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Modeling of Age Structure, 1_x Values and ASDRs for Male Population of Bangladesh

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Abstract: The purpose of the present study is to build mathematical models to age structure, the number of persons surviving at an exact age x (1_Xvalues) and age specific death rates (ASDRs) for male population of Bangladesh in 1981. For this, the secondary data of age structure for male population has been taken from 1981 census. While 1_X values and ASDRs for male population have been taken from Islam (2003). It is found that age structure follows modified negative exponential model while both 1_X values and ASDRs for male of Bangladesh in 1981 follows 4th degree polynomial model. To examine whether they are valid or not model validation technique, cross-validity prediction power (CVPP), P^2_{cv} is applied to those mathematical models.

Key words: Age structure, The number of persons surviving at an exact age x (1_x values), Age specific death rates(ASDRs) modeling cross- validity prediction power (CVPP) F-test

INTRODUCTION

In Population Studies especially in Demography in Bangladesh mathematical modeling have rarely been used. In modern era, mathematical model is a very sophisticated tool to express data in mathematics. Mathematical model is very important in differentiating among various important and unimportant variables to see the relationships among various demographic phenomenons. Model is finally very essential for the estimation of population estimations and projections. Model is indeed essentially an effort to find out structural relationships and their dynamic behaviors among the various elements in demographic processes. One can traditionally draw some graphs of the demographic parameters. But, for the parameters in the context of Bangladesh, very few of us know what types of functional form are more appropriate. Therefore, an effort has been made here to find what types of models are more appropriate to age structure, 1_x values and ASDRs for male population of Bangladesh in1981. Thus, the main objectives of this study are:

- To build up mathematical models to age structure,1_x values and ASDRs for male population of Bangladesh in 1981 and
- To apply Cross-Validity Prediction Power (CVPP), P^2_{cv} , to these models to check how much these models are valid or not. P^2_{cv}

MATERIALS AND METHODS

Sources of Data: To fulfill the above objectives in this study, the secondary data of age structure for male population taken from the 1981 census[1] and 1_x values, ASDRs for male taken from Rafiqul Islam [2] have been used as raw materials.

Mathematical Modeling: The age structure for different ages in years has been plotted in graph paper shown in Fig. 1. It is seen that there are some sort of distortions that is unexpected. Before going to use this data, an adjustment has been needed. For this an adjustment has been made here using the Package Minitab Release 12.1 by the latest smoothing method 4253H, Twice [3]. Hereafter, this smoothed data has been used to fit model for structure.

Using the scattered plot of ages and smoothed age structure for male population (Fig. 2), it is observed that population is modified negative exponentially distributed with respect to ages. Therefore, a modified negative exponential model is considered and the structure of the model is

$$y = c + e^{-(ax+b)} + u \quad [4]$$

where, x represents the age group; y represent population for male; a, b, c are unknown parameters and u is the disturbance term of the model.

From the scattered plot of ages and the number of persons surviving at an exact age x (l_x), it appears that l_x can be fitted by polynomial for different ages. Therefore, an nth degree polynomial model is treated and the form of the model is

$$y = a_0 + \sum_{i=1}^{n} a_i x^i + u$$

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where, x is age group; y is l_x a_0 is the constant; a_i is the coefficient of X^i (i =1, 2, 3, ..., n) and u is the error term of the model. Here a suitable n has been selected for which the error sum of square is minimum.

Again from the dotted plot of ages and ASDRs for male (Fig. 4), it seems that ASDRs can be fitted by polynomial model for different ages. In this case, an nth degree polynomial model is treated and the model of the nth degree polynomial is

$$y = a_0 + \sum_{i=1}^{n} a_i x^i + u$$

where, x is age group; y is ASDRs for male; a_0 is the constant; a_i is the coefficient of X^i (i =1, 2, 3, ..., n) and u is the disturbance term of the model. Here, we have to choose a suitable n so that the error sum of square is minimum.

These models have been estimated using the software statistica.

Model Validation: To examine how much those models are stable, the Cross Validity Prediction Power ρ^2_{cv} (CVPP), is applied.Here

$$P^2_{cv} = 1 - \frac{(n-1)(n-2)(n+1)}{n(n-k-1)(n-k-2)}(1 - R^2)$$

where, n is the number of cases, k is the number of predictor in the model and the cross-validated R is the correlation between observed and predicted values of the dependent variable [5]. The shrinkage of the model is equal to $|\rho^2_{cv} - R^2|$ where ρ^2_{cv} is cross validity prediction power and R^2 is the coefficient of determination of the model. Moreover, the stability of R^2 of the model is 1-shrinkage. The estimated CVPP, ρ^2_{cv}, corresponding to their R^2 and information of model fitting are shown in Table 2.

F-Test: To test the measure of the overall significance of the model as well as the significance of R^2, the F-test is applied to the model. The formula for F-test is as follows:

$$F = \frac{R^2/(K-1)}{(1-R^2)/(n\_K)}$$

with (k-1, n-k) degrees of freedom (d.f.);
where k is the number of parameters to be estimated, n is the number of cases and R^2 is the coefficient of determination in the model [6].

RESULTS AND DISCUSSION

The fitted model of age structure for male of Bangladesh in 1981 is

$$y = 542.3384+\exp(-(0.0408924)x+(8.93262)) \quad (1)$$

with coefficient of determination R^2 is 0.98982 and ρ^2_{cv} =0.987110.
The fitted model of the number of persons surviving at an exact age x (1^X values) for male of Bangladesh in 1981 is

$$y = (90119.23)+(-2611.484)x+(124.705)x^2+(-2.30211)x^3+(0.0125538)x^4 \quad (2)$$

providing coefficient of determination R^2 is 0.98109 and ρ^2_{cv} =0.968633.
The fitted model of ASDRs for male of Bangladesh in 1981 is

$$y=(0.0905521)+(-0.0178697)x+(0.0009607)x^2+(-0.0000191)x^3+(1.286392e-7)x^4 \quad (3)$$

giving proportion of variance explained (R^2)= 0.95848 and ρ^2_{cv} is 0.93113.
The estimated CVPP, ρ^2_{cv}, corresponding to their R^2 is shown in Table 2. From this table it is seen that all the fitted models (1), (2) and (3) are highly cross- validated and their shrinkages are 0.00271, 0.012457and 0.02735, respectively. These imply that the fitted models (i), (ii) and (iii) will be stable more than 98 , 96 and 93%, respectively. Moreover, from this table, it is shown that the parameters of the fitted model (1) and (2) are highly statistically significant with more than 98% of variance explained where as the parameters of the fitted model (3) are also significant explaining more than 95% variation. The stability of R^2 of these models are more than 99%, 98% and 97% respectively.
The calculated values of F-test of the models (1), (2) and (3) are 534.78 with (2, 11) d.f., 207.53 with (4, 16) d.f. and 92.34 with (4, 16) d.f. respectively where as the corresponding tabulated values of F-test are only 7.21, 4.77 and 4.77 at 1% level of significance, respectively. Therefore, from these statistics it is seen that these models and their corresponding R^2 are highly statistically significant.

CONCLUSIONS

In this study it is seen that age structure for male follows modified negative exponential model. While both the number of persons surviving at an exact age x (l_x values) and ASDRs for male follows 4th degree polynomial model. It should be noted that usual models i. e. Gompertz, Makeham, log-linear, semi-loglinear and logistic were also applied but seem to be worse fitted in terms of their shrinkages. Therefore, the results of these models were not shown here.

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