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A Class of Modified Calibration Ratio Estimators of Population Mean with Known Coefficient of Kurtosis in Stratified Double Sampling

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Abstract: This study proposes a class of ratio estimators of mean for calibration estimation that is more precise and efficient than the linear regression estimator under the stratified double sampling using coefficient of kurtosis of auxiliary variable. Some well-known estimators are obtained under certain prescribed conditions and shown to be special members of this class of estimators. Analytical and numerical results proved the efficacy of the new class of estimators over all existing modified estimators in stratified double sampling with appreciable gains in efficiency at its optimum condition.

INTRODUCTION

It is noted that the regression estimator of mean is the most efficient estimator. The ratio (and product) estimator of mean is equally good if the regression line is a straight line and passes through the origin. However in most practical situations the regression line does not pass through the origin.

To address this problem, most survey statisticians have carried out researches towards the modification of the existing ratio and product estimators to provide better alternatives and improve their precision. In the progression for improving the performance of the ratio estimators, many researchers like[1-12] among others have proposed different modified ratio estimators in sample surveys.

Keeping this in view, this study introduces modified class of ratio estimators for population mean in stratified double sampling based on the coefficient of Kurtosis of the auxiliary variable using the theory of calibration estimation. The concept of calibration estimator was introduced by Deville and Sarndal[13] in survey sampling. Many researchers such as[14-25] have defined some modified calibration estimators in survey sampling using population information of different parameters of the auxiliary variable such as the total, mean, variance, coefficient of variation, population coefficient of correlation to increase the efficiency of estimation of population parameter's of interest.

However, no attempt is made to use the population information of coefficient of kurtosis of the auxiliary variable to improve calibration estimators in survey sampling. This study is an attempt in this direction. The choice is obvious; coefficient of kurtosis and its functions are unaffected by extreme values or the presence of outliers. Further, it always has strong correlation with other population parameters like the mean and variance.

BASIC DEFINITIONS AND NOTATIONS

Consider a finite population U = (U_1, U_2,..., U_N) of size (N). Let (X) and (Y) denote the auxiliary and study variables taking values X_i and Y_i, respectively on the i-th unit U_i (i = 1, 2,..., N) of the population.

The theory of double sampling for stratification was first given by Neyman[26]. The population is to be stratified into H strata such that the h-th stratum consists of N_h units and:

$$\sum_{h=1}^{H} N_h = N,\ \sum_{h=1}^{H} n_h = n$$

From the N_h units a preliminary large sample of n'_h units is drawn by the Simple Random Sampling Without Replacement (SRSWOR) and the auxiliary character x_{hi} is measured only. A subsample of n_h is then selected from the given preliminary large sample of n'_h units by SRSWOR and both the study variable y_{hi} and the auxiliary variable x_{hi} are measured. Let,

$$\bar{x}'_h = \frac{1}{n'_h}\sum_{i=1}^{n'_h} x_{hi}, S'^2_{hx} = \frac{1}{n'_h - 1}\sum_{i=1}^{n'_h}(x_{hi}, \bar{x}'_h)^2$$

denote the first phase sample mean and variance respectively for the auxiliary variable. Similarly, let,

$$\bar{x}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} x_{hi}, S^2_{hx} = \frac{1}{n_h - 1}\sum_{i=1}^{n_h}(x_{hi}, \bar{x}_h)^2,$$
$$\bar{y}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} y_{hi}, S^2_{hy} = \frac{1}{n_h - 1}\sum_{i=1}^{n_h}(y_{hi}, \bar{y}_h)^2$$

denote the second phase sample means and variances for the auxiliary variable and study variable respectively. Let consider the following equations:

$$e_{hy} = \left(\frac{\bar{y}_h - \bar{Y}_h}{\bar{Y}_h}\right) \text{so that } \bar{y}_h = \bar{Y}_h\left(1 + e_{hy}\right)$$

$$e_{h\beta} = \left(\frac{\beta_{2h}(x) - B_{2h}(x)}{B_{2h}(x)}\right) \text{so that } \beta_{2h}(x) = B_{2h}(x)\left(1 + e_{h\beta}\right)$$

