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Statistical Model for Estimating the Rate of Spread of Human Immune-Deficiency: A Case Study of Ondo Kingdom

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Abstract: This research considered a statistical model for estimating the rate of spread of Human Immune-Deficiency Virus by making use of Ondo-West Local Government as a case study. The virus' modes of infection and some of the existing epidemic models were discussed. Ondo-West was randomly sampled out of the local government areas that make Ondo Kingdom. Records of samples were obtained from the two government-recognised Hospitals in the vicinity. Data on relevant variables comprising Year of Test, Age-group, Gender, as well as Percentage that tested positive were analysed. The study made use of Generating Function approach to solve the associated Birth process model. A reduction from normal population was numerically noticed through the use of Birth process model, Results of various age-groups (0-5), (6-22) and (23+), respectively defined as Infants, Youth and Adults were presented. It was found that the age-group (6-22) has the greatest risk, as out of an assumed 5000 members of each group, the Youth has 34 possible infected members, as against 6 and 27 for Infants and Adults, respectively.

Key words: Statistical model, spread, estimating, immune-deficiency, AIDS, HIV

INTRODUCTION

Acquired Immune Deficiency Syndrome (AIDS), resulting from an infection of Human Immuno-Deficiency Virus (HIV) has become a universal problem, spreading through every nook and cranny of the universe. Erroneous opinions of certain clans, that such a menace has never and will never pass through their lands are swiftly been eroded at an alarming rate. Hence, it becomes meaningfully valuable for one to find a way of quantifying the rate of spread, given an initial statistical record of its spread in any community, so that when an eventual authentic curative formular is procured by the medical profession, this model can help to estimate the numerical strength already infected and so, the quantity of the procured medicine in need by such community can easily be estimated.

Whenever a need arises to authenticate Mathematical theories, Mathematical modeling can not be pushed aside. This particular model has however been christened Statistical Model because it is being developed through the use of empirical data and many biological factors, like the incubation periods and social factors affecting the spread of HIV/AIDS are naturally random.

Considering the observations of Bashiru (2005) most past models have been more of deterministic approach. Lubobi (1994) and Anderson (1992) used deterministic model investigating hierarchical age-structured populations with intra-specific competition or predation. Becker (1989) also used deterministic approach in considering vaccination strategies in age-structured populations. Others noted for their works using deterministic approaches include Olowofeso *et al.* (1997), Anderson *et al.* (1986) and Lubobi (1991).

Particularly on contractible diseases, the SEIR-Model of Micheal the SIR-Model of Anderson *et al.* (1987), the SIA-Model of Gumel (2003), the improved MSEIR-Model of Bashiru (2005), the MTCT-Model of Waema (2002) and the Improved SIA-Model of Waema and Olowofeso (2005) are to mention a few. However, even though deterministic models may serve as guide in parameter estimation, basis for precision in estimation for the purpose of quantifying variation in data can be formed on Stochastic Models, especially when analyzing data on infectious diseases.

Thus, this model, having been carefully developed, will surely help in the estimation of the infection concerning the three categories of age-groups of Infants, Youth and Adults.

Though, the purpose of the model in this study aims at universal acceptance, the case study is however limited to Ondo-West Local Government of Ondo

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State, Nigeria for reasons of financial constraint and Africans' unreadiness to divulge useful facts.

The objectives of this reaearch are to formulating a Stochastic Model for estimating the rate of spread of HIV/AIDS in Ondo Kingdom of Ondo State, using the model so formulated for the prediction of the spread of HIV and examining the reliability of the model by comparing the results with available past records in our hospitals.

MATERIALS AND METHODS

Relevant data were collected from the two government-recognized hospitals in Ondo Kingdom. The data collected were classified by Year of Test, Age-group, Sexual gender, population that reported for test and percentage that tested positive.

