} + \sum_{i=0}^{1}\frac{\partial}{\partial x}\beta_i(x) f_{n+i} + \sum_{i=1}^{3}\frac{\partial}{\partial x}\beta_{\frac{i}{6}}(x) f_{n+\frac{i}{6}} \qquad (6)$$

Where:

$$\alpha_{\frac{2}{6}} = 3 - \frac{6\,(x-x_n)}{h}$$

$$\alpha_{\frac{3}{6}} = \frac{6\,(x-x_n)}{h} - 2$$

$$\beta_0 = \frac{(x-x_n)^2}{2} - \frac{2(x-x_n)^3}{h} + \frac{47(x-x_n)^4}{12h^2} - \frac{18(x-x_n)^5}{5h^3} + \frac{6(x-x_n)^6}{5h^4} - \frac{9h(x-x_n)}{160} + \frac{h^2}{480}$$

$$\beta_{\frac{1}{6}} = +\frac{18(x-x_n)^3}{5h} - \frac{54(x-x_n)^4}{5h^2} + \frac{297(x-x_n)^5}{25h^3} - \frac{108(x-x_n)^6}{25h^4} - \frac{773h(x-x_n)}{3600} + \frac{103h^2}{3600}$$

$$\beta_{\frac{2}{6}} = \frac{81(x-x_n)^4}{8h^2} - \frac{9(x-x_n)^3}{4h^2} - \frac{27(x-x_n)^5}{2h^3} + \frac{27(x-x_n)^6}{5h^4} - \frac{71h(x-x_n)}{576} + \frac{137h^2}{2880}$$

$$\beta_{\frac{3}{6}} = \frac{2(x-x_n)^3}{3h} - \frac{10(x-x_n)^4}{3h^2} + \frac{27(x-x_n)^5}{5h^3} - \frac{12(x-x_n)^6}{5h^4} - \frac{49h(x-x_n)}{2160} + \frac{11h^2}{2160}$$

$$\beta_1 = \frac{11(x-x_n)^4}{120h^2} - \frac{(x-x_n)^3}{60h} - \frac{9(x-x_n)^5}{50h^3} + \frac{3(x-x_n)^6}{25h^4} + \frac{11h(x-x_n)}{43200} - \frac{h^2}{43200}$$

Evaluating Eq. 5 at the non-interpolating point, i.e, x_n, $x_{n+1/6}$, x_{n+1} and Eq. 6 at all points produce the discrete schemes and its derivative. The combination of the discrete schemes and its derivative at x_n gives Eq. 7 in a matrix form as:

$$A^{[3]_2} Y_m^{[3]_2} = B^{[3]_2} R_1^{[3]_2} + \left[D^{[3]_2} R_2^{[3]_2} + E^{[3]_2} R_3^{[3]_2} \right] \qquad (7)$$

$$A^{[3]_2} = \begin{pmatrix} 0 & -3 & 2 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & -4 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{6}{h} & \frac{-6}{h} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{6}{h} & \frac{-6}{h} & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{6}{h} & \frac{-6}{h} & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{6}{h} & \frac{-6}{h} & 0 & 0 & 0 & 1 & 0 \\ 0 & \frac{6}{h} & \frac{-6}{h} & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad Y_m^{[3]_2} = \begin{pmatrix} y_{n+\frac{1}{6}} \\ y_{n+\frac{2}{6}} \\ y_{n+\frac{3}{6}} \\ y_{n+1} \end{pmatrix}$$

$$B^{[3]_2} = \begin{pmatrix} -1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad R_1^{[3]_2} = \begin{pmatrix} y_n \\ y'_n \end{pmatrix}, \quad D^{[3]_2} = \begin{pmatrix} \frac{h^2}{480} \\ \frac{-h^2}{12960} \\ \frac{-(3h^2)}{80} \\ \frac{-(9h)}{160} \\ \frac{(43h)}{12960} \\ \frac{-(5h)}{2592} \\ \frac{h}{480} \\ \frac{-(91h)}{480} \end{pmatrix}, \quad R_2^{[3]_2} = \left(f_n \right)$$