$$e'_{h\beta} = \left(\frac{\beta'_{2h}(x) - B_{2h}(x)}{B_{2h}(x)}\right) \text{so that } \beta'_{2h}(x) = B_{2h}(x)\left(1 + e'_{h\beta}\right)$$

$$E\left(e_{hy}\right) = E\left(e_{h\beta}\right) = E\left(e'_{h\beta}\right) = 0; E\left(e^2_{hy}\right) = \gamma_h C^2_{hy}; E\left(e^2_{h\beta}\right) = \gamma_h C^2_{h\beta};$$

$$E\left(e'^2_{h\beta}\right) = \gamma'_h C^2_{h\beta}; E\left(e_{hy}e_{h\beta}\right) = \gamma_h \rho_{hy\beta} C_{hy}; E\left(e_{hy}e'_{h\beta}\right) = \gamma'_h \rho_{hy\beta} C_{hy} C_{h\beta};$$

$$E\left(e'_{h\beta}e_{h\beta}\right) = \gamma'_h C^2_{h\beta}; \gamma'_h = \left(\frac{1}{n'_h} - \frac{1}{N_h}\right); \gamma_h = \left(\frac{1}{n_h} - \frac{1}{N_h}\right); C^2_{h\beta} = \frac{S^2_{hx}}{\bar{B}_h^2};$$

$$C^2_{hy} = \frac{S^2_{hy}}{\bar{Y}_h^2}; \alpha_h = \rho_{hy\beta}\frac{C_{hy}}{C_{h\beta}}; \rho_{hxy} = \frac{S_{hxy}}{S_{hx}S_{hy}}; \bar{B}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} W_h \beta_{2h}(x);$$

$$B = \sum_{i=1}^{n_h} W_h \beta_{2h}(x); \omega_h = \frac{n'_h}{n'}; W_h = \frac{N_h}{N}; B' = \sum_{i=1}^{n_h} \omega_h \beta'_{2h}(x)$$

where the parameters are defined wherever they appear as the following:

- $\bar{y}_h$ = The second phase sample stratum mean of the study variable
- $\bar{Y}_h$ = The second phase population stratum mean of the study variable
- $\bar{x}'_h$ = The first phase sample stratum mean of the auxiliary variable
- x_h = The second phase sample stratum mean of the auxiliary variable
- $\bar{X}_h$ = The second phase population stratum mean of the auxiliary variable
- s'^2_{hx} = The first phase sample stratum variance of the auxiliary variable
- s^2_{hx} = The second phase sample stratum variance of the auxiliary variable
- S^2_{hx} = The second phase population stratum variance of the auxiliary variable
- $\beta'_{2h}(x)$ = The first phase sample coefficient of kurtosis of the auxiliary variable
- $\beta_{2h}(x)$ = The second phase sample coefficient of kurtosis of the auxiliary variable
- $B_{2h}(x)$ = The second phase population coefficient of kurtosis of the auxiliary variable
- $C^2_{h\beta}$ = The coefficient of variation of the auxiliary variable
- C^2_{hy} = The coefficient of variation of the study variable
- $\rho_{hy\beta}$ = The correlation coefficient between the mean of the study variable and coefficient of kurtosis of the auxiliary variable
- ρ_{hxy} = The correlation coefficient between the auxiliary variable and the study variable

SUGGESTED ESTIMATOR

Solanki *et al*.[27], using information from known values of means of the auxiliary variable, proposed the estimator:

$$t_{(\alpha,\delta)} = \bar{y}\left\{2 - \left(\frac{\bar{x}}{\bar{X}}\right)^{v} \exp\left[\frac{\xi\left(\bar{x} - \bar{X}\right)}{\left(\bar{x} + \bar{X}\right)}\right]\right\} \quad (1)$$

