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New Types of Openness and Closed Graphs in Topological Space

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Abstract: ∝-unequivocally θ-continuity function and (∝, θ)-closed graphs was examined by Chae *et al*. The goal of this study is to research a few of new portrayal and properties of ∝-unequivocally θ-continuity and (∝, θ)-closed graphs. Besides we characterize new sort of a function called ∝, θ-open function which is more grounded than quasi ∝-open and ∝,-open and we acquire a few portrayals and properties for it.

Key words: Characterize, properties, portrayals, function, θ-continuity, ground

INTRODUCTION

The concept of ∝-open sets was introduced and investigated by Njastad (1965). Latterly, the concept of ∝-unequivocally θ-continuity function has studied by Chae *et al*. (1995). We know from Chae *et al*. (1995) that the type of ∝-unequivocally θ-continuity function is stronger than a unequivocally θ-continuity function (Noiri, 1980) and a unequivocally ∝-continuous function (Faro, 1987).

In this study we aim to investigate further properties and characterizations of ∝-unequivocally θ-continuity functions as well as θ-closed graph (Chae *et al*., 1995) and new types of function define called ∝, θ-open functions which is stronger than quasi ∝-open and hence, unequivocally ∝-open, some characterizations and properties are obtain for it.

Preliminaries: All through this study just X speaks to a topological space.

Definition 2.1: Let be an subset of a topological space (X, τ) then is called:

- Regular open if $A = (\bar{A})^{\circ}$ (Njastad, 1965)
- ∝-pen if $A \subseteq \overline{(A^{\circ})}^{\circ}$ (Levine, 1963)
- Semi-open if $A \subseteq \overline{A^{\circ}}$ (Levine, 1963)
- θ-open if for each x∊A, there exist an open set U in X such that $x \in U \subset \overline{U} \subset A$ (Velicko, 1968)
- θ-semi-open if for each x∊A, there exist an semi-open set U in X such that $x \in U \subset \overline{U} \subset A$ (Noiri and Kang, 1984)

The supplements of the sets said above are their individual closed sets.

Definition 2.2: The set $\propto\bar{A}$ = {p∊X: A∩H≠∅ for each ∝-open set H containing p}.

Definition 2.3: "A filter base Ψ is said to be θ-convergent (Velicko, 1968) (resp. ∝- convergent to a point x∊X if for each open (resp. ∝-open) set G containing x, there exist an F∊Ψ such that $F \subset \bar{F}$ (resp". F⊂G).

Definition 2.4; (Maheshwari *et al*., 1982): "A subset A of a topological space (X, τ) is called a feebly open set in X if there exist an open set U such that U⊂A⊂sCl(U) where is the semi-closure operator".

Remark 2.5; (Jankovic, 1985): A subset A of a topological space (X, τ) is called ∝-open if and only if it is feebly open. It is notable that for a space (X, τ), X can be retopologized by the family τ^{α} of all ∝-open sets of X (Maheshwari *et al*., 1982; Thakur, 1980) and furthermore the family τ^{θ} of all θ-open set of X (Velicko, 1968) that is τ^{θ} (called θ-topology) and τ^{α} (called an α-topology) are topologies on X and it is clearly that $\tau^{\theta} \subset \tau \subset \tau^{\alpha}$. The family of all ∝-open (resp. θ-open and feebly-open) arrangements of X is indicated by ∝0(X) (resp. θ0(X) and F0(X)).

Definition 2.5; (Noiri and Kang, 1984): A function f: X→Y is said to be unequivocally θ-continuous if for each x∊X and each open set H of Y containing f (x), there exist an open set G of X containing x such that $f(\bar{G}) \subset H$.

Definition 2.6; (Noiri and Kang, 1984): A function f: X→Y is said to be unequivocally θ–continuous if for each open set H of Y, f^{1} (H) is θ-open X in if and only if each closed set F of Y f^{1} (F), is θ-closed in X .

Definition 2.7; (Maheshwari *et al*., 1983): A function f: X→Y is said to be unequivocally ∝-continuous (resp. faintly continuous (Long and Herrington, 1982), completely ∝- irresolute and unequivocally ∝-irresolute (Faro, 1987) if for each open (resp. θ-open, ∝-open and ∝-open) set H of Y, f^{1} (H) is ∝-open (resp. open, regular open and open) in X.

