OnLine Journal of Earth Sciences 2 (2): 36-47, 2008
ISSN: 1991-7708

Satellite Altimetry in the Czech Republic: Status 2007

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Abstract: The aim of this presentation is to inform about works in satellite altimetry in the Czech republic during the last 15 years and to provide an outline of the results. The results come from two groups, one is from CEDR (Center for Earth Dynamics Research) and the second is from Military Geographic and Hydrometeorologic Office. We also cooperated with NOAA and NIMA. We worked out-and used repeatedly-a method based on long-term averaged crossover altimetry data (from missions ERS 1, 2, Envisat and Geosat). With these (generated from orbits using tested geopotential models) we computed Latitude Lumped Coefficient (LLC) discrepancies, compared them to the LLC errors projected from the covariance matrices of these models and from the comparison assessed the accuracy of the gravity models. We used single and dual satellite crossover data (SSC and DSC) to refine gravity field models. We also utilized the DSC between low and higher altimetry orbits to detect possible offsets among the terrestrial reference frames of the altimetry missions involved. The potential of geoid W_0 has been derived by means of combining altimetry, levelling, GPS height and gravity data and in turn, an offset between different world height systems has been estimated. We also studied the theory of bistatic satellite altimetry, namely the distribution of reflecting points and the accuracy of their determined surface heights and most recently the determination of the specular reflecting points on the Earth represented not only by a sphere but also an oblate ellipsoid.

Key words: Satellite altimetry, accuracy of the earth gravity field models, single- and dual-satellite crossovers, world height systems, bistatic satellite altimetry

INTRODUCTION

Accuracy assessment of gravity field models by means of crossover altimetry data: It is one thing is to compute an Earth gravity field model-a set of harmonic geopotential coefficients C_{lm}, S_{lm} (also known as Stokes parameters), of degree l and order m. Just as demanding is to estimate their actual accuracy (not only internal precision from the covariance matrix); this becomes more and more difficult as their apparent errors decrease (Klokočník *et al*., 2004; Novák *et al*., 2006). Various methods are in use (Lerch, 1991, Lemoine *et al*., 1998). We have developed and used a method based on crossover satellite altimetry data, very often independent of the global solutions and sufficiently accurate, at least in the pre-GRACE era.

The radial direction is crucial for altimetry. The formulae for the radial, static gravity field induced error as a time series, are well known (Colombo, 1984; Wagner, 1989). But the time series of error does not provide any useful insight into its source on the orbit or in altimeter corrections because direct altimetry is dominated by the much larger uncertainty of the detailed geoid at the surface. That is revealed in differences of altimeter heights at crossovers (eliminating the influence of the geoid error) and (in comparison) displaying the geographical distribution of the radial error due to the geopotential in latitude and longitude. This last is accomplished using (Rosborough's, 1986) transformation (Rosborough and Tapley, 1987) by projecting the variance-covariance matrices of the tested gravity models to yield expected orbit geopotential errors of SSCs (single-satellite crossovers) geo-graphically. But such information, while useful as an overview and used many times, says nothing about the spectral quality of the geopotential errors. Following the natural formulation of spherical harmonics, this ‘Rosborough spectrum’ is most readily displayed by order in terms of so-called Latitude Lumped Coefficients (LLC). They were defined (for SSC)

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in Klokočník and Kobrle (1992) and Klokočník *et al.* (1992) and applied in accuracy tests of a number of global gravity field models: GEM-T2, JGM2, 3, EGM96, TEG4, GRIM5-S1, -C1, POEM GS 01, PGM 2000A, EIGEN-1S, 2 (Wagner and Klokočník, 1994; Klokočník and Wagner, 1994; Wagner *et al.*, 1995a, b; Wagner *et al.*, 1997b; Klokočník *et al.*, 1996, 1998, 1999, 2002, 2004) with an extension to dual-satellite crossovers (Klokočník *et al.*, 2000a). Recently we tested EIGEN-3p (also called 03S), EIGEN-GRACE 02S, TUGRAZ 04, UTCP 03 and EIGEN-GL04C (results not yet published).

Following Klokočník and Kobrle (1992) and Klokočník *et al.* (1992), for the SSCs, we may write

$$\Delta X(\phi,\lambda)=\sum_{m=1}^{m=m_{max}}\begin{bmatrix}\delta C_m(\phi,a,I)\sin m\lambda\\ +\delta S_m(\phi,a,I)\cos m\lambda\end{bmatrix} \quad (1)$$

Where, $\delta C_m(\phi,\alpha,I)$, $\delta S_m(\phi,\alpha,I)$ are corrections to the LLCs for crossovers due to the full geopotential:

$$C_m=\sum_{l=m}^{l=l_{max}}2Q_{lm}C_{lm} \qquad S_m=-\sum_{l=m}^{l=l_{max}}2Q_{lm}S_{lm} \quad (2)$$

Where, I is the orbital inclination, a the semimajor axis of the satellite orbit and the Qs are the resulting influence functions (Rosborough, 1986) in metric units and ϕ is geocentric latitude. The harmonic coefficients C_{lm}, S_{lm} and the influence functions Q_{lm} (I,a,ϕ) are fully normalized.

Latitude Lumped Coefficients (LLC), based on long-term averaged SSC data, can be (and were) used to test many global gravity models (namely their lower degree and order portion), as described in detail in Klokočník *et al.* (2000b, 2002). A short review of the method is available as a poster on request (Klokočník *et al.*, 2006).

Our method consists of the following three steps:

Step 1: We use observed SSCs after all available altimeter corrections, then convert them to observed Latitude Lumped Coefficient (LLC) discrepancies according to Eq. 1 by a least squares adjust-ment (with some constraints on land areas without altimetry data).