Then as a boundary condition, if:

$$\left(\frac{\bar{x}}{\bar{X}}\right)^{v} \to 1 \text{ and } \left\{\exp\left[\frac{\xi(\bar{x}-\bar{X})}{(\bar{x}+\bar{X})}\right]\right\} \to 1; \text{ since } E(\bar{x}) = \bar{X}$$

in Eq. (1), then $E[t_{(\alpha,\delta)}] \to \bar{Y}$ making the estimator $t_{(\alpha,\delta)}$ unbiased. This justifies the use of the number two in Solanki *et al*.[27] estimator. In stratified random sampling this estimator is defined as:

$$\hat{\bar{Y}}_R = \sum_{h=1}^{H} w_h \bar{y}_h \left\{2 - \left(\frac{\bar{x}_h}{\bar{X}_h}\right)^{v_h} \exp\left[\frac{\xi_h(\bar{x}_h - \bar{X}_h)}{(\bar{x}_h + \bar{X}_h)}\right]\right\} \tag{2}$$

If φ_h and:

$$\tau_h \left(\frac{\beta_{2h}(x)}{\beta'_{2h}(x)}\right)^{\upsilon_h} \exp\left[\frac{\xi_h(\beta_{2h}(x) - \beta'_{2h}(x))}{(\beta_{2h}(x) + \beta'_{2h}(x))}\right]$$

are used to replace 2 and:

$$\left(\frac{\bar{x}_h}{\bar{X}_h}\right)^{v_h} \exp\left[\frac{\xi_h(\bar{x}_h - \bar{X}_h)}{(\bar{x}_h + \bar{X}_h)}\right]$$

respectively in (2), the class of estimators:

$$\hat{\bar{Y}}_R^* = \sum_{h=1}^{H} w_h^* \bar{y}_h \left\{\varphi_h - \tau_h \left(\frac{\beta_{2h}(x)}{\beta'_{2h}(x)}\right)^{\upsilon_h} \exp\left[\frac{\xi_h(\beta_{2h}(x) - \beta'_{2h}(x))}{(\beta_{2h}(x) + \beta'_{2h}(x))}\right]\right\} \tag{3}$$

is a modification of (2) with extension to stratified double sampling using information from known values of coefficient of kurtosis of the auxiliary variable where φ_h, τ_h, ν_h and ξ_h are suitably chosen scalars such that φ_h and τ_h satisfies the condition:

$$\varphi_h + \tau_h = 1; -\infty < r < \infty \tag{4}$$

and w^*_h are calibration weights chosen such that achi-square loss functions of the form:

$$L(\omega_h^*, W_h) = \sum_{h=1}^{H} \frac{(\omega_h^* - W_h)^2}{W_h Q_h} \tag{5}$$

is minimized subject to thecalibration constraints of the form:

$$\sum_{h=1}^{H} \omega_h^* \beta_{2h}(x) = \sum_{h=1}^{H} \omega_h \beta'_{2h}(x) \tag{6}$$

where, $\beta_{2h}(x)$ is the coefficient of kurtosis of the auxiliary variable X. Minimizing the chi-square loss functions (5) subject to the calibration constraints (6), gives the calibration weights for stratified double sampling as follows:

$$\omega_h^* = W_h + \frac{W_h Q_h \beta_{2h}(x)}{\sum_{h=1}^{H} W_h Q_h \beta_{2h}^2(x)} \left[\sum_{h=1}^{H} \omega_h \beta'_{2h}(x) - \sum_{h=1}^{H} W_h \beta_{2h}(x)\right] \tag{7}$$

So that setting $Q_h = \beta_{2h}^{-1}(x)$:

$$\omega_h^{*2} = W_h^2 \left(\frac{B'}{B}\right)^2 \tag{8}$$

Where:

$$B' = \sum_{h=1}^{H} \omega_h \beta'_{2h}(x)$$

And:

$$B = \sum_{h=1}^{H} W_h \beta_{2h}(x)$$

Variance expression for the proposed class of estimators: This section derives the estimator of variancefor the proposed class ofestimatorsusing the large sample approximation (LASAP) method. Expressing (3) in terms of the e's gives:

$$\hat{\bar{Y}}_R^* = \sum_{h=1}^{H} w_h^* \bar{Y}_h (1 + e_{hy}) \left\{\varphi_h - \tau_h \left[(1 + e_{h\beta})(1 + e'_{h\beta})^{-1}\right]^{\nu_h} \exp\left[\xi_h \frac{(e_{h\beta} - e'_{h\beta})}{2}\left[1 + \frac{(e_{h\beta} + e'_{h\beta})}{2}\right]^{-1}\right]\right\}$$

