Pakistan Journal of Social Science, 3 (8): 1060-1064 , 2005

Study the Differentials, Patterns, Trends and Mathematical Modeling of Expectation of Life (E_x) of Bangladesh

Md. Rafiqul Islam
Department of Population Science and Human Resource Development, Rajshahi University, Bangladesh

Abstract: The purpose of the present study is to study the differentials, patterns and trends of expectation of life (e_x) of Bangladesh during 1961-1991. For this, the expectation of life (e_x) of Bangladesh in the census years 1961, 1974, 1981 and 1991 have been used in this study. A few mathematical models have been fitted to e_x and $f(x)=x+e_x$ only for male of Bangladesh in 1961 and it is found that they follow polynomial models. The Cross-Validity Prediction Power (CVPP) and F-test are also applied to these models to check the validity and significance of the models.

Key words: Expectation of life (e_x), mathematical modeling, coefficient of determination (R^2), cross-validity prediction power (CVPP), F-test

INTRODUCTION

Bangladesh is one of the most developing countries in the world. So various types of difficulties like other developing countries are also do here. One of them is expectation of life. The expectation of life of the peoples of Bangladesh are still very low like others developing regions than developed regions in the world. The expectation of life is actually the end product of a life table. This is also the summary description of a life table. The expectation of life may transmute from country to country, developed country to less developed country, sub region to sub region of a country, religious to religious, occupation to occupation, sex to sex, age groups to age groups and time to time. In general, it increases in some ages and then started to decrease up to last age group. However, it is observed that the expectation of life at 1 year of age is higher than that at birth. It is due to the reason that the risk of dying is extremely high in the first year of life and once the age of 1 year is attained the chance of survival increases. So, the main aims and objectives of this study are:

- To study the sex differentials of life expectancy at different ages in the censuses and observe the trends and patterns for male and female during 1961- 1991 and
- To fit some mathematical models to e_x and $f(x)=x+e_x$ for male of Bangladesh in 1961and apply Cross-Validity Prediction Power (CVPP) to check the validity of the models.

MATERIAL AND METHODS

In the present study e_x values have been taken from Islam[1] in which eight abridged life tables constructed by Widowhod method using marital status information of the censuses of 1961[2], 1974[3],1981[4]and 1991[5]. These e_x values are demonstrated in Table 1.

In this study the meaning of expectation of life at age x (e_x) is the average number of years of a survivor to age x is expected to live. It is given by $e_x = \frac{T_x}{l_x}$; where T_x is the total person-years lived at age x and l_x is the number of persons surviving at an exact age x[6].

The function f(x) is equal to $x+e_x$ where x is age group in years and e_x in years is life expectancy at age x.

The expectation of life at age x (e_x) for male of Bangladesh in 1961 have been plotted in Fig. 7 in which it appears that e_x can be fitted by polynomial model for different ages. Therefore, an nth degree polynomial model is treated and the model of the nth degree polynomial function is

$$y = a_0 + \sum_{i=1}^{n} a_i x^i + u$$

where, x is age group; y is e_x; a_0 is the constant; a_i is the coefficient of x^i (i =1, 2, 3, ..., n) and u is the error term of the model. Here we have to select a suitable n so that the error sum of square is minimum.

The $f(x)=x+e_x$ for male of Bangladesh in 1961 (Fig. 8). It seems that f(x) can be fitted by polynomial model for different ages. Therefore, an nth degree polynomial model is treated and the form of the model is

$$y = a_0 + \sum_{i=1}^{n} a_i x^i + u$$

where, x is age group; y is f(x); a_0 is the constant; a_i is the coefficient of x^i (i =1, 2, 3, ..., n) and u is the error term of the model. Here n has been chosen for which the error sum of square is minimum.

Table 1: The Expectation of life (e_x) for male and female of bangladesh in the census years 1961, 1974, 1981 and 1991