Step 2: We compute the corresponding LLC errors for the geopotential used for the observed orbit from its variance-covariance matrix of spherical harmonic coefficients (usually tentatively scaled or already calibrated):

$$\begin{aligned}\sigma^2_{C_m}&=4\sum_{l_1=m}^{l_{max}}\sum_{l_2=m}^{l_{max}}Q^S_{l_1m}\,Q^S_{l_2m}\,\mathrm{COVAR}\left(C_{l_1m},C_{l_2m}\right),\\ \sigma^2_{S_m}&=4\sum_{l_1=m}^{l_{max}}\sum_{l_2=m}^{l_{max}}Q^S_{l_1m}\,Q^S_{l_2m}\,\mathrm{COVAR}\left(S_{l_1m},S_{l_2m}\right).\end{aligned} \quad (3)$$

Step 3: Finally, we compare results of STEP 1 and STEP 2 in a statistical way since STEP 1 yields actual discrepancies while STEP 2 only expected squared errors. We can do this comparison of LLCs (observed/computed) for each latitude band but normally we summarize the result by geopotential order for all bands. This comparison either confirms the (tentative) scale/-calibration factor of the covariance matrix tested or leads to suggestions for its change.

One example is in Fig. 1a-c, namely STEP 1 (upper left), STEP 2 (upper right) and STEP 3 (below). Recent testing of the newest gravity models based solely or dominantly on the CHAMP/GRACE data revealed a decrease of the accuracy at few lowest orders, namely at order m = 2 (Kostelecký *et al.*, 2006). Figure 2 shows more about these new results.

Is satellite altimetry precise enough to test the newest gravity models with extensive sets of the CHAMP data? Radial orbit improvement with the CHAMP data is remarkable-from about 1 meter with GRIM5S1 to several centimeters with EIGEN 2, 3p and to subcentimeter level with the newest EIGEN 04, for the orbit of CHAMP.

The radial error may decrease also for the other orbits with inclinations and semimajor axes different from those of CHAMP. For the ERS orbit type, SSC error was about 4 cm with GRIM5S1 and is about 3 cm with EIGEN 1S, for Geosat ~11/8 cm, respectively. Let us consider a signal to noise (s/n) ratio, where s means the SSC errors from the covariance projections and n means the SSC data errors. Due to the long-term averaging of SSC (sometimes several years), we are able to keep n below 1 cm, for the best ERS1/2, Envisat and Geosat data available in 2003, n is about 0.6/0.9 cm. Thus, s/n was and still is high enough to have our test statistically significant.

Is satellite altimetry precise enough to test the newest gravity models with extensive sets of the GRACE data? Gravity field models with the GRACE data are already more precise than satellite crossover altimetry. However, common trends appear in the results of testing with the LLCs for CHAMP and GRACE based models, Fig. 2. It means that also for the GRACE based models there is a problem of the accuracy at lowest orders of harmonics.

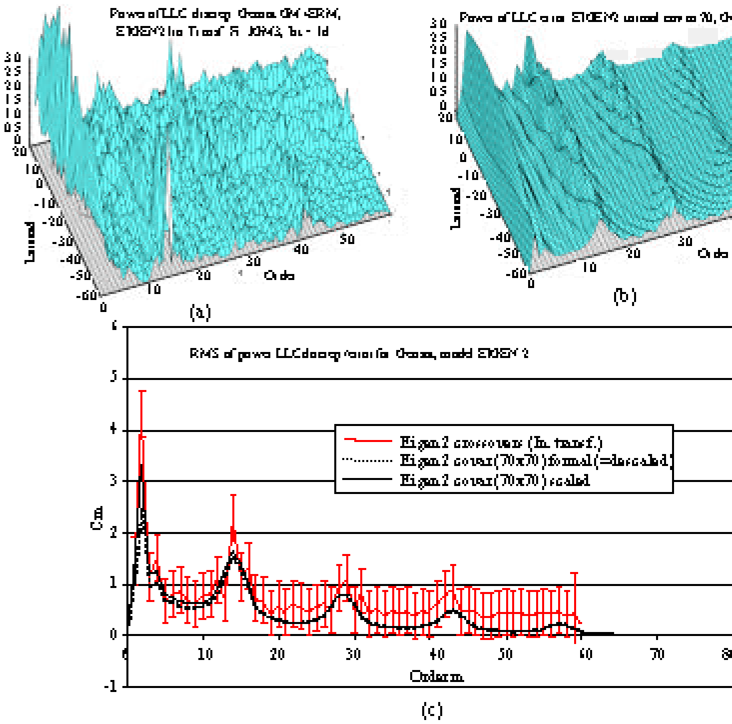

Fig. 1 a-c: Example of application of accuracy assessment of a gravity model (EIGEN 2) using SSC (Geosat) with LLC, for more details see Eq. 1-3. (Klokočník *et al.*, 2002)