Now, it is assumed that $|e_{h\beta}|<1$ so that expanding:

$$(1 + e'_{h\beta})^{-1}, \left[(1 + e_{h\beta})(1 + e'_{h\beta})^{-1}\right]^{\nu_h} \left(1 + \frac{(e_{h\beta} + e'_{h\beta})}{2}\right)^{-1}$$

And:

$$\exp \frac{\xi_h (e_{h\beta} - e'_{h\beta})}{2} \left(1 + \frac{(e_{h\beta} + e'_{h\beta})}{2}\right)^{-1}$$

as a series in power of $e_{h\beta}$, multiplying out and retaining terms of the e's to the second degree, gives:

$$\left[\hat{\bar{Y}}_R^* - \bar{Y}\right] = \sum_{h=1}^{H} w_h^* \bar{Y}_h \Big[e_{hy} + \tau_h \frac{(2\nu_h + \xi_h)}{2} \Big\{e_{h\beta} - e'_{h\beta} + e_{hy} e_{h\beta} - s e_{hy} e'_{h\beta} + \frac{(2\nu_h + \xi_h)}{2} e_{h\beta} e'_{h\beta} + \frac{(2\nu_h + \xi_h - 2)}{4} e_{h\beta}^2 - \frac{(2\nu_h + \xi_h + 2)}{4} e'^2_{h\beta}\Big\}\Big] \tag{9}$$

Taking expectation of both sides of (9) gives the bias of $\hat{\bar{Y}}_R^*$ to the first order of approximation (i.e., to terms of order $o(n_h^{-1})$) as:

$$\text{Bias}(\hat{\bar{Y}}_R^*) = \sum_{h=1}^{H} w_h^* \bar{Y}_h (\gamma_h - \gamma'_h) \left[\tau_h \frac{(2\nu_h + \xi_h)}{2} \left\{\alpha_h - \frac{(2\nu_h + \xi_h - 2)}{4}\right\} C_{h\beta}^2\right] \tag{10}$$

Where:

$$\alpha_h = \rho_{hy\beta}C_{hy} / C_{h\beta}$$

If $\nu_h = -1/2\xi_h$, then Bias $(\hat{\bar{Y}}_R^*)$ the is equal to zero. Therefore, the estimator $\hat{\bar{Y}}_R^*$ with is almost unbiased. Squaring both sides of (9) and retaining terms to the second degree, gives:

$$\left(\hat{\bar{Y}}_R^* - \bar{Y}\right)^2 = \sum_{h=1}^{H} w_h^{*2}\,\bar{Y}_h^2 \begin{bmatrix} e_{hy}^2 + \tau_h^2\left(\dfrac{2\nu_h+\xi_h}{2}\right)^2\left(e_{h\beta}^2 + e_{h\beta}^{'2} - 2e_{h\beta}e_{h\beta}^{'}\right) \\ -\tau_h\left(2\nu_h+\xi_h\right)\left(e_{hy}e_{h\beta} - e_{h\beta}e_{h\beta}^{'}\right) \end{bmatrix} \tag{11}$$

Taking expectation of both sides of (11) and using the results in (8), gives the variance of $\hat{\bar{Y}}_R^*$ to the first order of approximation as:

$$V\left(\hat{\bar{Y}}_R^*\right) = \left(\frac{B^{'}}{B}\right)^2 \sum_{h=1}^{H} w_h^2\,\bar{Y}_h^2 \begin{bmatrix} \gamma_h C_{hy}^2 + \left(\dfrac{2\nu_h+\xi_h}{4}\right)\left(\gamma_h - \gamma_h^{'}\right) \\ \left[\tau_h^2\left(2\nu_h+\xi_h\right) - 4\tau_h\alpha_h\right]C_{h\beta}^2 \end{bmatrix} \tag{12}$$