Definition 2.8; (Noiri, 1973): A function f: X→Y is said to be semi-open (resp. ∝-open (Maheshwari *et al.*, 1983), quasi ∝-open (Thivagar, 1991; Abdul Jabbar, 2000), θs-open (Abdul-Jabbar, 2000) weakly θs-open and s**-open (Ali, 2003) function if the image of each open, (resp. open ∝-open, open, θ-open and semi-open) set of G of X, f (G) is semi-open (resp. ∝-open, open, θ-semi-open, θ-semi-open and open) in Y.

Definition 2.9; (Lee *et al.*, 1985): A function f: X→Y is said to be pre-feebly-open (resp. unequivocally ∝-open (Thivagar, 1991), ∝**-open (Ali, 2003) function if the image of each ∝-open set of G of X, f (G) is ∝-open in Y.

Definition 2.10; (Baker, 1986): Let A be an subset of a topological space (X, τ) then A is called θ-neighborhood of a point x in X if there exist an open set U such that $x \in U \subset \overline{U} \subset A$.

Definition 2.11; (Lee *et al.*, 1985): "A function f: X→Y is said to be" θ-open function if for each x∈X and each θ-neighborhood A of X, F (A) is -neighborhood f (x).

Definition 2.12; (Singal and Arya, 1969): A space X is said to be practically regular if for each regular closed set of X and each point x≠R, there exist disjoint open set U and V such that R⊂U and x∈V.

Definition 2.13; (Faro, 1987): A space X is said to be ∝-hausdorff if for any x, y∈X, x≠y, there exist ∝-open set G and H such that x∈G, y∈H and G∩ H = ϕ.

Definition 2.14: "A space X is said to be θ-compact (resp. ∝-compact (Jankovic *et al.*, 1988) if and only if every cover of X by θ-open (resp. ∝-open) sets has a finite subcover".

Definition 2.15; (Porter and Thomas, 1969): "A subset A of a topological space (X, τ) is said to be quasi H-closed relative to X if $\{E_i : i \in I\}$ each cover of A by open sets of X, there exist a finite subset I_0 of I such that $A \subset \cup\{\overline{E_i} : i \in I_0\}$".

Definition 2.16; (Porter and Thomas, 1969): "A space X is said to be quasi H-closed if X is quasi H-closed relative to X".

Definition 2.17; (Noiri, 1975): A function f: X→Y is said to be θ-closed (resp. s**-closed (Long and Herring, 1977), semi-closed (Dube *et al.*, 1998), θs-closed (Abdul-Jabbar, 2000), almost unequivocally θs-closed and unequivocally θs-closed graph if and only if for x∈X each and each y∈Y such that y≠f (x), there exist an open (resp. semi-open, semi-open, semi-open, semi-open and semi-open) U containing x in X and an open (resp. open, semi-open, open, open and open) set V containing f(x) in Y such that $(\overline{U} \times \overline{V}) \cap G(f) = \varphi$ {resp. $(U \times V) \cap G(f) = \varphi$, $(U \times V) \cap G(f) = \varphi$, $((\overline{U}) \times V) \cap G(f) = \varphi$, $(\overline{U} \times (\overline{V})^{\circ}) \cap G(f) = \varphi$ and $(\overline{U} \times (\overline{V})) \cap G(f) = \varphi$}.

MATERIALS AND METHODS

∝-Unequivocally θ-coherence

Definition 3.1: By Chae *et al.* (1995) "A function f: X→Y is said to be ∝-unequivocally θ-coherence if for each x∈X and each ∝-open set H of Y containing f (x), there exist an open set U of X containing x with the end goal that $f(\overline{U}) \subset H$".

Theorem 3.1: For a function f: (X, τ)→(Y, γ) the accompanying proclamations are proportionality:

- f is ∝-unequivocally θ-coherence
- f: (X, τ^{θ}→(Y, γ)is unequivocally ∝-irresolute

Theorem 3.2: In the event that a function f: X→Y∝-unequivocally θ-coherence at that point for each x∈X and each ∝-open set H of Y containing f (x), there exist θ-open set N of X containing x with the end goal that f (N)⊂H. The evidence of the above theorems are not hard and along these lines, they are precluded.