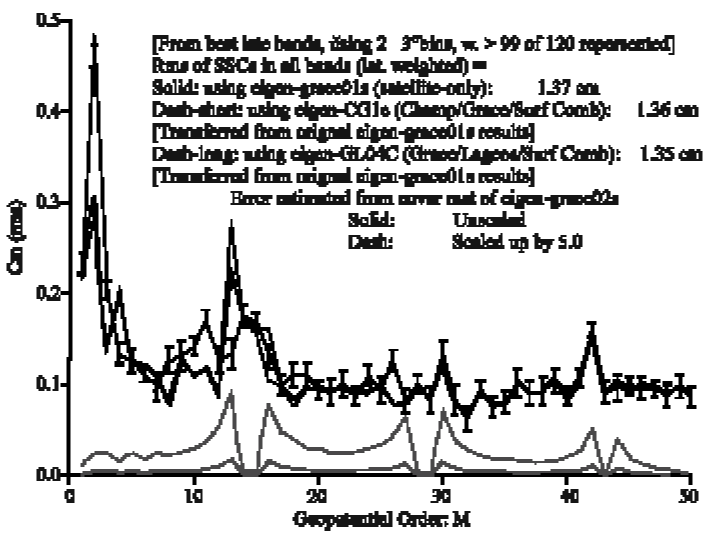

Fig. 2: Measured and projected errors in LLCs from Envisat SSC with three GRACE models. First model (EIGEN-GRACE01S) is based only on the GRACE data, the second model (EIGEN-CG01C) is a combination solution from CHAMP, GRACE and surface data and the third model (EIGEN-GL04C) uses GRACE, LAGEOS and surface data. The LLC errors projected from the covariance matrix of EIGEN-GRACE02S (a model similar to other models used) are shown in grey, first directly from the formal covariance matrix, second from that matrix scaled by factor 5. We see a significant discrepacy between the smaller projected LLC errors from covariances and the larger ones derived from observed altimetric SSCs, namely for the lowest orders. The problem is still open

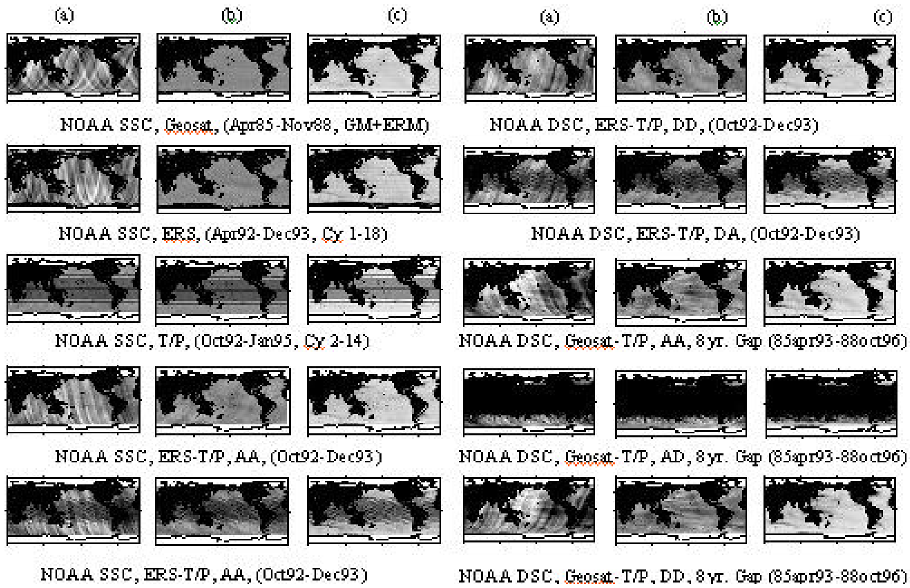

Fig. 3: Illustration of inversion using SSC and DSC from ERS 1, T/P and Geosat (Wagner *et al*., 2000) to try to refine older JGM 3. Columns (a) contain the data, i.e., SSC or DSC, columns (b) show their precision and columns (c) depict residuals after the inversion. Reader can found in (Wagner *et al*., 2000) the series of color figures for all SSC and combinations of DSC used in that analysis

SINGLE AND DUAL SATELLITE CROSSOVERS DATA (SSC and DSC) TO REFINE GRAVITY FIELD MODELS

In our older experiments, we were interested in the role of SSC and DSC data, independent of existing gravity models, in determination of C_{lm}, S_{lm}. We took JGM 3 model (now very old) and added SSC and DSC between T/P, ERS 1 and Geosat. We tried to refine JGM 3 (Wagner *et al*., 2000). The experiment can be repeated with new CHAMP/GRACE-only models, provided that we would have excellent SSC/DSC, corrected for all accessible environmental error say from JASON, ERS-2 or ENVISAT. Then, we can expect detectable changes in values of some of lower degree and order harmonic coefficients (Fig. 3).

Moreover, in Wagner *et al*. (2000), we explain (we think for the first time) the case of statistical expectation for semi-independent solutions in LSE, taking fully independent and completely dependent solutions as special cases.

USE OF DSC BETWEEN LOW AND HIGH ORBITS TO DETECT OFFSET AMONG TERRESTRIAL FRAMES BETWEEN ALTIMETRY MISSIONS

There is one type (AD) of the Single-Satellite Crossovers (SSC) between the Ascending (A) and Descending (D) track of the given orbit, just four types (AD, DA, AA and DD) of the Dual-Satellite Crossovers (DSC) between the relevant tracks of a pair of satellite orbits and just twelve their combinations (AA+AD, AA-AD, AA+DA, AA-DA, DD+DA, DD-DA, DD+AD, DD-AD, AD+DA, AD-DA, DD+AA and DD-AA), taking the DSC in pairs (Klokočník and Wagner, 1999). Each quantity has different sensitivity to the gravity field parameters and in turn, also different gravity induced error (e.g., Klokočník *et al*., 1993, 1995, 1999, Wagner *et al*., 1997b). In the first use of DSC, Shum *et al*. (1990) intended to surrogate SSC of the lower orbit by DSC between lower and higher satellite (but they used only the AD type). More generally, there are very interesting properties of some of the combinations mentioned above, which might be used as a diagnostic tool in the study of residual error effects in environmental corrections of altimetry data.