The variance $V\left(\hat{\bar{Y}}_R^*\right)$ in (12) is minimized when:

$$\begin{aligned} \nu_h &= \left(\frac{\alpha_h - \xi_h\tau_h}{2\tau_h}\right) \\ &= \nu_{h,opt}\,(\text{say}) \end{aligned} \tag{13}$$

So that:

$$V\left(\hat{\bar{Y}}_{R,opt}^*\right) = \left(\frac{B^{'}}{B}\right)^2 \sum_{h=1}^{H} w_h^2\,\bar{Y}_h^2\left[\gamma_h C_{hy}^2 - \left(\gamma_h - \gamma_h^{'}\right)\alpha_h^2 C_{h\beta}^2\right] \tag{14}$$

Membership of the proposed class of estimators

Stratified random sampling estimator: If $\varphi_h = 1$, $\tau_h = 0$, $\nu_h = \nu_h$ and $\xi_h = \xi_h$; then the proposed estimator (4) reduces to the stratified random sampling estimator in stratified double sampling given by:

$$\hat{\bar{Y}}_{R,1}^* = \sum_{h=1}^{H} w_h^*\,\bar{y}_h$$

with variance estimator given by:

$$V\left(\hat{Y}_{R,1}^*\right) = \left(\frac{B^{'}}{B}\right)^2 \sum_{h=1}^{H} w_h^2\,\bar{Y}_h^2\gamma_h C_{hy}^2$$

Classical ratio estimator: If $\varphi_h = 0$, $\tau_h = -1$, $\nu_h = -1$ and $\xi_h = 0$; then the proposed estimator (4) is a modification of classical ratio estimator in stratified double sampling given by:

$$\hat{\bar{Y}}_{R,2}^* = \sum_{h=1}^{H} w_h^*\,\frac{\bar{y}_h}{\beta_{2h}(x)}\beta_{2h}^{'}(x)$$

with variance estimator given by:

$$V(\hat{\bar{Y}}_{R,2}^*) = \left(\frac{B^{'}}{B}\right)^2 \sum_{h=1}^{H} w_h^2\,\bar{Y}_h^2\left[\gamma_h C_{hy}^2 + \left(\gamma_h - \gamma_h^{'}\right)\{1 - 2\alpha_h\}C_{h\beta}^2\right]$$

Classical product estimator: If $\varphi_h = 0$, $\tau_h = -1$, $\nu_h = 0$ and $\xi_h = -1$; then the proposed estimator (4) is a modification of classical product estimator in stratified double sampling given by:

$$\hat{\bar{Y}}_{R,3}^* = \sum_{h=1}^{H} w_h^*\,\frac{\beta_{2h}(x)}{\beta_{2h}^{'}(x)}\bar{y}_h$$

with variance estimator given by:

$$V(\hat{\bar{Y}}_{R,3}^*) = \left(\frac{B^{'}}{B}\right)^2 \sum_{h=1}^{H} w_h^2\,\bar{Y}_h^2\left[\gamma_h C_{hy}^2 + \left(\gamma_h - \gamma_h^{'}\right)\{1 + 2\alpha_h\}C_{h\beta}^2\right]$$