Theorem 3.3: For a function f: (X, τ)→(Y, γ) the accompanying articulations are proportionality:

- f is ∝-unequivocally θ-coherence
- For each point x∈X and each fiter base Ψ in X θ-converging to x, the fiterbase f (Ψ) converges to f (x) in (Y, α0(Y))
- For each point x∈X and each net $\{x_\lambda\}_{\lambda \in \nabla}$ in X θ-converging to x, the net $\{f(x_\lambda)\}_{\lambda \in \nabla}$ converges to f (x) in (Y, α0(Y))
- For each point x∈X and each fiterbase Ψ in X θ-converging to x, the fiterbase f (Ψ) α-converges to f (x) in (Y, γ)
- For each point x∈X and each net $\{x_\lambda\}_{\lambda \in \nabla}$ in X θ-converging to x, the net $\{f(x_\lambda)\}_{\lambda \in \nabla}$ α-converges to f (x) in (Y, γ)

Proof: (i)⇔(ii)⇔(iii) and (i)⇔(iv)⇔(v) follows, immediately from Definition 3.1 and Theorem 2 of (Chae *et al.*, 1995).

Lemma 3.1; (Andrijevic, 1984): Let X be a topological space and A⊆X. At that point the accompanying are hold:

- αCl (E) = E∪Cl (Int (Cl (E)))
- αInt (E) = E∪Int (Cl (Int (E)))

Theorem 3.4: For a function f: X→Y the accompanying articulations are comparability:

- f is ∝-unequivocally θ-coherence
- f $(Cl_\theta(A))\subset(Cl\,(Int\,(Cl\,(f\,(A)))$, for every subset an of X
- $Cl_\theta(f^{-1}(E))\subset f^{-1}(Cl(Int(Cl(f(E)))))$, for every subset an of E of Y
- $f^{-1}(Cl(Int(Cl(f(E))))\subset Int_\theta(f^{-1}(E))$, for every subset an of E of Y

Proof: This follows from Lemma 3.1 and Theorem 2 of (Chae *et al.*, 1995).

Theorem 3.5: If a function f: X→Y is ∝-unequivocally θ-coherence and if E is an open subset of X, then f|E:E→ is ∝-unequivocally θ-coherence in the subspace E.

Proof: Let H be any ∝-open subset of Y. Since, f is ∝-unequivocally θ-coherence. Therefore, by [7, theorem 2], $f^{-1}(H)\epsilon\theta 0(X)$, so by Lemma 1.2.9 of (Abdul-Jabbar, 2000) $(f|E)^{-1}(H) = f^{-1}(H)\cap E\theta 0(E)$. This implies that f|E:E→Y is ∝-unequivocally θ-coherence.

Theorem 3.6: For any two functions, f: X→Y and g: Y→Z, the accompanying are valid:

- f is ∝-unequivocally θ-coherence and g is ∝-continuous, then g∘f is unequivocally θ-coherence
- f is faintly continuous and g is ∝-unequivocally θ-coherence, then g∘f is unequivocally ∝-irresolute

Theorem 3.7: "Let f: X→Y be faintly continuous and θ-open function and g: Y→Z be a function. Then g∘f: X→Z ∝-unequivocally θ-continuous if and only if g is ∝-unequivocally θ-coherence".

Proof: Let g∘f∝-unequivocally θ-coherence and $H\epsilon\alpha 0(Z)$. Then $(g\circ f)^{-1}(H) = f^{-1}(g^{-1}(H))\epsilon\theta 0(X)$. Since, f is θ-open function, $f(f^{-1}(g^{-1}(H)))\ \epsilon\theta 0(Y)$. Hence, $g^{-1}(H)\epsilon\theta 0(Y)$. Thus, g is ∝-unequivocally θ-coherence. It is easy to prove the opposite and is thus omitted.

Theorem 3.8: If g: Y→Z be a one to one ∝-open function on Yonto Z and g∘f: X→Z is ∝-unequivocally θ-continuous. Then f is unequivocally θ-coherence.

Proof: Suppose g is ∝-open function. Let H be an open subset of Y, since, g is one to one a nd onto, then the set g(H) is an ∝-open subset of Z, since, g∘f is ∝-unequivocally θ-coherence, it follows that $(g\circ f)^{-1}(g(H)) = f^{-1}(g^{-1}(g(H))) = f^{-1}(H)$ is θ-open in X. Thus, f is unequivocally θ-continuous.

Theorem 3.9: If X is almost regular and f: X→Y is completely ∝-irresolute function f is ∝-unequivocally θ-coherence.