Age Group	1961		1974		1981		1991	
x	Male	Female	Male	Female	Male	Female	Male	Female
0	43.42518	43.75311	45.15313	45.80202	48.33692	49.81527	55.13	55.83656
1	50.99708	50.51651	53.91502	54.10124	55.06998	56.21018	59.62	61.34717
2	53.64232	53.2301	56.42797	56.70088	57.00176	58.20782	60.57	62.58348
3	54.65610	54.35299	57.30306	57.65675	57.57019	58.83692	60.60	62.74302
4	54.90577	54.69111	57.44855	57.85967	57.53177	58.84116	59.65	62.44332
5	54.73412	54.58551	57.20167	57.65403	57.17022	58.5041	59.06	61.90346
10	51.41084	51.3987	53.71899	54.25089	53.48003	54.8531	54.95	57.84568
15	47.05134	47.11436	49.29753	49.8778	48.99051	50.38666	50.31	53.22194
20	42.73012	43.06678	44.91232	45.70573	44.54143	46.08858	45.71	48.70729
25	38.45408	39.12691	40.56737	41.6209	40.13022	41.86779	41.15	44.2451
30	34.23952	35.15237	36.27837	37.50827	35.7744	37.63246	36.63	39.78558
35	30.10443	31.21996	32.06113	33.43393	31.48515	33.44345	32.16	35.37226
40	26.10834	27.29189	27.97157	29.36979	27.32583	29.28192	27.82	31.00417
45	22.3143	23.39351	24.06867	25.34313	23.36027	25.17633	23.66	26.71385
50	18.76533	19.5931	20.39195	21.4125	19.63775	21.18141	19.77	22.54421
55	15.55748	16.09353	17.02981	17.75881	16.26108	17.47008	16.25	18.64317
60	12.63539	12.97313	13.93288	14.45024	13.19187	14.1179	13.12	15.08439
65	10.15187	10.31242	11.23996	11.55918	10.57707	11.21045	10.56	11.95994
70	8.052094	8.018461	8.900003	9.002796	8.355919	8.680908	8.51	9.229112
75	6.336097	6.12519	6.912444	6.811502	6.525223	6.56004	7.06	6.916395
80+	2.500042	2.5	2.5	2.500026	2.500032	2.500026	2.50	2.500018

Table 2: Information on model fitting and estimated cross validity prediction power (ρ^2_{cv}) of the predicted equations of expectation of life (e_x) and f(x)=x+ e_x for male of Bangladesh in 1961

Models	n	k	R^2	ρ^2_{cv}	Shrinkage	Parameters	p-value
1	21	3	0.98322	0.975441	0.007779		
						a_0	0.0000
						a_2	0.00001
						a_3	0.004855
						a_4	0.05428
2	21	4	0.95320	0.922371	0.030829		
						a_0	0.0000
						a_1	0.000071
						a_2	0.00095
						a_3	0.002040
						a_4	0.00586

Using the software STATISTICA, the mathematical models have been estimated. The results on model fitting shown in Table 2.

Model validation: The cross validity prediction power (CVPT), ρ^2_{cv} is applied to check how much those models are stable over the population. Here

$$\rho^2_{cv} = 1 - \frac{(n-1)(n-2)(n+1)}{n(n-k-1)(n-k-2)}(1-R^2)$$

where, n is the sample size or number of cases, k is the number of predictors in the model and the cross validated R is the correlation between observed and predicted values of the dependent variable[8]. The estimated CVPP, ρ^2_{cv}, corresponding to their R^2 is launched in Table 2.

F-test: The formula for F-test is given below:

$$F = \frac{R^2/(k-1)}{(1-R^2)/(n-k)}$$

where, n is the number of cases, k is the number of parameters to be estimated and R^2 is the coefficient of determination in the model[9].

RESULTS AND DISCUSSION

The 2nd and 3rd columns of Table 1 and Fig. 1 shows the expectation of life at exact ages x for male and female of Bangladesh in 1961, respectively. It is seen that both curves show almost all-identical pattern in the whole age range excepting some sort of distortions in some ages. It is observed that the expectation of life at birth (e^m_0=43.43) for male is less than that of female (e^f_0=43.75). Where as, male expectation of life in the age interval (1, 10) years is slightly higher than that of female. It is also seen that female is higher than that of male in the age groups (15, 65) years where as male is slightly higher than that of female in the interval (70, 75) years. But, they are approximately same in the last age group. It is also found that the highest expectation of life for male and female are 54.91 years and 54.69 years in ages 4