We used DSC of the lowest degrees to test possible offset between coordinate systems of the higher, more precise altimetry mission with better determined orbit (like TOPEX-/Poseidon) and a lower flying, less accurate mission (like Geosat) (e.g., Bosch *et al*., 1998). This task can be repeated for any pair of altimetry missions (like Jason and Envisat or GFO). The DSCs and the offset detection and removal is useful to interconnect series of SSCs from different (desparate) missions even with time gaps (like T/P-Geosat).

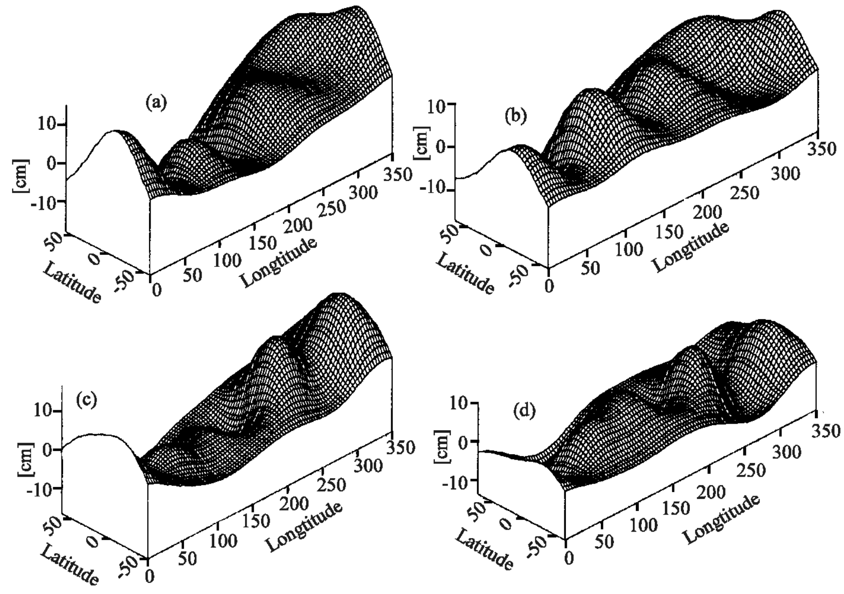

Fig. 4: The example of applications of DSC from two altimetry missions (Geosat and TOPEX/Poseidon) to find relative coordinate frame offset of the lower and less accurate with respect to the higher and more precise (taking as errorless) mission (here Geosat vs T/P, model JGM3, NOAA data. a) AA duals before removing geocenter shift, b) AA duals after removing geocenter shift, c) DD duals before removing geocenter shift, d) DD duals after removing geocenter shift. More in (Wagner *et al.*, 1997a, b; Kostelecký and Klokočník, 1999)

Here we recall our results from Wagner *et al.* (1997a,b) and we show one of examples of our analysis (1997a) on Fig. 4a-d.

For the DSC, type AA we have

$$\Delta X_{12}^{AA:l=1,m=1} = \left({}^{1,2}dQ_{11}^{C}S_{11} + {}^{1,2}dQ_{11}^{S}C_{11}\right) \sin\lambda + \left({}^{1,2}dQ_{11}^{C}C_{11} + {}^{1,2}dQ_{11}^{S}S_{11}\right)\cos\lambda \qquad (4)$$

which can be transformed to the well known formula for the offset between two coordinate systems:

$$-\Delta h = \Delta X_{12.(rel)}^{m=0,1} = \begin{bmatrix} dx\cos\phi\cos\lambda + \\ dy\cos\phi\sin\lambda + dz\sin\phi \end{bmatrix} + [1 \text{ cpr corrections}] \qquad (5)$$

One example of our analysis with TOPEX and Geosat is shown in Figs. 4a-d and the following values for the offset were found $dx = +1.0\pm0.1$, $dy = -7.0 \pm0.2$.

DSC residuals and their combinations are based on observations, thus they consist of various gravitational and non-gravitational effects. The non-gravitational parts of the DSC comes from the offset, but also from the remaining errors after the altimetry corrections are applied, including empirical one like 1 cpr and from residual errors of tidal models, too. Unknown amount of the DSC signal suffers from an aliasing between the non-gravitational and gravitational signal. Finally, there is also white data noise.

As an analogy of gravity-induced geographically correlated (not depending on sense of track and time) and variable (anticorrelated) portions of radial component or error, we may introduce non-gravitational correlated and variable components. We will call non-gravitational correlated error such an error of non-gravitational origin which does not depend on sense of track and is (quasi)-constant over some long-term interval. The variable (anticorrelated) error of non-gravitational origin will be such an error which varies with track and time. Fast variables within orbit arc or during the period used to compute the crossovers should be averaged out, slower variables will have some effect on the crossover residuum.

The coordinate frame offset between two missions is typical example of geographically correlated non-