Proof: Let H be an ∝-open subset of Y, since, f is completely ∝-irresolute function, then $f^{-1}(H)$ is regular open in X and from the fact that a space X is almost regular if and only if fore each x∈X and each regular open set $f^{-1}(H)$ containing x, there exist a regular open set 0 such that $x\epsilon 0\subset\bar{O}\subset f^{-1}(H)$ [31, theorem 2.2]. Therefore is θ-open in X and by [7, theorem 2], f is ∝-unequivocally θ-continuous.

Lemma 3.2; (Chae *et al.*, 1986): Let $\{X_\lambda:\lambda\epsilon\Delta\}$ be a family of spaces and $U_{\lambda i}$ be subset of $X_{\lambda i}$ for each I = 1, 2, ..., n. Then $U=\Pi_{i=1}^{n}U_{\lambda i}\times\Pi_{\lambda\neq\lambda i}X$ is ∝-open in $\Pi_{\lambda\epsilon\Delta}X_\lambda$ if and only if $U_{\lambda i}\epsilon\alpha 0(X_{\lambda i})$ for each i = 1, 2, ..., n.

Theorem 3.10: Let g_λ: $X_\lambda\to Y_\lambda$ be a function for each $\lambda\epsilon\Delta$ and g: $\Pi X_\lambda\to\Pi Y_\lambda$ a function defined by $g(\{x_\lambda\}) = \{g_\lambda(x_\lambda)\}$ for eac $\{x_\lambda\}\epsilon\Pi X_\lambda$. If g is ∝-unequivocally θ-coherence, then g_λ is ∝-unequivocally θ-coherence for each $\lambda\epsilon\Delta$.

Proof: "Let $\beta\epsilon\Delta$ and $V_\beta\epsilon\alpha 0(Y_\beta)$. Then, by Lemma 3.2, $V = V_\beta\times\Pi_{\lambda\neq\beta}Y_\lambda$ is ∝-open in ΠY_λ and $g^{-1}(V) = g^{-1}_\beta(V_\beta)\times\Pi_{\lambda\neq\beta}Y_\lambda$ is ∝-open in ΠX_λ. From Lemma 3.2, $g^{-1}_\beta(V_\beta)\epsilon\theta 0(X)$." Therefore, g_β is ∝-unequivocally θ-coherence.

Remark 3.1: It was known in [6, example 2.2] that $V\epsilon\propto 0(X\times Y)$ may not, generally, be a union of sets of the form A×B in the product space X×Y where $A\epsilon\propto 0(X)$ and $B\epsilon\propto 0(Y)$. Therefore, the converse of theorem 3.10 may not be true, generally.

Theorem 3.11: Let g: $X\to Y_1\times Y_2$ be ∝-unequivocally θ-coherence where X,Y_1 and Y_2 are any topological spaces. Let f_i:$X\to Y_i$ defined as follows: For x∈X, g(x) = (x_1, x_2), $f_i(x) = x_i$ for i = 1, 2. Then f_i:$X\to Y_i$ is ∝-unequivocally θ-continuous for i = 1, 2.

Proof: Let x be any point in X and H_1 be any ∝-open set in Y_1 containing $f_1(x) = x_1$, then by Lemma 3.2, $H_1\times Y_2$ is ∝-open $Y_1\times Y_2$ which contain (x_1, x_2). Since, g is ∝-unequivocally θ-continuous, therefore, there sexist an open set U containing x such that $g(Cl(U))H_1\times Y_2$. Then $f_1(Cl(U))\times f_2(Cl(U))\subset H_1\times Y_2$. Therefore, $f_1(Cl(U))H_1$. Hence, f_1 ∝-unequivocally θ-coherence. Similar statement for f_2 is ∝-unequivocally θ-coherence.

Lemma 3.3: Let X_1, X_2, ..., X_n be n topological spaces and $X=\Pi_{i=1}^{n}X_i$. Let $E_i\epsilon\theta 0(X_i)$ for i = 1, 2, ..., n then $\Pi_{i=1}^{n}E_i\in\theta 0(\Pi_{i=1}^{n}X_i)$.