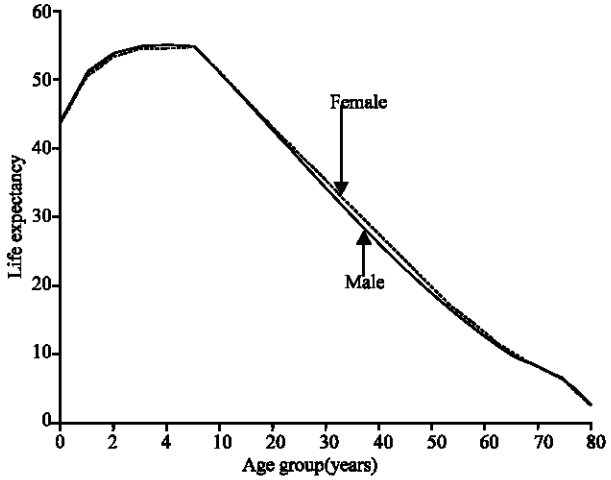

Fig. 1: The life expectancy for male and female of Bangladesh in 1961

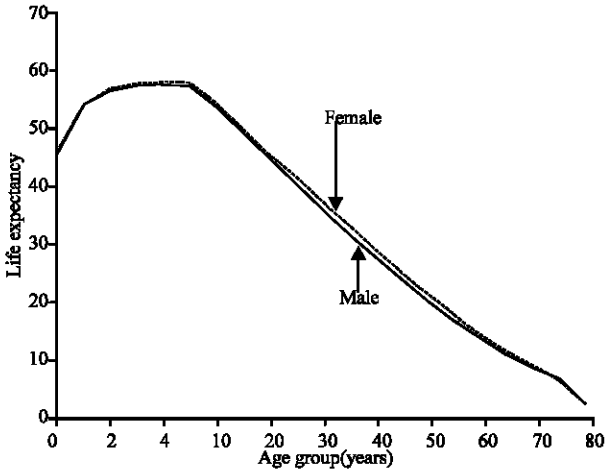

Fig. 2: The life expectancy for male and female of Bangladesh in 1974

years, respectively in which male is higher than that of female.

The 4th and 5th columns of Table 1 and Fig. 2 exhibit the expectation of life at exact ages x for male and female of Bangladesh in 1974, respectively. It is observed that both curves show almost all same pattern in which each of them is increasing with the increase of ages 0 to 4 years and then each of which is starting to decrease more fastly to the last ages. It is found that female expectation of life at birth (e^f_0=45.80) is higher than that of male (e^m_0=45.15). At each age in the continuous age interval (0, 80+) yours excepting the age 75 years, female expectation of life is higher than that of male. The highest female expectation of life (e^f_4=57.86) is also higher than the male expectation of life (e^f_4=57.45) in the same age group 4 years.

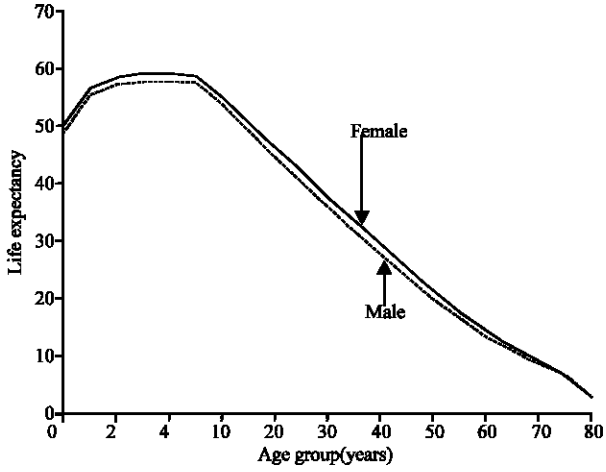

Fig. 3: The life expectancy for male and female of Bangladesh in 1981.

The columns 6th and 7th of Table 1 and Fig. 3 display the male and female expectation of life at exact ages x of Bangladesh in 1981 census, respectively. It is observed that the curve of expectation of life for male increases in the age interval (0, 3) years and then start to decrease gradually to the last age. Moreover, female curve increases with the increase of ages 0 to 4 years and then start to decrease smoothly to infinity age. It is also seen that both curves show approximately similar pattern. It is found that the female expectation of life is greater than that of male at each age in the whole age interval (0, 80+) years. It is seen that female expectation of life at birth (e^f_0=49.82) is also higher than that of male (e^m_0=48.34) . The highest female expectation of life (e^f_4=58.84) in the age 4 years is also larger than that of male (e^f_3=57.57) in the age 3 years.

The last two columns of Table 1 and Fig. 4 indicate the male and female expectation of life of Bangladesh in 1991 census, respectively. Female life expectancy is higher than that of male at each age in the whole age range (0, 80+) years excepting the ages 75 years (Fig .4 and Table 1). It is found that female expectation of life at birth (e^f_3=63.74) is also larger than that of male (e^f_3=60.60) in the same age group 3 years. It is seen that both functions strictly increase in the age interval (0, 3) years and then they strictly decrease in the age group (3, 80+) years excepting 75 years.