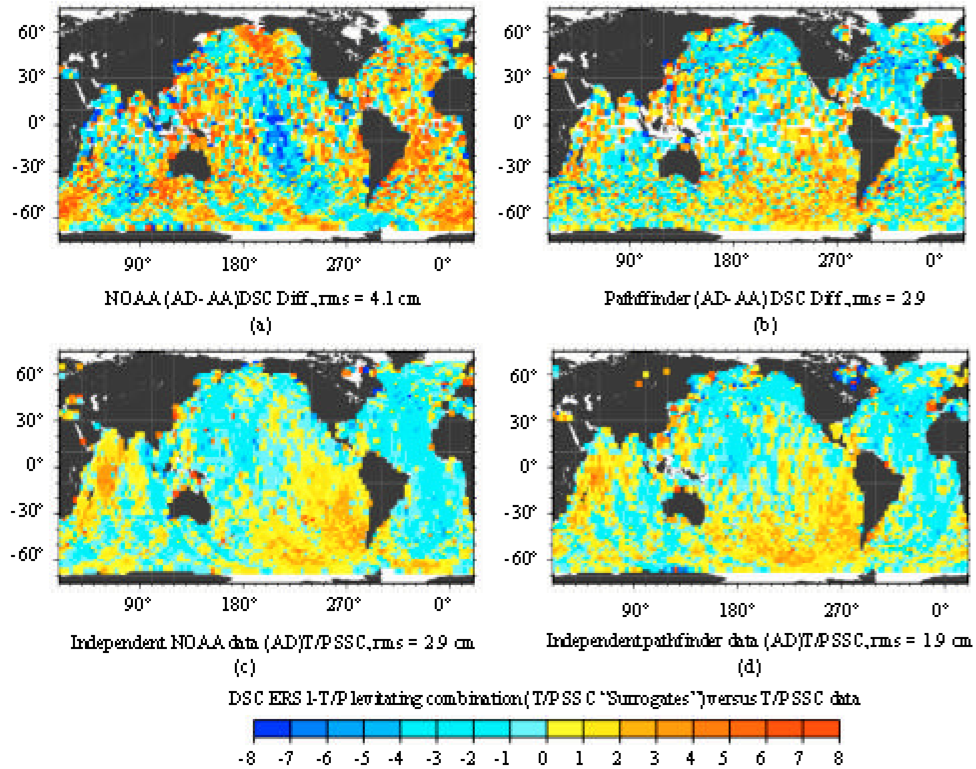

Fig. 5: The „ levitating" combinations (AD-AA) of ERS 1 and T/P, where the crossovers for ERS 1 are based on DGM E04 gravity model (Delft) and the crossovers for T/P on JGM 3 (NOAA). One source of data is NOAA (Fig. 5a) upper left, another is from Pathfinder NASA (Fig. 5b), upper right. The combinations (Fig. 5 a and b) provide a T/P ‚surrogate'. They compare to the T/P SSC ‚originals' (Fig. 5c and d; bottom). These figures support the hypothesis that the NOAA data of T/P suffered from an inconsistency, probably in media corrections, recommended to NOAA for T/P prior to 1997 (in contrast with those used later)

gravitational phenomenon. It does not depend on type of the DSCs (i.e. on sense of satellite's track and on time). Also mean sea surface would belong to this category. Media corrections are typical variable components as they differ for the crossovers between the tracks. The long-term 'month-to-month' averaging is effective mostly for seasonal effects. Long-term effects (like a quasi-permanent part of the ocean currents and partly ENSO effects) cannot be averaged by this way and a part of them remains, in our terminology, as geographically correlated non-gravitational signal.

More about combinations of DSC and their applications is in Klokočník *et al*. (1999, 2000a), Klokočník and Wagner (1999) and Wagner *et al*. (1997b). They are found useful for error analysis in conjuction with applications of DSC in geodesy and oceanography. One kind of combination, when applied to the dual missions in a high-low orbit configuration, approximates the dominant mean part of the total radial perturbations of the lower orbit. Another kind yields no geopotential error and can help elucidate oceanographic changes between passes, among other error of altimetry corrections. A third kind is a surrogate of SSC data and can be used to check the significance of the background (non-geopotential) errors that are not common to the 2 kinds of crossovers. The levitating combinations (with zero gravity effect), namely (DD-DA) and (AD-AA) of ERS1 and T/P and their comparison with SSC of T/P are shown on Fig. 5a-c, taken from Klokočník *et al*. (2000a), sect 3.5.

GEOPOTENTIAL AND DIFFERENCES AMONG HEIGHT SYSTEMS

Here we note on use of satellite altimetry for determination of gravity potential of the geoid and definition of the world height systems. We start with formula for the gravity potential in spherical coordinates

Table 1: Mean (1993-1996) geopotential values over oceans and difference Δh between the mean sea levels related to $W = W_0 = 62\ 636\ 855.611\ m^2s^{-2}$

Ocean	Number of sites	$\bar{W}_{EGM96}$ $[m^2s^{-2}]$	$\bar{W}_{EGM96}$ $[m^2s^{-2}]$	$\Delta\bar{h}$ [m]	$\Delta\bar{W}$ $[m^2s^{-2}]$	$\delta\bar{W}$ $[m^2s^{-2}]$
Pacific (P)	388 237	62 636 854.102±0.010	-1.509	0.153±0.001	-1.349	0.160±0.003
Atlantic (A)	182 787	62 636 858.201±0.014	2.590	-0.263±0.001	2.323	-0.267±0.005
Indian (I)	156 175	62 636 856.275±0.020	0.664	-0.065±0.002	0.647	-0.017±0.006
P+A+I	727 199	62 636 855.625±0.008	0.014	-0.001±0.001	0.0240	0.010±00002

Taken from Burša *et al.* (1999)

$$W(r,\varphi,\lambda) = \frac{GM}{r}\sum_{l=2}^{\infty}\sum_{m=0}^{l}\frac{a_e^{\,l}}{r^l}\begin{pmatrix}C_{lm}\cos m\lambda\\ +S_{lm}\sin m\lambda\end{pmatrix} P_{lm}(\sin\varphi) + \frac{1}{2}\omega^2 r^2\cos^2\varphi = f(r,\varphi,\lambda;GM,\omega,C_{lm},S_{lm}) \quad (6)$$

where, r, φ, λare spherical coordinates of the point, GM geocentric gravity constant, C_{lm}, S_{lm} fully normalized Stokes' parameters (harmonic coefficients) of the Earth's gravity field, P_{lm} (sin φ) are fully normalized Lagrange associated functions and ω is rotational velocity of the Earth.