Proof: Let $(x_1, x_2, ..., x_n) \in \prod_{i=1}^{n} E_i$ then $x_i \in E_i$ for i = 1, 2, ..., n. Since, $E_i \in \theta O(X_i)$, for i = 1, 2, ..., n. Then there exist open sets U_i for I = 1, 2, ..., n such that $x_i \in U_i \subset \overline{U_i} \subset E_i$ for i = 1, 2, ..., n. Therefore, $(x_1, x_2, ..., x_n) \in U_1 \times U_2 \times ... \times U_n \subset \overline{U_1} \times \overline{U_2} \times ... \times \overline{U_n} = Cl x_1 \times x_2 \times ... \times x_n (U_1 \times U_2 \times ... \times U_N) \subset \prod_{i=1}^{n} E_i$ and $\prod_{i=1}^{n} U_i \in \tau(\prod_{i=1}^{n} E_i)$ is θ-open set in $\prod_{i=1}^{n} X_i$.

Theorem 3.12: Let X_1, X_2, ..., X_n and Z be topological spaces and $\prod_{i=1}^{n} X_i \to Z$. If given any point p of Let X_1, X_2, ..., X_n be n topological spaces and $\prod_{i=1}^{n} X_i$ and given any ∝-open set U in Z containing f(p), there exist θ-open set E_i in X_i for i = 1, 2, ..., n such that $p \in \prod_{i=1}^{n} E_i$ and $f(\prod_{i=1}^{n} E_i) \subset U$. Then f is ∝-unequivocally θ-coherence.

Proof: Let $p \in \prod_{i=1}^{n} X_i$ and U be any ∝-open set in Z containing f(p), there exist θ-open set E_i in X_i for i = 1, 2, ..., n such that $p \in \prod_{i=1}^{n} E_i$ and $f(\prod_{i=1}^{n} E_i) \subset U$. Since, $E_i \in \theta O(X_i)$ for i = 1, 2, ..., n. Therefore, by Lemma 3.3 $\prod_{i=1}^{n} E_i \in \theta O(\prod_{i=1}^{n} X_i)$ for i = 1, 2, ..., n. Thus, f is ∝-unequivocally θ-coherence.

RESULTS AND DISCUSSION

αθ-Open function: In this area new kind of function call αθ-open function study and we discover some portrayal and properties for it.

Definition 4.1: A function f: X→y is call αθ-open if and only if for each α-open set G in X, f(G)∈θO(Y). On the off chance that takes after quickly that each αθ-open function is quasi α-open and thus, unequivocally α-open, the opposite is not valid as observed from the accompanying illustration.

Example 4.1: Let X = {a, b, c, d} and τ = {x, ∅, {a}, {c}, {a, b}, {a, c}, {a, b, c}, {a, c, d}}. The identity function i: (X, τ)→(X, τ) is unequivocally α-open but is not αθ-open function, since, {a}∈αO(X, τ) but f({a}) = {a}∉θO(X, τ). We discover a few portrayals and properties of αθ-open function.

Theorem 4.1: For any bijection function f: Y→X, the accompanying are proportionate:

- The inverse function is ∝-unequivocally θ-coherence
- f: Y→X is αθ-open function

The following lemmas are used in sequel.

Lemma 4.1; (Abdul-Jabbar, 2000): The accompanying is valid, for each subset E of X:

$$X / Cl_\theta(E) = Int_\theta(X / E)$$

Lemma 4.2: The accompanying is valid for every subset E of X:

$$X / Cl_\theta(E) = \alpha Int_\theta(X / E)$$

Theorem 4.2: For a function f: Y→X the accompanying are equal:

- f is αθ-open function
- f(αInt(E))⊂Int_θ(f(E)), for each subset E of X
- αInt(f^{-1}(W))⊂f^{-1}(int_θ(W)), for each subset W of Y
- f^{-1}(Cl_θ(W))⊂αCl(f^{-1}(W)), for each subset W of Y