Bahl and Tuteja estimator 1: If $\varphi_h = 0$, $\tau_h = -1$, $\nu_h = 0$ and $\xi_h = -1$; then the proposed estimator (4) is a modification of Bahl and Tuteja[28] exponential ratio estimator in stratified double sampling given by:

$$\hat{\bar{Y}}_{R,4}^* = \sum_{h=1}^{H} w_h^*\,\bar{y}_h \exp\left[\frac{\left(\beta_{2h}^{'}(x) - \beta_{2h}(x)\right)}{\left(\beta_{2h}(x) + \beta_{2h}^{'}(x)\right)}\right]$$

with variance estimator given by:

$$V(\hat{\bar{Y}}_{R,4}^*) = \left(\frac{B^{'}}{B}\right)^2 \sum_{h=1}^{H} w_h^2\,\bar{Y}_h^2\left[\gamma_h C_{hy}^2 + \frac{\left(\gamma_h - \gamma_h^{'}\right)}{4}\{1 - 4\alpha_h\}C_{h\beta}^2\right]$$

Bahl and Tuteja estimator 2: If $\varphi_h = 0$, $\tau_h = -1$, $\nu_h = 0$ and $\xi_h = 1$; then the proposed estimator (4) is a modification of Bahl and Tuteja[28] exponential ratio estimator in stratified double sampling given by:

$$\hat{\bar{Y}}_{R,5}^* = \sum_{h=1}^{H} w_h^*\,\bar{y}_h \exp\left[\frac{\left(\beta_{2h}(x) - \beta_{2h}^{'}(x)\right)}{\left(\beta_{2h}(x) + \beta_{2h}^{'}(x)\right)}\right]$$

with variance estimator given by:

$$V(\hat{\bar{Y}}_{R,5}^*) = \left(\frac{B^{'}}{B}\right)^2 \sum_{h=1}^{H} w_h^2\,\bar{Y}_h^2\left[\gamma_h C_{hy}^2 + \frac{\left(\gamma_h - \gamma_h^{'}\right)}{4}\{1 + 4\alpha_h\}C_{h\beta}^2\right]$$

Kadilar and Cingi estimator: If $\varphi_h = 0$, $\tau_h = -1$, $\nu_h = -2$ and $\xi_h = 0$; then the proposed estimator (4) is a modification of Kadilar and Cingi[29] estimator in stratified double sampling given by:

$$\hat{\bar{Y}}^*_{R,6} = \sum_{h=1}^{H} w^*_h \bar{y}_h \left(\frac{\beta'_{2h}(x)}{\beta_{2h}(x)}\right)^2$$

with variance estimator given by:

$$V(\hat{\bar{Y}}^*_{R,7}) = \left(\frac{B'}{B}\right)^2 \sum_{h=1}^{H} w_h^2 \bar{Y}_h^2 \left[\gamma_h C_{hy}^2 - 4\left(\gamma_h - \gamma'_h\right)\{1+\alpha_h\} C_{h\beta}^2\right]$$

Solanki *et al.*[27] estimator: If $\varphi_h = 2$, $\tau_h = 1$, $\nu_h = \nu_h$ and $\xi_h = \xi_h$; then the proposed estimator (4) is a modification of Solanki *et al*.[27] exponential ratio estimator in stratified double sampling given by:

$$\hat{\bar{Y}}^*_{R,7} = \sum_{h=1}^{H} w^*_h \bar{y}_h \left\{2 - \left(\frac{\beta_{2h}(x)}{\beta'_{2h}(x)}\right)^{\upsilon_h} \exp\left[\frac{\xi_h\left(\beta_{2h}(x) - \beta'_{2h}(x)\right)}{\left(\beta_{2h}(x) + \beta'_{2h}(x)\right)}\right]\right\}$$

with variance estimator given by:

$$V(\hat{\bar{Y}}^*_{R,7}) = \left(\frac{B'}{B}\right)^2 \sum_{h=1}^{H} w_h^2 \bar{Y}_h^2 \left[\begin{array}{l}\gamma_h C_{hy}^2 - \dfrac{\left(2\nu_h + \xi_h\right)}{4}\left(\gamma_h - \gamma'_h\right) \\ \left\{\left(2\nu_h + \xi_h\right) - 4\alpha_h\right\} C_{h\beta}^2\end{array}\right]$$

EMPIRICAL STUDY

To judge the relative performances of the proposed calibration estimator over members of its class in stratified double sampling, the data set in Table 1 was considered. Two measuring criteria; variance and percent relative efficiency were used to compare the performance of each estimator.