For the potential we can write

$$W_{geoid}(r_{geoid},\varphi,\lambda) = f\begin{pmatrix}r_{geoid},\varphi,\lambda;\\ GM,\omega,C_{lm},S_{lm}\end{pmatrix} \quad (7)$$

and 'altimetry equation' yields

$$r_{geoid}(\varphi,\lambda) = r_s(\varphi,\lambda) - \Delta_1 - h(\varphi,\lambda) - \Delta_2 \quad (8)$$

where, $r_s(\varphi,\ \lambda)$ is the geocentric height of altimetric satellite determined from precise orbit determination, Δ_1 includes calibration, tidal and SSH corrections, h(φ, λ) is altimetric measu-rement and $Ä_2$ is a „geometric correction" (altimetric measurement is directed to plumb line, we use spherical coordinates).

Theoretically, one point with altimetry data is enough to obtain a solution for W_{geoid}, but in practice we have available many sites for the solutions. The values of W_{geoid}, based on Eq. (7) and (8) and the reference gravity field model EGM96, were determined as a mean value of individual point solutions for particular oceans and then for the whole Earth. If Eq. (7) and (8) are used separately for different oceans, then we can determine differences between the mean gravity potential $W_{geoid} = W_0$ and W_{ocean} and the mean height differences Äh between individual oceans-see Table 1, taken from (Burša *et al*., 1999). The T/P data in time interval 1993-1996 was used. The mean value determined by Burša *et al*. (1998) is:

Table 2: Differences of geopotential at several tide gauge stations and corresponding vertical shifts ΔH_i (reproduced from Burša *et al*., 2001)

LVD i	k	ΔW_{ik} $[m^2s^{-2}]$	ΔH_{ik} [cm]
AHD	NAVD 88	+9.68±0.51	-97.4±5.1
NAP	NAVD 88	+4.3±0.29	-44.2±2.8
KHD	NAVD 88	+4.38±0.14	-43.6±1.3
N60	NAVD 88	+5.68±0.65	-56.9±6.6
NAP	AHD	-5.25±0.57	+53.2±5.8
KHD	AHD	-5.30±0.51	+53.8±5.3
N60	AHD	-4.00±0.82	+40.5±8.3
KHD	NAP	-0.05±0.30	+0.6±3.1
N60	NAP	+1.25±0.71	-12.7±7.2
N60	KHD	+1.30±0.66	-1.33±6.7

$$W_{geoid} = (62\ 636\ 855.611 \pm 0.5)\ m^2 s^{-2}.$$

"Altimetric" gravity potential of the geoid W_{geoid} is a fundament for determination of the "common world height system" if we use additional results from "GPS/levelling". Principle of this method is obvious from Fig. 6.

For a terrestrial point P the geopotential W_P is Burša *et al.* (2001)

$$W_P(h_P,\varphi_P,\lambda_P) = f\begin{pmatrix}h_P,\varphi_P,\lambda_P;\\ GM,\omega,C_{lm},S_{lm}\end{pmatrix} \quad (9)$$

Where, ellipsoidal coordinates are determined by GPS positioning. The value of W_{origin} is computed by gravimetry and by levelling from W_P.

For the height anomaly ÄH of individual height system the following equation is valid

$$\Delta H = \frac{W_{geoid} - W_{origin}}{\gamma} \quad (10)$$

where, γ is mean value of gravity. Results from Burša *et al.* (2001, 2004) are here presented in Table 2 and 3. The differences between the 5 basic world height systems are in Table 2 and have accuracy better than 0.1 m.

If we know „sea level height" $h_{P(levelling)}$ of the point P, geometric access to solve this problem is possible. From Fig. 6 we have

$$\Delta H = h_P - h_{P(levelling)} - h_{P(geoid)} \quad (11)$$

Table 3: Geopotential values at local vertical datums LVD. The quantity äH$_{0i}$ is the vertical shift of the LVD origin, related to the reference surface W-W $_{0}$ (Burša *et al*., 2004)

Country	LVDi	Number of GPSLS	EGM96R [cm]	W_{0i} [m^2s^{-2}]	W_{0i}-W_0 [m^2s^{-2}]	δH_{0i} [m]
Argentina	La Plata	32	24	62 636 852.73±1.49	-3.27±1.40	-0.33±0.14
Belgium	Oostende	40	19.4	62 636 881.66±1.99	+25.66±0.85	+0.02±0.09
Baltic region	N60	25	5.5	626 36 855.83±0.99	-0.17±0.85	+0.02±0.09
Czech	KHD	175	15.0	62 636 857.40±1.57	+1.40±1.49	-0.14±0.15
Republic	Kronstadt					
Federal rep. Of NAP						
	Amsterdam					
Germany	KHD	179	6.4	62 636 857.51±0.83	+1.51±0.66	-0.14±0.15
	Kronstadt					
Hungary	NWP-3	299	15.0	62 636 858.48±1.56	+2.48±1.48	-0.25±0.15
	Jakarta					
Indonesia	NAP	12	15.0	62 636 846.63±2.07	-9.37±2.01	+0.95±0.20
	Amsterdam				+1.35±1.56	
Netherlands	KHD	11	7.2	62 636 857.09±1.13	+1.09±1.01	-0.11±0.10
	Kronstadt					
Poland	NAP	91	3.0	62 636 857.08±0.65	+1.08±0.41	-0.11±0.04
	Amsterdam					
Scandinavia	KHD	54	20.5	62 636 856.80±2.17	+0.80±2.11	0.08±0.21
	Montevideo					
Uruguay	1948.0	10	7.0	62 636 855.19±2.14	-0.81±2.09	+0.08±0.21
Venezuela	La Guaira	16	3.0	62 636 853.95±1.80	-2.05±1.73	+0.21±0.18