Proof: (a)⇒(b) Suppose f is αθ-open function and E⊂X. Since, αInt⊂E, f(αInt(E))∈θO(Y) and f(αInt(E))⊂f(E) and hence, f(αInt(E))⊂Int_θ(f(E)). Let W⊂Y. Then f^{-1}(W)⊂X, therefore, we apply (b), we obtain f(αInt(f^{-1}(W))) ⊂int_θ(f(f^{-1}(W))). Then αInt(f^{-1}(W))⊂f^{-1}(int_θ(W)). (c)⇒(d): let W⊂Y, then apply (c) to Y/W, we get ∝Int(f^{-1}(W/Y))⊂f^{-1}(Int_θ(Y/W)). Then ∝Int(X/f^{-1}(W))⊂f^{-1}(Cl_θ(W)) which implies that X/∝Cl(f^{-1}(W))⊂X/f^{-1}(Cl_θ(W)). Hence f^{-1}(Cl_θ(W)) ⊂∝Cl(f^{-1}(W)). (d)⇒(a): let G be any ∝-open set in X. Then Y/f(g)⊂Y, apply (d), we obtain f^{-1}(Cl_θ(Y/f(G)))⊂∝Cl(f^{-1}(Y/f(G))). Then f^{-1}(Y/Int_θ(f(G)))⊂∝Cl(X/G). Which implies that X/f^{-1}(Int_θ(f(G)))⊂X/IntG = X/G. Therefore, G⊂f^{-1}(Int_θ(f(G))). Then f(G)⊂Int_θ(f(G)). Therefore, f(G)∈θO(Y). Which completes the proof.

Remark 4.1: Let f: X→Y be a bijective function. Then, f is ∝θ-open function if and only if f(F)∈θC(Y), for each ∝-closed set F in X.

Theorem 4.3: If Y is regular space, then each s**-open function is ∝θ-open.

Proof: Given G a chance to be any ∝-open subset of X, then it is semi-open. Since, f is s**-open function. Therefore, f(G) is open in Y. But Y is regular space, then by [1, Lemma 1.2.8] f(G) is θ-open in Y. Which completes the proof.

Theorem 4.4: In the event that f: X→Y is θ-open function and E⊂X is an open set in X, at that point the f|E:E→Y is ∝θ-open function.

Proof: Let H be any ∝-open set in the open subspace E. At that point, by [15, Theorem 3.7], H is ∝-open in X. Since, f is ∝θ-open function. In this way, f(H) is θ-open in Y. Hence, f|E is ∝θ-open function.

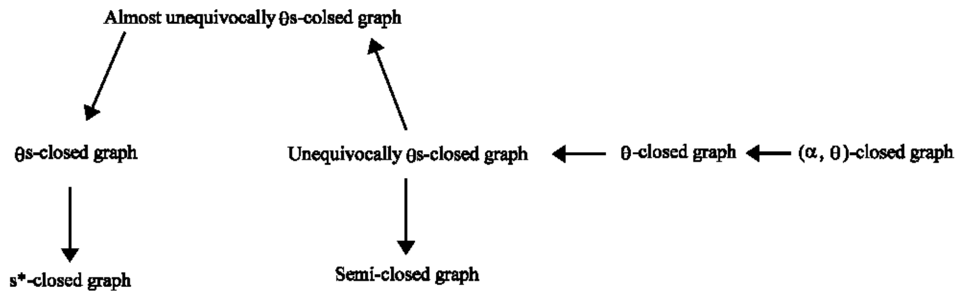

Fig. 1: Growth of the graph

Theorem 4.5: Given f: X→Y be a function and $\{E_\alpha: \alpha\epsilon\nabla\}$ be an open cover of X. If the restriction $f|E_\alpha$: E_α→Y is ∝θ-open function for each α∈∇, then f is ∝θ-open function.

Proof: Give H a chance to be any ∝-open set in X. In this manner, by [15, Theorem 3.4], H∩E∝ is ∝-open in the subspace E∝ for each α∈∇. Since, f|Eα is ∝θ-open function (f|Eα) (H∩E∝) is θ-open in Y and hence, f(H) = ∪{(f|E∝)(H∩E∝):α∈∇} is θ-open in Y. This demonstrate f is ∝θ-open function.

Remark 4.1: Unmistakably θ-compact and quasi H-closed equivalent from theorem 2.11 of (Ahmed and Yunis, 2002).

Theorem 4.6: In the event that f: X→Y is ∝θ-open function and f(F) is θ-compact relative to Y, then F is ∝-compact subspace relative to X.

Proof: Let $\{E_\alpha:\alpha\epsilon\nabla\}$ be an open cover of F, then $\{f(E_\alpha):\alpha\epsilon\nabla\}$ is cover for f(F). Since, f is ∝θ-open function. Therefore, $f(E_\alpha)$:∈θ0(Y) for each α∈∇. Since, f(F) is θ-compact relative to Y. Therefore, there exist a finite subfamily $\{f(E_{\alpha i}):i = 1, 2, ..., n\}$ such that $f(F)\subset Y_{i=1}^{n} f(E_{\alpha i})$. Hence, $F\subset Y_{i=1}^{n} f(E_{\alpha i})$. Therefore, F is ∝-compact subspace relative to X.