Formulation of birth process: Every new HIV infection is regarded an addition to the HIV/AIDS family. So, the pure birth process is considered as follows:

Let the rate of birth within time interval $(t, t + \Delta t)$ $= \lambda$
Let the probability of more than one birth in time interval $(t, t + \Delta t)$ $= o(\Delta t)$
If the probability of unit at time t $= P_n$
Probability of 1 birth within time interval $(t, t + \Delta t)$ $= \lambda_n(t)\,\Delta t + o(\Delta t)$
Probability of 0 birth within time interval $(t, t + \Delta t)$ $= 1-\lambda_n(t)\,\Delta t + o(\Delta t)$

Considering two consecutive possible independent events,

$$P_n(t, t+\Delta t) = [1-\lambda_n(t)\,\Delta t + o(\Delta t)]\,P_n(t) + [\lambda_{n-1}(t)\,\Delta t + o(\Delta t)]\,P_{n-1}(t)\,o(\Delta t)$$
$$= [1-\lambda_n(t)\,\Delta t]P_n(t) + [\lambda_{n-1}(t)\,\Delta t]\,P_{n-1}(t) + o(\Delta t)^3$$

Since it is assumed that more than 1 person can not be infected simultaneously from the same source at the same time,

Then, $o(\Delta t) \to 0$.
Thus, $P_n(t, t+\Delta t) = [1-\lambda_n(t)\,\Delta t]\,P_n(t) + \lambda_{n-1}(t)\,P_{n-1}(t)\,\Delta t$
$= P_n(t) - \lambda_n(t)\,P_n(t) + \lambda_{n-1}(t)\,P_{n-1}(t)\,\Delta t$
$\Rightarrow P_n(t, t+\Delta t) - P_n(t) = [-\lambda_n(t)\,P_n(t) + \lambda_{n-1}(t)\,P_{n-1}(t)]\,\Delta t$
$\Rightarrow \dfrac{Pn(t, t+\Delta t) - Pn(t)}{\Delta t} = [-\lambda_n(t)\,P_n(t) + \lambda_{n-1}(t)P_{n-1}\,\Delta(t)]$

But

$$\frac{P_n(t, t+\Delta t) - P_n(t)}{\Delta t} = P_n^1(t)$$

$$\Rightarrow P_n^1(t) = -\lambda_n(t)\,P_n(t) + \lambda_{n-1}(t)\,P_{n-1}(t) \quad (1)$$

Now, since λ = rate and n = population at time t; then, mean = $n\lambda$.
Now, let $\lambda_n = n\lambda$
Then, Eq. 1 becomes

$$\Rightarrow P_n^1(t) = -n\lambda(t)\,P_n(t) + (n-1)\,\lambda(t)\,P_{n-1}(t) \text{ for } n>1 \quad (2)$$

For n = 1, we have
$P_1^1(t) = -\lambda(t)P_1(t) + 0$
$\therefore$ Eq. 2 holds for $n \geq 1$

$$\text{i.e., } P_n^1(t) = -n\lambda(t)P_n(t) + (n-1)\,\lambda(t)P_{n-1}(t) \quad (3)$$

To solve Eq. 3, we consider the probability generating function (p.g.f).

$$\theta(z,t) = \sum_{n=1}^{\infty} P_n(t)z^n$$

That is, multiplying (3) by z^n to have

$$P_n^1(t)\,z^n = -n\lambda(t)P_n(t)\,z^n + (n-1)\,\lambda(t)\,P_{n-1}(t)\,z^n \quad (4)$$

Summing this over n, we have

$$\sum_{n=1}^{\infty} P_n^1(t)z^n = -\lambda\sum_{1}^{n} nP_n(t)z^n + \lambda\sum_{1}^{\infty}(n-1)P_{n-1}(t)z^n \quad (5)$$

$$\text{Now, let } \theta(z,t) = \sum_{1}^{\infty} P_n(t)z^n \quad (6)$$

$$\text{Then } \frac{\partial\theta}{\partial t} = \sum_{1}^{\infty} P_n^1(t)z^n \quad (7)$$

$$\text{and } \frac{\partial\theta}{\partial z} = \sum_{1}^{\infty} nP_n(t)z^{n-1} \quad (8)$$

Multiplying (8) by λz and λz^2 in succession, we have

$$\lambda z\frac{\partial\theta}{\partial z} = \lambda\sum_{1}^{\infty} nP_n(t)z^n \quad (9)$$

and

$$\lambda z^2\frac{\partial\theta}{\partial z} = \lambda\sum_{1}^{\infty} nP_n(t)z^{n+1} \quad (10)$$