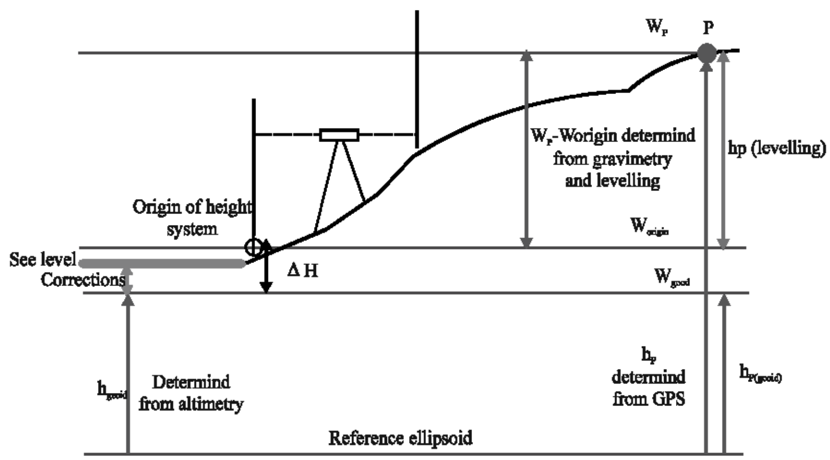

Fig. 6: Value of ÄH is estimated by two ways (dynamical and geometrical), as was described by Eq. 6-8

where, $h_{P(geoid)}$ is the ellipsoidal height of geoid in P. It is determined by inversion of Eq. (6) for r_{geoid}. If detailed gravimetric data is at disposal, then the gravity potential, Eq. (6), can be completed by additional potential T.

AHD-Australian Vertical Datum, NAP-Normal Amsterdamsch Peil, KHD-Kronstadt Height Datum, NAVD88-North American Vertical Datum 1988, N60-Finnish Height Datum

THEORETICAL INVESTIGATION OF BISTATIC SATELLITE ALTIMETRY

Principle of bistatic satellite altimetry: Instead of one satellite, two satellites cooperate and sender S_2 sends signal to receiver S_1 directly (d_{12}) and indirectly, the indirect signal being reflected off the ocean surface ($d_1 + d_2$), Fig. 7. The observed quantity in Bistatic Altimetry (BA) is the time or distance difference between those two (s/c "bistatic radar value").

The method was suggested by (Martin-Neira, 1993). It was tested and is used on the ground at lakes or from aircrafts (e.g., Treuhaft *et al*., 2001; Lowe *et al*, 2002). The most precise GPS lake altimetry measurements from a fixed site receiver retrieved lake surface height with 2 cm precision (Zuffada *et al*., 2002, 2004). We have to mention the short space technology experiment on the SLR-2 Space Shuttle mission (in 1994); it recorded about 15 hours of BA data from GPS, gathered in the high resolution mode. Technology experiments on CHAMP and SAC-C were also performed, but scientifically useful

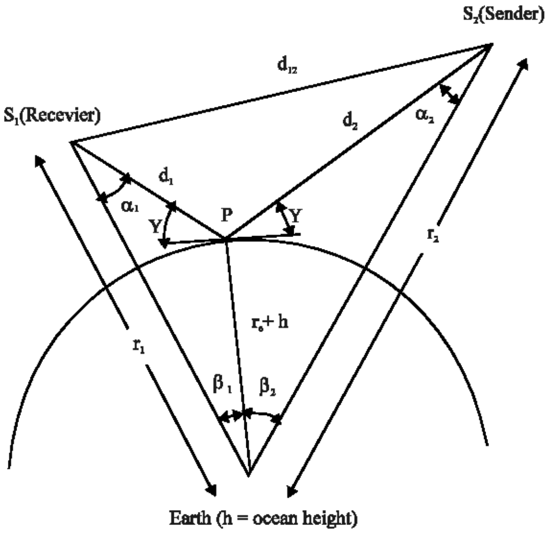

Fig. 7: The concept of satellite bistatic altimetry

space data is not yet available. A dedicated bistatic altimetry mission is highly required (e.g., Komjathy *et al.*, 1999; Le Traon *et al.*, 2002, 2003; Ruffini *et al.*, 1999, 2000; Wiehl *et al.*, 2003; Zuffada *et al.*, 2002, 2004).

We studied geometry and accuracy of the BA concept assuming specular reflecting points on the ocean/sea surface (Wagner and Klokočník, 2003; Kostelecký *et al.*, 2005).

Geometry: In conventional satellite altimetry all orbits have been in repeat tracks with cycle periods less than about 1 month since Seasat except for the Geodetic Missions (GM). But these GM were specifically designed to generate detailed mean sea surfaces, not to monitor ocean dynamics (e.g., Lemoine *et al.*, 1998). Long term simulations show that complementing a traditional satellite-altimeter (short cycle) ERM with the capability of bistatic altimetry will yield a dense pattern of returns (spatially and temporally) that could be used for either geodetic or oceanographic purposes.