From Fig. 5 and 6, it is shows that the expectation of life for male and female exhibit almost all similar pattern. It is observed that with passing of time the peaks of the curves of the expectation of life for male and female are showing increasing trend, that is, they show upward

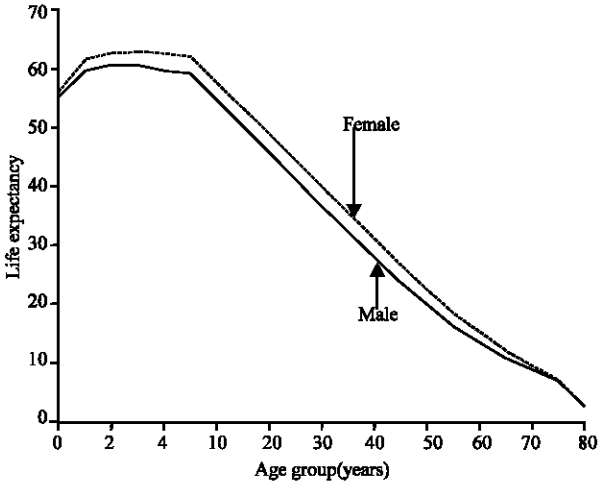

Fig. 4: The Life Expectancy for Male and Female of Bangladesh in 1991.

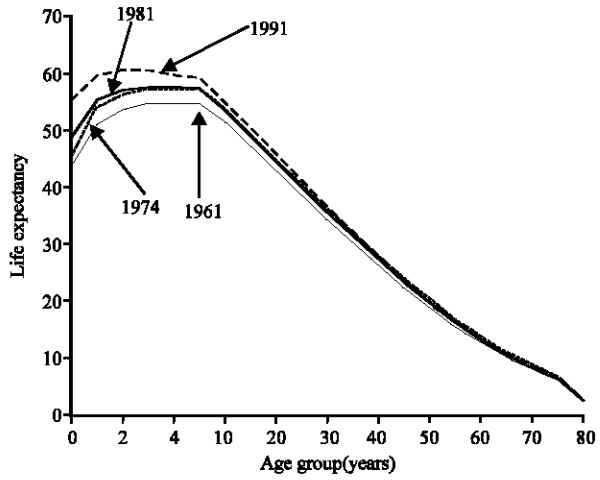

Fig. 5: Life Expectancy for Male of Bangladesh During 1961-1991.

pattern with respect to time. The expectation of life at birth for male and female are 43.43 and 43.75, 45.15 and 45.80, 48.34 and 49.82, 55.13 and 55.84 in 1961, 1974, 1981 and 1991 censuses, respectively in which both are increasing trend over time. It is also found that female is always higher than that of male at birth during study period.

The polynomial model is assumed for the expectation of life (e_x) for male of Bangladesh in 1961 and the fitted equation is:

$$y = 52.5666-0.0339904x^2+0.0005451x^3-0.0000027x^4 \quad (1)$$

giving $R^2 = 0.98322$ and ρ^2_{cv} is 0.975441. It is to be noted that the parameter i.e. the coefficient of x is insignificant,