Corollary 4.1: If f: X→Y is ∝θ-open surjective and Y is θ-compact space, then X is ∝-compact space.

Theorem 4.7: A function f:X→Y is ∝θ-open if and only if for each subset S of Y and any ∝-closed set F in X containing $f^{-1}(S)$, there exist a θ-closed set M in Y containing S such that $f^{-1}(M)\subset F$.

Proof: Assume that f is ∝θ-open function. Let S⊂Y and F be an ∝-closed set in X containing $f^{-1}(S)$. Put M = Y/f(X/F), then M is θ-closed in Y and since, $f^{-1}(S)\subset F$, we have S⊂M. Since, f is ∝θ-open function and F is ∝-closed in X, M is θ-closed in Y. It follows that $f^{-1}(M)\subset F$.

Conversely, let G be any ∝-open subset of X and put S = Y/f. Then X/G is ∝-closed set containing $f^{-1}(G)$. By hypothesis, there exist a θ-closed set M in Y containing S such that $f^{-1}(M)\subset X/G$. Thus, we have f(G)⊂Y/M. On the other hand, we have f(|G) = Y/S⊃Y/M and hence, f(G) = Y/M. Consequently, f(G) is θ-open in is ∝θ-open function.

Function with (∝, θ)-closed graph: In this area, we examine new properties of (∝, θ)-closed graph (Chae *et al*., 1995). Definition 5.1 (Chae *et al*., 1995). Let, f(G) = {(x, f(x)):x∈X} be the graph of f: X→Y, then is said to be (∝, θ)-closed with respect to X×Y, if for each point (x,y)∉G(f), there exist an open set U and an ∝-open set H containing x and y, respectively, such that $f(\bar{U})\cap H = \varphi$. The accompanying diagram is a growth of the graph 4.1.1 of (Abdul-Jabbar, 2000). None of the suggestions is reversible (Fig. 1).

Example 5.1: Let X = {a, b, c} and, τ = {φ, X, {a}, {a, b}, {a, c}}, then the function f:(X, τ)→(Y, τ) defined as: f(x) = a, for each x∈X has θ-closeed graph which has not (∝ θ)-closed graph.

Theorem 5.1: If f: X→Z is a function with (∝θ)-closed graph and f: X→Z ∝-unequivocally θ-coherence functions, then the set {(x, y): f(x) = g(y)} is θ-closed in X×Y.

Proof: Let E = {(x, y): f(x) = g()y}. If (x, y)∈X×Y/E, then f(x)≠g(y). Hence, (x, g(y))∈(X, Z)G(f). Since, f has (∝θ)-closed graph. Therefore, there exist open set U and ∝-unequivocally θ-coherence of g implies that there is an open set V of X such that $g(\bar{V})\subset H$. Therefore, we have $f(\bar{U})\times g(\bar{V}) = \varphi$. This established that $f(\bar{U})\times g(\bar{V})\cap E = \varphi$ which implies that (x, y)$\notin Cl_\theta E$. E is θ-closed in X×Y.

Corollary 5.1: If is an Hausdorff space and f, g: X→Y are is ∝-unequivocally θ-coherence functions, then the set {(x, y): f(x) = g(y)} is θ-closed in X×Y.

Theorem 5.2: If f: X→Y is any function with θ-closed point inverses such that the image of each closure of open set is ∝-closed, then has (∝θ)-closed graph.

Proof: Let (x, y)∈X×YG(f). Then x∉f^{-1}(y) and since, f^{-1}(y) is θ-closed, there exist an open set U containing x such that $\overline{U} \cap f^{-1}(y) = \varphi$. .It follows that $f(\overline{U})$ is ∝-closed. Therefore, there is an ∝-open set H in Y containing y such that $f(\overline{U}) \cap H = \varphi$. Thus, f has (∝θ)-closed graph.

Theorem 5.3: Let f: X→Y be given function with (∝θ)-closed graph, then for each x∈X, {f(x) = ∩{∝Cl(f($\overline{U}$)): U is an open set of x}has.

Proof: Let the graph of the function be (∝θ)-closed. Then it is claimed that for each x∈X, {f(x) = ∩{∝Cl(f($\overline{U}$)): U is an open set of x}.