$$E^{[3]_2} = \begin{pmatrix} \frac{(103h^2)}{3600} & \frac{(137h^2)}{3880} & \frac{(11h^2)}{2160} & \frac{-h^2)}{43200} \\ \frac{(7h^2)}{2700} & \frac{(197h^2)}{8640} & \frac{h^2}{405} & \frac{-h^2}{129600} \\ \frac{(313h^2)}{1800} & \frac{-(433h^2)}{1440} & \frac{(341h^2)}{1080} & \frac{(329h^2)}{21600} \\ \frac{-(773h)}{3600} & \frac{-(71h)}{576} & \frac{-(49h)}{2160} & \frac{(11h)}{43200} \\ \frac{-(13h)}{180} & \frac{-(493h)}{2880} & \frac{-(4h)}{405} & \frac{-h}{25920} \\ \frac{(43h)}{3600} & \frac{-(211h)}{2880} & \frac{-(131h)}{6480} & \frac{(17h)}{129600} \\ \frac{-(11h)}{900} & \frac{(131h)}{2880} & \frac{(13h)}{270} & \frac{-(7h)}{43200} \\ \frac{(623h)}{720} & \frac{-(4243h)}{2880} & \frac{(268h)}{2160} & \frac{(1183h)}{8640} \end{pmatrix} \text{ and } R_3^{[3]_2} = \begin{pmatrix} f_{n+\frac{1}{6}} \\ f_{n+\frac{2}{6}} \\ f_{n+\frac{3}{6}} \\ f_{n+1} \end{pmatrix}$$

Multiplying Eq. 7 by the inverse of $A^{[3]_2}$ gives the following one step with generalized three off-step points block method:

$$I^{[2]_3} Y_m^{[3]_2} = \bar{B}^{[3]_2} R_1^{[3]_2} + \bar{D}^{[3]_2} R_2^{[3]_2} + \bar{E}^{[3]_2} R_3^{[3]_2} \tag{8}$$

Where:

$$I^{[2]_3} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \bar{B}_1^{[3]_2} = \begin{pmatrix} 1 & \frac{h}{6} \\ 1 & \frac{h}{3} \\ 1 & \frac{h}{2} \\ 1 & h \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$$

$$\bar{D}^{[3]_2} = \begin{pmatrix} \frac{187h^2}{25920} \\ \frac{h^2}{60} \\ \frac{5h^2}{192} \\ \frac{h^2}{60} \\ \frac{193h}{3240} \\ \frac{22h}{405} \\ \frac{7h}{120} \\ -\frac{2h}{15} \end{pmatrix}, \bar{E}^{[3]_2} = \begin{pmatrix} \frac{211}{21600} & -\frac{73}{17280} & \frac{}{864} & -\frac{7}{259200} \\ \frac{29}{675} & -\frac{7}{1080} & \frac{}{405} & \frac{-}{16200} \\ \frac{63}{800} & \frac{9}{640} & \frac{-}{160} & \frac{-}{9600} \\ \frac{9}{25} & -\frac{9}{40} & \frac{}{3} & \frac{3}{200} \\ \frac{57}{400h} & -\frac{23}{480h} & \frac{83}{6480h} & -\frac{19}{64800h} \\ \frac{17}{75h} & \frac{1}{20h} & \frac{1}{405h} & \frac{-1}{8100h} \\ \frac{81}{400h} & \frac{27}{160h} & \frac{17}{240h} & \frac{-1}{2400h} \\ \frac{27}{25h} & -\frac{27}{20h} & \frac{19}{15h} & \frac{41}{300h} \end{pmatrix}$$

Equation 8 can be written as:

$$y_{n+\frac{1}{6}} = y_n + \frac{y_n' h}{6} + \frac{187h^2}{25920} f_n - \frac{7h^2}{259200} f_{n+1} + \frac{211h^2}{21600} f_{n+\frac{1}{6}} - \frac{73h^2}{17280} f_{n+\frac{2}{6}} + \frac{h^2}{864} f_{n+\frac{3}{6}}$$

$$y_{n+\frac{2}{6}} = y_n + \frac{h}{3} y_n' + \frac{h^2}{60} f_n - \frac{h^2}{16200} f_{n+1} + \frac{29h^2}{675} f_{n+\frac{1}{6}} - \frac{7h^2}{1080} f_{n+\frac{2}{6}} + \frac{h^2}{405} f_{n+\frac{3}{6}}$$