Since the 2 consecutive possibilities P_n and P_{n-1} have been considered from the beginning,

Then, let n → n - 1 in (10) to have

$$\lambda z^2 \frac{\partial \theta}{\partial z} = \lambda \sum_{1}^{\infty} (n-1) P_{n-1}(t) z^n \qquad (11)$$

Putting (7), (9) and (11) in (5),

$$\sum_{n=1}^{\infty} P_n^1(t) z^n = -\lambda \sum_{1}^{n} n P_n(t) z^n + \lambda \sum_{1}^{\infty} (n-1) P_{n-1}(t) z^n$$

becomes

$$\frac{\partial \theta(z,t)}{\partial t} = -\lambda z \frac{\partial \theta}{\partial z} + \lambda z^2 \frac{\partial \theta}{\partial z}$$

$$\Rightarrow \frac{\partial \theta(z,t)}{\partial t} = -\lambda z(1-z) \frac{\partial \theta}{\partial z}$$

$$\Rightarrow \frac{\partial \theta}{\partial t} + \lambda z(1-z) \frac{\partial \theta}{\partial z} = 0 \qquad (12)$$

This Eq. 12 is a linear probability differential equation with auxiliary equations

$$\frac{dt}{1} = \frac{dz}{\lambda z(1-z)} = \frac{d\theta}{0}$$

To solve these, First consider

$$\frac{dt}{1} = \frac{d\theta}{0}$$

$$\Rightarrow \int d\theta = \int 0 dt$$

$$\Rightarrow \theta = k \qquad (1.13)$$

Then, consider

$$\frac{dt}{1} = \frac{dz}{\lambda z(1-z)}$$

$$\Rightarrow \int \lambda dt = \int \frac{dz}{z(1-z)}$$

Which when solved by partial fraction, leads to

$$\lambda t + c_1 = l_n z - l_n (1-z) + c_2$$

$$\Rightarrow \lambda t = l_n \frac{kz}{(1-z)}$$

$$\Rightarrow \ell^{\lambda t} = \frac{kz}{(1-z)} \qquad (14)$$

Combining (13) and (14), we have

$$\ell^{\lambda t t} = \frac{z\theta}{(1-z)}$$

This implies that $z\theta = (1 - z)\, \ell^{\lambda t t}$

$$\text{Thus, } \theta = \frac{(1-z)}{z} \ell^{\lambda t}$$

Therefore,

$$\theta(z,t) = \theta\left(\left(\frac{1-z}{z}\right) \ell^{\lambda t t} \right) \qquad (15)$$

Equation 15 is the most general solution for Birth Process and this forms the Birth Process Model that can be used to estimate the number of infected persons (θ) after time interval (t) when the rate of infection (λ) and initial population (z) are known.

RESULTS AND DISCUSSION

The formulated Birth Process Model

$$P_n^1(t) = -n\, \lambda(t)\, P_n(t) + (n-1)\, \lambda\, (t) P_{n-1}(t)$$

Gave way to obtain the partial differential equation

$$\frac{\partial \theta}{\partial t} + \lambda z(1-z) \frac{\partial \theta}{\partial z} = 0$$

and this partial differential equation was solved to have the Birth Process Model general solution

$$\theta(z,t) = \theta\left(\left(\frac{1-z}{z}\right) \ell^{\lambda t t} \right)$$

Empirical Illustrations:

As earlier stated,

λ = Rate of HIV infection,
t = Time interval of testing (say 1 year)
z = Initial population,
θ = Population getting infected

Thus, suppose for age (0-5), averagely,

λ = 0.158,
t = 1,
z = 5000

Table 1: Percentage of infection among tested individuals

	Age group			Sex	
Year	0-5	6-22	23+	Male	Female
2000	0.17	0.17	0.16	0.18	0.15
2001	0.17	0.22	0.21	0.24	0.18
2002	0.15	0.22	0.17	0.16	0.18
2003	0.14	0.15	0.12	0.12	0.13
Average	0.158	0.190	0.165	0.175	0.160

$$\text{Then}\,\theta(5000,1) = (\frac{1-5000}{5000})\ell^{0.158(1)} = -1.17 \cong -1$$

(Negativity implies reduction from normal population)

Thus, after 1 year, 1 person gets infected form the population; provided that the rate of infection and population remain constant (Table 1).