For if not, so, let y≠f(x) such that y∈∩, {∝Cl(f($\overline{U}$)): U is an open set of x}. Which implies that $y \in \propto Cl(f(\overline{U}))$ for each open set of x; it means that, for each ∝-open set V of y in Y, $V \cap f(\overline{U}) \neq \varphi$. Thus, we obtain that (x, y)∉G(f) and there exist U and V such that $V \cap f(\overline{U}) \neq \varphi$ which implies that is contradiction. Thus, y = f(x).

Theorem 5.4: Let f: X→Y be a function with (∝θ)-closed graph. If is quasi H-closed in X, then f(E) is has ∝-closed in Y.

Proof: Let E be a quasi H-closed in X. Suppose that f(E) is not ∝-closed in Y. Let y∉f(E). Therefore, y≠f(x) for each x∈E. Since, f has (∝θ)-closed graph. Therefore, there exist open set U_x and ∝-open set H_x containing x and y, respectively such that $f(\overline{U_x}) \cap H_x = \varphi$, for each x∈E. The family Q = {U_x: x∈E} is an open cover of E. Since, E is quasi H-closed, there exist a finite subfamily {$U_{x(1)}$, ..., $U_{x(n)}$} of Q such that $E \subset Y_{i=1}^{n}(\overline{U_{x(i)}})$. Put $H = I_{i=1}^{n} H_{x(i)}$. Then:

$$f(E) \cap H \subset Y_{i=1}^{n} f\left(\overline{U_{x(i)}}\right) \cap H \subset Y_{i=1}^{n} f\left(\overline{U_{x(i)}}\right) \cap \left(I_{i=1}^{n} H_{x(i)}\right) = \varphi$$

Since, H is an ∝-open set containing y, y∉∝Cl(f(E)). Therefore, ∝Cl(f(E))⊂f(E).

Corollary 5.2: The image of any quasi H-closed space in any space is ∝-closed under functions with (∝θ)-closed graphs.

Theorem 5.4: Let f:X→Y be a given function. Then is G(f) is (∝θ)-closed graph if and only if for each filter base **Ψ** in X θ-converges to some p in X, f(**Ψ**)∝-cconverges to some q in Y, f(p) = q.

Proof: Suppose that Then G(f) is (∝θ)-closed graph and let **Ψ** = {E_α:α∈∇} be a filter based in X such that **Ψ**θ-converges to p and f(**Ψ**)∝-converges to q. If f(p)≠q, then (q, p)∉G(f). Thus, there exist open set U⊂X and ∝-open set V⊂Y containing p and q, respectively, such that $(\overline{U} \times V) \cap G(f) = \varphi$. Since, **Ψ**∝-converges to p and f(**Ψ**)∝-converges to q, there exist an E_α∈**Ψ** such that $E_\alpha \subset \overline{U}$ and $f(E_\alpha) \subset V$. Consequently, $(\overline{U} \times V) \cap G(f) \neq \varphi$ which is a contradiction.

Conversely, assume G(f) that is not (∝θ)-closed graph. Then, there exist a point (p, q)∉G(f) such that for each open set U⊂X and ∝-open set V⊂Y containing p and q, respectively, such that $(\overline{U} \times V) \cap G(f) \neq \varphi$. Define:

$$\Psi_1 = \{\overline{U_\alpha} : U_\alpha \text{ is an open set containing p and } \alpha \in \nabla_1\}$$

$$\Psi_2 = \{V_\beta : V_\beta \text{ is an } \alpha\text{-open set containing q and } \beta \in \nabla_2\}$$

$$\Psi_3 = \{E(\alpha, \beta) : E(\alpha, \beta) = (\overline{U_\alpha} \times V_\beta) \cap G(f), (\alpha, \beta) \in \nabla_1 \times \nabla_2\}$$

Ψ = {**Ψ***(α, β):(α, β)∈$\nabla_1 \times \nabla_2$} where **Ψ***(α, β) = {x∈U_x:(x, f(x))∈E(**Ψ***(α, β)}. Then **Ψ** is a filter base in X with property that **Ψ** ∝-converges to p and f(**Ψ**)∝-converges to q and f(p)≠q.

Corollary 5.3: A function f: X→Y be has (∝θ)-closed graph if and only if for each net X_γ in X such that X_γ→θp∈X and f(x_γ)→αq∈Y, f(p) = q.

CONCLUSION

This study briefly described the θ-open function, Quasi θ-open and θ-open their properties in this research.

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