$$y_{n+\frac{2}{6}} = y_n + \frac{h}{2} y_n' + \frac{5h^2}{192} f_n - \frac{h^2}{9600} f_{n+1} + \frac{63h^2}{800} f_{n+\frac{1}{6}} - \frac{9h^2}{640} f_{n+\frac{2}{6}} + \frac{h^2}{160} f_{n+\frac{3}{6}}$$

$$y_{n+1} = y_n + hy_n' + \frac{h^2}{60} f_n + \frac{3h^2}{200} f_{n+1} + \frac{9h^2}{25} f_{n+\frac{1}{6}} - \frac{9h^2}{40} f_{n+\frac{2}{6}} + \frac{h^2}{3} f_{n+\frac{3}{6}}$$

$$y'_{n+\frac{1}{6}} = y_n' + \frac{193h}{3240} f_n - \frac{19h}{64800} f_{n+1} - \frac{57h}{400} f_{n+\frac{1}{6}} + \frac{23h}{480} f_{n+\frac{2}{6}} + \frac{83h}{6480} f_{n+\frac{3}{6}}$$

$$y'_{n+\frac{2}{6}} = y_n' + \frac{22h}{405} f_n - \frac{h}{8100} f_{n+1} - \frac{17h}{75} f_{n+\frac{1}{6}} + \frac{h}{20} f_{n+\frac{2}{6}} + \frac{h}{405} f_{n+\frac{3}{6}}$$

$$y'_{n+\frac{3}{6}} = y_n' + \frac{7h}{120} f_n - \frac{h}{2400} f_{n+1} - \frac{81h}{400} f_{n+\frac{1}{6}} + \frac{27h}{160} f_{n+\frac{2}{6}} + \frac{17h}{240} f_{n+\frac{3}{6}}$$

$$y'_{n+1} = y_n' - \frac{2h}{15} f_n - \frac{41h}{300} f_{n+1} + \frac{27h}{25} f_{n+\frac{1}{6}} - \frac{27h}{20} f_{n+\frac{2}{6}} + \frac{19h}{15} f_{n+\frac{3}{6}}$$

MATERIALS AND METHODS

Properties of the method

Order of method: The linear difference operator L associated with Eq. 8 is defined as:

$$L[y(x);h] = Y_m^{[3]_2} - \bar{B}^{[3]_2} R_1^{[3]_2} - h^2 \left[\bar{D}^{[3]_2} R_2^{[3]_2} + \bar{E}^{[3]_2} R_3^{[3]_2} \right] \quad (9)$$

where y(x) is an arbitrary test function continuously differentiable on (a, b). The elements of Y_m and $R_3^{[3]_2}$ are expanded in Taylor series, respectively and their terms are collected in powers of h to give:

$$\begin{bmatrix}
\sum_{j=0}^{\infty} \frac{\left(\frac{1}{6}\right)^j h^j}{j!} y_n^j - y_n - \frac{1h}{6} y_n' - \frac{187h^2}{25920} y_n'' - \frac{211}{21600} \sum_{j=0}^{\infty} \frac{\left(\frac{1}{6}\right)^j h^{j+3}}{j!} y_n^{j+3} + \frac{73}{17280} \sum_{j=0}^{\infty} \frac{\left(\frac{2}{6}\right)^j h^{j+3}}{j!} y_n^{j+3} \\
-\frac{1}{864} \sum_{j=0}^{\infty} \frac{\left(\frac{3}{6}\right)^j h^{j+3}}{j!} y_n^{j+3} + \frac{7}{259200} \sum_{j=0}^{\infty} \frac{h^{j+3}}{j!} y_n^{j+3} \sum_{j=0}^{\infty} \frac{\left(\frac{2}{6}\right)^j h^j}{j!} y_n^j - y_n - \frac{2h}{6} y_n' - \frac{h^2}{60} y_n'' - \frac{29}{675} \\
\sum_{j=0}^{\infty} \frac{\left(\frac{1}{6}\right)^j h^{j+2}}{j!} y_n^{j+2} + \frac{7}{1080} \sum_{j=0}^{\infty} \frac{\left(\frac{2}{6}\right)^j h^{j+2}}{j!} y_n^{j+2} - \frac{1}{405} \sum_{j=0}^{\infty} \frac{\left(\frac{3}{6}\right)^j h^{j+2}}{j!} y_n^{j+2} + \frac{1}{16200} \sum_{j=0}^{\infty} \frac{h^{j+2}}{j!} y_n^{j+2} \\
\sum_{j=0}^{\infty} \frac{\left(\frac{3}{6}\right)^j h^j}{j!} y_n^j - y_n - \frac{(3h)}{6} y_n' - \frac{5h^2}{192} y_n'' - \frac{63}{800} \sum_{j=0}^{\infty} \frac{\left(\frac{1}{6}\right)^j h^{j+2}}{j!} y_n^{j+2} + \frac{9}{640} \sum_{j=0}^{\infty} \frac{\left(\frac{2}{6}\right)^j h^{j+2}}{j!} y_n^{j+2} - \frac{1}{160} \\
\sum_{j=0}^{\infty} \frac{\left(\frac{3}{6}\right)^j h^{j+2}}{j!} y_n^{j+2} + \frac{1}{9600} \sum_{j=0}^{\infty} \frac{h^{j+2}}{j!} y_n^{j+2} \sum_{j=0}^{\infty} \frac{h^j}{j!} y_n^j - y_n - hy_n' - \frac{h^2}{60} y_n'' - \frac{9}{25} \\
\sum_{j=0}^{\infty} \frac{\left(\frac{1}{6}\right)^j h^{j+2}}{j!} y_n^{j+2} + \frac{9}{40} \sum_{j=0}^{\infty} \frac{\left(\frac{2}{6}\right)^j h^{j+2}}{j!} y_n^{j+2} - \frac{1}{3} \sum_{j=0}^{\infty} \frac{\left(\frac{3}{6}\right)^j h^{j+2}}{j!} y_n^{j+2} - \frac{3}{200} \sum_{j=0}^{\infty} \frac{h^{j+2}}{j!} y_n^{j+2}
\end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Comparing the coefficients of h^j and y^j gives the order of the method to be $(5, 5, 5, 5)^T$ with error constants:

$$\bar{C}_7 = \begin{bmatrix} 5.875776e^{-8} \\ 1.383536e^{-7} \\ 2.239032e^{-7} \\ -6.062610e^{-6} \end{bmatrix}$$

Zero stability: According to Lambert (1973), the method Eq. 8 is said to be zero s if no root of the first characteristic polynomial $\Pi(z)$ has modulus >1 and the multiplicity must equal one when the modulus of a root is one. For our method, it is illustrated as follows:

$$\prod(z) = \left| zI^{[2]_3} - \bar{B}^{[3]_2} \right| = \left| z\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right| = z^3 \left(z\text{-}1 \right)$$

Table 1: Comparison of the new method with (Jator and Li, 2009) for solving problem 1

x	Exact solution	Computed solution in our method	Error in our method, p = 5	Errors by Jator and Li (2009), p = 5
0.1	1.0500417292784914	1.0500417292786968	$2.0539e^{-13}$	$1.1839e^{-10}$
0.2	1.1003353477310756	1.1003353477319662	$8.9062e^{-13}$	$2.3750e^{-10}$
0.3	1.1511404359364668	1.1511404359379553	$1.4885e^{-12}$	$4.2485e^{-10}$
0.4	1.2027325540540821	1.2027325540548390	$7.5695e^{-13}$	$6.1629e^{-10}$
0.5	1.2554128118829952	1.2554128118792789	$3.7163e^{-12}$	$1.0233e^{-9}$
0.6	1.3095196042031119	1.3095196041865977	$1.6514e^{-11}$	$1.4483e^{-9}$
0.7	1.3654437542713964	1.3654437542249667	$4.6429e^{-11}$	$2.5549e^{-9}$
0.8	1.4236489301936019	1.4236489300828896	$1.1071e^{-10}$	$3.7221e^{-9}$
0.9	1.4847002785940520	1.4847002783497647	$2.4428e^{-10}$	$7.3287e^{-9}$
1.0	1.5493061443340550	1.5493061438132436	$5.2081e^{-10}$	$1.1337e^{-8}$

which implies z = 0, 0, 0, 1. Hence, our method is zero stable.

RESULTS AND DISCUSSION

Consistency

Definition (3.1): The hybrid block method Eq. 8 is said to be consistent if the order of the method is ≥ 1, i.e., $p \geq 1$. Obviously, the conditions stated in definition (3.1) are satisfied, hence, the developed method is consistent.