The Percent Relative Efficiency (PRE) of an estimator φ with respect to the conventional ratio estimator in stratified double sampling $\left[\hat{\bar{Y}}_R\right]$ is defined by:

$$PRE[\phi, \hat{\bar{Y}}_R] = \frac{V\left[\hat{\bar{Y}}_{,R}\right]}{V[(\phi)]} \times 100$$

The variance of the conventional ratio estimator of population mean for double sampling for stratification defined by Cochran[30] is given by:

$$V\left[\hat{\bar{Y}}_R\right] = \sum_{h=1}^{H} w_h^2 \left[\gamma_h S_{hy}^2 + \gamma'_h\left(S_{hy}^2 + R^2 S_{hx}^2 - 2RS_{hxy}\right)\right] = 3349.277$$

Also, the variance of the conventional regression estimator of population mean for double sampling for stratification defined by Cochran[30] is given by:

$$V\left[\hat{\bar{Y}}_{REG}\right] = \sum_{h=1}^{H} w_h^2 S_{hy}^2 \left[\gamma_h + \gamma'_h\left(1 - \rho_{hxy}^2\right)\right] = 1370.9815$$

Table 1: Data Statistics adapted from Clement (2018)

Parameters	Stratum 1	Stratum 2	Stratum 3
N_h	52	76	82
n'_h	15	20	28
n_h	4	5	7
$\bar{X}_h$	6.813	10.12	7.967
$\bar{Y}_h$	417.33	503.375	3.40
S_{hx}	4.00	11.52	6.20
S_{hy}	273.45	75.56	37.48
S_{hxy}	83.42	143.08	56.682
γ'_h	0.0474	0.0368	0.0235
γ_h	0.2308	0.1868	0.1307
ρ_{hxy}	0.703	0.738	0.805
$\rho_{hx\beta}$	0.86	0.764	0.726
C_{hy}	0.66	0.15	0.11
$C_{h\beta}$	0.53	1.92	1.45
α_y	1.07	0.06	0.05
$\bar{B}_h$	7.49	5.99	4.28
$\beta_{2h}(x)$	22.642	36.385	28.682
$\beta'_{2h}(x)$	32.42	26.385	24.682

Table 2: Performance of estimators

Estimator	Variance	PRE $[\theta, \hat{\bar{Y}}_R]$
$\hat{\bar{Y}}_R$	3349.277	100
$\hat{\bar{Y}}^*_{R,opt}$	449.169	745.66
$\hat{\bar{Y}}^*_{R,1}$	1020.76	328.12
$\hat{\bar{Y}}^*_{R,2}$	16,420.43	20.40
$\hat{\bar{Y}}^*_{R,3}$	22,517.28	14.87
$\hat{\bar{Y}}^*_{R,4}$	6,084.865	55.04
$\hat{\bar{Y}}^*_{R,5}$	7156.996	46.80
$\hat{\bar{Y}}^*_{R,6}$	80,909.972	4.14
$\hat{\bar{Y}}^*_{R,7}$	37,956.320	8.82
$\hat{\bar{Y}}_{REG}$	1370.9815	244.2978

The percent relative efficiency of the estimators with respect to the conventional ratio estimator in stratified double sampling $\left[\hat{\bar{Y}}_R\right]$ is presented in Table 2.

CONCLUSION

This study proposes a class of ratio estimators of mean for calibration estimation under the stratified double sampling using coefficient of kurtosis of auxiliary variable. Some well-known estimators are obtained under certain prescribed conditions and shown to be special members of the proposed class of estimators. Analytical and numerical results clearly showed that the new estimator is more precise and efficient than the conventional ratio and regression estimators of mean in double sampling for stratification by Cochran[30] and all existing modified estimators in stratified double sampling under review with appreciable gains in efficiency at its optimum condition.

It is observed that the new estimator is very attractive and should be preferred in practice as it provides consistent and more precise parameter estimates than existing modified estimators in stratified double sampling.

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