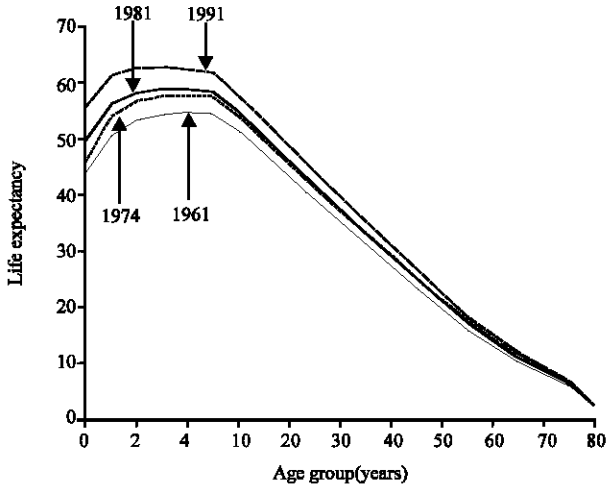

Fig. 6: Life expectancy for female of Bangladesh during 1961-1991

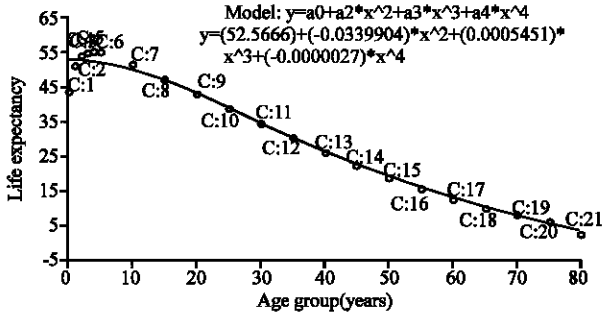

Fig. 7: Observed and fitted life expectancy for male of Bangladesh 1961

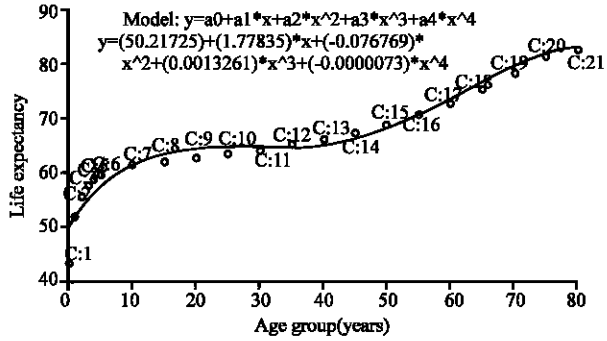

Fig. 8: Observed and fitted f(x) for male of Bangladesh 1961

so, the term containing x is kept out in the fitted model of e_x.

The polynomial model is assumed for f(x) for male of Bangladesh in 1961 and the fitted equation is:

$$y = 50.21725+1.77835x-0.0764769x2+ 0.0013261x3-0.0000073x4 \quad (2)$$

providing R^2 is 0.95320 and ρ^2_{cv} is 0.922371.

Table 2 shows that all the parameters of the fitted models are highly significant with significant proportion of variance explained. It is seen that all the fitted models in equations (1) and (2) are highly cross-validated and their shrinkages are 0.007779 and 0.030829, respectively. These imply that the fitted models in equations (1) and (2) will be stable more than 97 and 92%, respectively. Furthermore, the stability of R^2 of these models are more than 99 and 96%, respectively.

Moreover, in this study F-test is also applied to verify the measure of the overall significance of the model as well as the significance of R^2. The calculated values of F-test of models (i) and (ii) are 332.04 with (3, 17) degrees of freedom (df) and 81.47 with (4, 16) df, respectively. But, the tabulated values of F-test of the models (i) and (ii) are only 5.18 with (3, 17) df and 4.47 with (4, 16) df at 1% level of significance, respectively. Therefore, from this statistics we see that models and R^2 of the models are highly statistically significance once more.

It is very interesting study because if one would like to know how much more time he/she will live, he/she can know from this study. If any one stay now in age x group and its corresponding life expectancy is e_x then he/she live more e_x years in averages.

CONCLUSIONS

The expectation of life for male and female of Bangladesh at different censuses exhibit more or less similar pattern. It is observed that the peaks of the curves for male and female are showing increasing trend over time, that is, they display upward pattern with respect to time. It is seen that both e_x and f(x) follow polynomial model of degree four but the term containing x is absent in the fitted model of e_x because of inconsistency of the parameter. It should be mentioned here that the usual models, i.e. Gompertz model, Makeham model and logistic model were also applied but which are worse fitted with respect to their shrinkages.

REFERENCES

1. Islam and Md. Rafiqul, 2003. Modeling of demographic parameters of Bangladesh: An empirical forecasting. Unpublished Ph.D Thesis, Rajshahi University L.C.
2. Nomani, H., 1964 Population Census of Pakistan 1961. Vol. 2 (East Pakistan), Karachi, Government of Pakistan.
3. BBS, 1997. Population Census of Bangladesh 1974. National Volume, Government of the People's Republic of Bangladesh, Dhaka.
4. BBS, 1984. Bangladesh Population Census 1981. National Series, Government of the People's Republic of Bangladesh, Dhaka.
5. BBS,1994 Bangladesh Population Census 1991. Vol. 1, National Series, Government of the People's Republic of Bangladesh, Dhaka.
6. Shryock, H.S. and J.S. Siegel *et al.,* 1975. The methods and materials of Demography. Vol. II. US Government Printing Office, Washington.
7. Montgomery, C Douglas and A Peck, Elizabeth, 1982. Introduction to Linear Regression Analysis. John Wiley and Sons, New York.
8. Stevens, J., 1996 Applied multivariate Statistics for the Social Sciences. 3rd (Edn.). Lawrence Erlbaum Associates, Inc. Publishers, New Jersey.
9. Gujarati and N. Damodar, 2003. Basic Econometric. 3rd Edn., McGraw Hill, Inc., New York.