For example, we computed number and distribution of reflecting points for 24 GPS satellite and CHAMP or SAC-C for 180 day intervals of BA observations with given reasonable rate of observations. We found several times higher number of BA points than would belong to traditional nadir radar altimetry. We have also one hypothetical example for 24 GPS-T/P configuration, although T/P is not equipped for BA (Kostelecký *et al.*, 2005). We wanted to show the geographic distribution of the reflecting points; noting the distribution of reflection tracks in specific 1 degree bin in the North Atlantic which (being close to the northern edge of the Gulf Stream) may show strong eddy and ring activity. This area is not

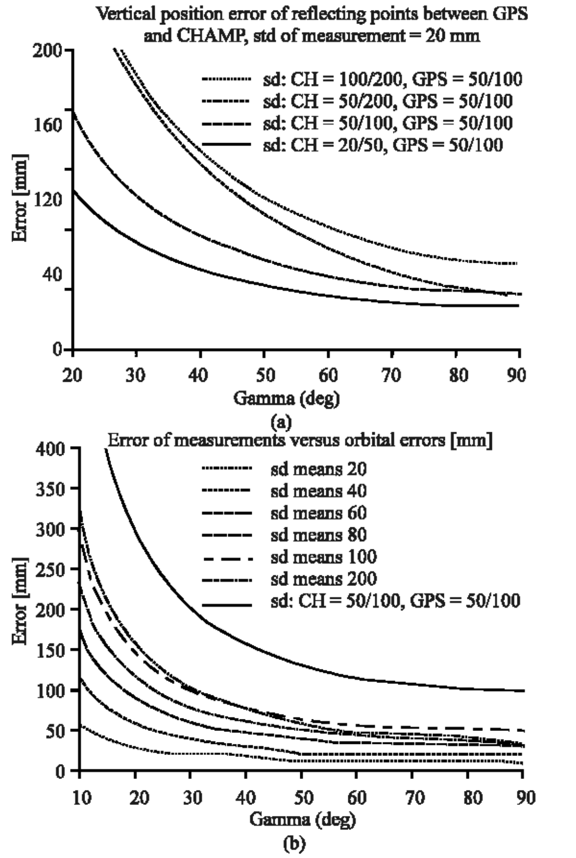

Fig. 8 : Accuracy estimates of BA concept accounting for the measurement and orbit errors. The angle gamma is defined in Fig. 7 and 8a on left: Various orbit errors of CHAMP and a constant orbit error of GPS satellites and measurement error, (8b) or right: Various measurement errors for fixed orbit errors

covered at all in the conventional T/P mission. With reflection tracks from all 24 GPS satellites, downward-looking receivers in a hypothetical "T/P" orbit would pick up a remarkably dense amount of sea height information. The key to the power of bistatic altimetry is the dispersion of the returns away from the lower-satellite track which fills in the limited coverage of the conventional mission. This dispersion is a function of both the altitude of the receiver orbit and the minimum reflection horizon angle permitted.

Accuracy: For the accuracy estimates, we consider not only measurement errors (in d_{12}) but also orbit errors (in the state vectors of S_1, S_2) and we ask how large error can be expected in the vertical (radial) position of a theoretical reflecting point P in a ‚model situation‘ (on spherical Earth).

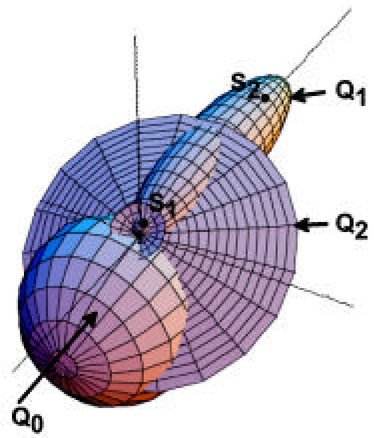

Fig. 9: Illustration to one method of solving position of reflecting point on rotational ellipsoid representing the Earth (Olivík *et al*., 2006), using three quadrics: the quadric Q_1 with foci in S_1 and S_2, the straight circular cone Q_2 with vertex in S_1 and the quadric Q_0 as a reference geocentric ellipsoid of the earth

Numerical results (Kostelecký *et al*., 2005), here in Fig. 8 a and b, were computed with this data:

- GPS orbit error is min/max = 50/200 mm radially, 100/500 mm cross-radial (Mader, priv. commun. 2002; materials of Interntl. GPS Service, 2000).
- CHAMP orbit error is min/max = 20/100 mm, radially, 50/200 mm cross-radial (Reigber, priv. commun. 2002).
- Delay measurement errors min/max = 20/200 mm (minimum estimate based on GPS-fixed ground antennae experiment at 500 m over Crater Lake, Truehaft *et al*., 2001; this figure may be optimistic for a receiving spacecraft in motion at 400km altitude [CHAMP]). The range from 200-20 mm expresses our wishes for the future accuracy level.

Obviously the accuracy degrades slowly with the off-nadir angle to about 50 degrees, then rapidly (Fig. 8 a). Thus, we can expect many BA points off-nadir with promising accuracy, comparable to that of the traditional nadir altimetry. The orbit and the measurement errors compete in the total error budget of the reflecting point position as is shown in Fig. 8 b.

Position of reflecting points on ocean: The spherical approximation of the reflecting surface (the ocean) is sufficient for geometry and accuracy reatment, but in reality we need to compute position of the reflecting point on geoid. Olivík *et al*. (2006) have developed three methods that are able to do it under different assumptions (one example in Fig. 9).

ACKNOWLEDGEMENT

Close, intensive and extremely fruitful cooperation with Carl A. Wagner and others from NOAA, with R. Scharroo (Delft at that time) and Ch. Reigber, P. Schwintzer and Ch. Foerste (GFZ) is highly acknowledged. This investigation has partly been done in frame of the grant # A30074 from GA AV of the Czech Republic and supported by the project LC 506 (CEDR) by Ministry of Education, Youth and Sports of the Czech Republic.

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