Illustrations

Age Group (0-5): Infants-Estimated Number of Infection (θ) with respect to time.

Let z = 5000,
λ = 0.158

t	1	2	3	4	5	6	7	8	9	10
θ	1	1	1	2	2	3	3	4	4	5

Thus, within ten years, about 26

$$(\text{i.e.}\sum_{t=1}^{10}\theta)$$

individuals within the age of (0-5) would have been infected out of a population of 5000 infants, if rate of infection (λ = 0.158) remain constant (Fig. 1).

Age group (6-22): Youth-estimated number of infection (θ) with respect to time.
Let z = 5000, λ = 0.190

t	1	2	3	4	5	6	7	8	9	10
θ	1	1	2	2	3	3	4	5	6	7

Thus, within ten years, about 34

$$(\text{i.e.}\sum_{t=1}^{10}\theta)$$

individuals within the age of (6-22) would have been infected out of a population of 5000 youth, if rate of infection (λ = 0.190) remain constant (Fig. 2).

Age group (23+): Adults-estimated number of infection (θ) with respect to time.
Let z = 5000, λ = 0.165

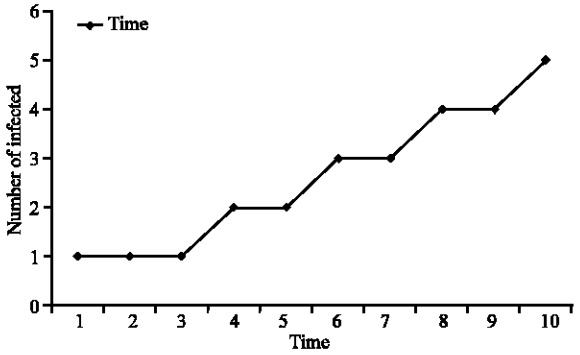

Fig. 1: Number of infants aged (0-5) Infected When Z = 5000 and λ = 0.158

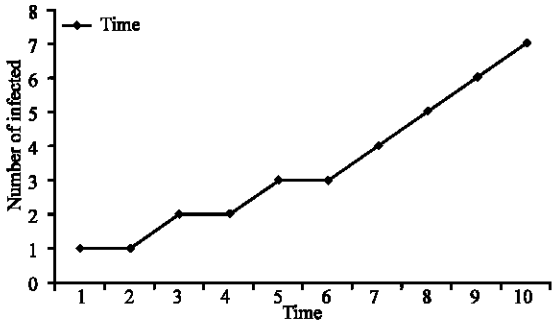

Fig. 2: Number of Youth Infected within Ages (6-22 when z = 5000 and λ = 0.190

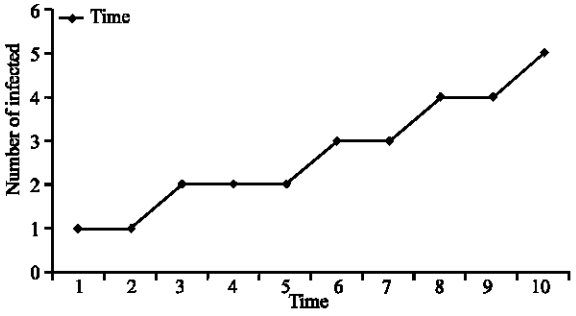

Fig. 3: Number of Adults Ages (23+) Infected When Z = 5000 and λ = 0.165

t	1	2	3	4	5	6	7	8	9	10
θ	1	1	2	2	2	3	3	4	4	5

Thus, within ten years, about 27 individuals

$$(\text{i.e.}\sum_{t=1}^{10}\theta)$$

within the age of (23+) would have been infected out of a population of 5000 infants, if rate of infection (λ = 0.165) remain constant (Fig. 3).