Convergence

Theorem (3.2); Henrici (1962): Consistency and zero stability are sufficient conditions for a linear multistep method to be convergent. Following (3.2), convergence is established by the zero stability and consistency of method.

Region of absolute stability: In order to find the region of absolutestability, the use of locus boundary technique is adopted. The method (8) is said to be absolutely s if for a given h all roots of the characteristic polynomial $\pi(z, h) = \rho(z)\text{-}h^2\sigma(z)$ satisfies $|z_t|<1$.

Substituting test equation $y = -\lambda^2 y$ in (8) where $\bar{h} = \lambda^2 h^2$ and $\lambda = df/dy$, replacing $r = \cos\theta - i \sin\theta$ and considering real part gives:

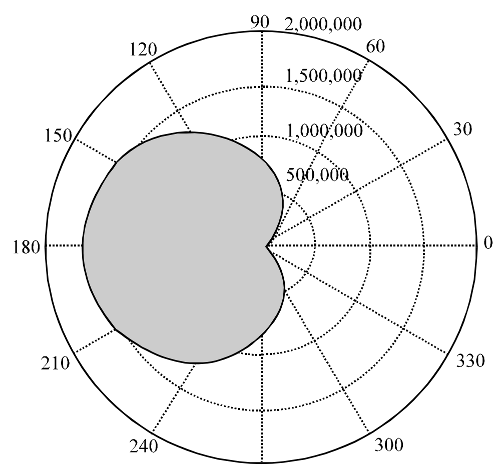

Fig. 1: Region stability of new method

$$\bar{h}\left(\theta, h\right) = \frac{\left(55987200\left(\cos\left(t\right)\text{-}1\right)\right)}{\left(\cos\left(t\right)\text{-}67\right)} \qquad (10)$$

Hence, the interval of stability gives $(0, 6.9134e^{8})$ as illustrated in Fig. 1 and 1 as a region in coordinate system.

Table 2: Comparison of the new method with (Badmus, 2014) for solving problem 2

x	Exact solution	Computed solution in our method	Error in our method, p = 5	Erorr in Badmus (2014), p = 8
1.003125	1.00307652585769610	1.0030765258576959	$2.220e^{-16}$	$1.645e^{-7}$
1.006250	1.0060575030835164	1.0060575030837120	$1.956e^{-13}$	$6.603e^{-7}$
1.009375	1.0089449950888376	1.0089449950894156	$5.779e^{-13}$	$4.414e^{-6}$
1.012500	1.0117410181679887	1.0117410181691262	$1.137e^{-12}$	$1.299e^{-5}$
1.018750	1.0170664942356729	1.0170664942384247	$2.751e^{-12}$	$1.637e^{-5}$
1.021875	1.0195997547562881	1.0195997547600779	$3.789e^{-12}$	$5.051e^{-5}$
1.028125	1.0244165187384029	1.0244165187446923	$6.289e^{-12}$	$2.829e^{-5}$
1.025000	1.0220491636294322	1.0220491636344038	$4.971e^{-12}$	$3.860e^{-5}$
1.028125	1.0244165187384029	1.0244165187446923	$6.289e^{-12}$	$7.490e^{-5}$
1.031250	1.0267035775008062	1.0267035775085420	$7.735e^{-12}$	$1.458e^{-5}$

Numerical examples: In finding the accuracy of our method, the following second order ODEs are tested. The new block method solved the same problems the existing methods solved in order to compare results in terms of error.

Problem 1: $y''-x(y')^2 = 0$, $y(0) = 1$, $y'(0) = 1/2$.

Exact solution: $y(x) = 1+1/2\ln(2+x/2-x)$ with $h = 0.05$.

Problem 2: $y''+(6/x)y'+(4/x^2)y = 0$, $y(1) = 1$, $y'(1) = 1$ (2).

Exact solution: $y(x) = 5/3x-2/3x^4$ with $h = 1/320$.

CONCLUSION

A single step hybrid block method for solving second order initial value problems directly has been developed. It has been proven that the developed method is consistent, zero stable, convergent with order five. The generated numerical results demonstrated in table above have revealed clearly that the developed method performs better than the previous methods in term of errors.

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