Male population: Estimated Number of Infection (θ) with respect to time.
Let z = 5000, $\lambda = 0.175$

t	1	2	3	4	5	6	7	8	9	10
θ	1	1	2	2	2	3	3	4	5	6

Thus, within ten years, about 29

$$(\text{i.e.} \sum_{t=1}^{10} \theta)$$

individuals would have been infected out of a population of 5000 men, if rate of infection ($\lambda = 0.175$) remain constant (Fig. 4).

Female population: Estimated number of infection (θ) with respect to time.
Let z = 5000, $\lambda = 0.160$

t	1	2	3	4	5	6	7	8	9	10
θ	1	1	2	2	2	3	3	4	4	5

Thus, within ten years, about 27

$$(\text{i.e.} \sum_{t=1}^{10} \theta)$$

individuals would have been infected out of a population of 5000 women, if rate of infection ($\lambda = 0.160$) remain constant (Fig. 5).

The results obtained from testing the birth process model conform appropriately with the available records in the hospitals visited. The earlier works done only presented the equations obtained. But for Waema (2002) alone who provided numerical illustration in Mother To Child Transmission (MTCT) Model, no other author gave illustrations to such equations in their reports. As can be seen from the graphs, this model easily makes it possible to calculate the number that will likely be infected after a given period of time interval if the initial population and the known rate of infection remained unchanged for the Birth Process.

All things been equal, except something drastic is done against the spread of the deadly disease, the whole population may get infected in no distant time at this rate of infection.

The youth (School age group (6-22)) has a greater risk, as out of an assumed population of 5000 considered for each group, the youth has the greatest infection of 34 as against 26 and 27 for infants and adults, respectively.

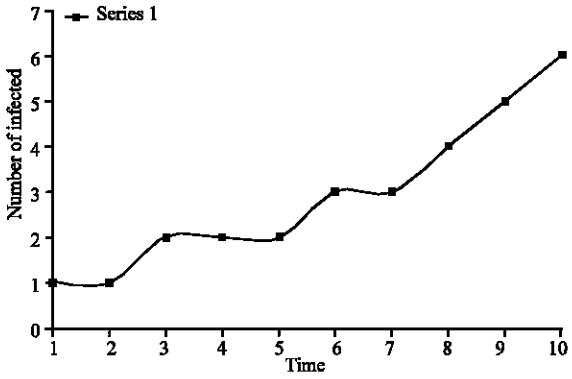

Fig. 4: Number of infected male when Z = 5000 and $\lambda = 0.175$

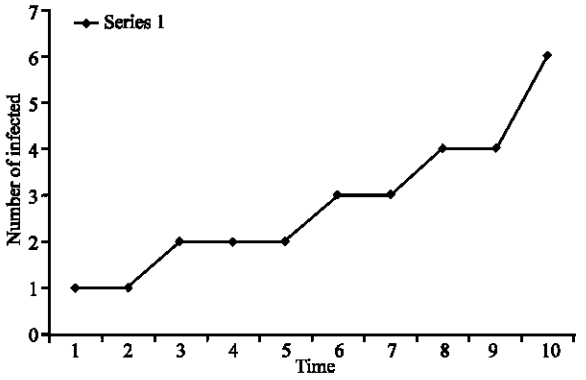

Fig. 5: Number of infected female when Z = 5000 and $\lambda = 0.160$

Considering the infection gender-wise, the Males stand a greater risk, as 29 of them (against 27 females) are possibly infected out of an assumed population of 5000 considered for each group.

CONCLUSION

Conclusively, no group is safe from the infection as it is observed that the graphs of the different groups rose year after year with respect to time. Hence, all hands must be on deck in intensifying efforts to curb the spread of the epidemic. It is however hoped that testing records in other parts of the country will further confirm the appropriateness of this Birth Process Equation.

REFERENCES

Anderson, R.M. *et al.*, 1986. A Preliminary Study of the Transmission Dynamics of the Immunodeficiency Virus (HIV). The Causative Agents of AIDS. IMA J. Math. Applied Med. Biol., 3: 229-263.

Anderson, R.M. *et al*., 1987. Transmission Dynamics of HIV Infection. Nature, 326: 37-142.

Anderson, Roy, 1992. The Transmission Dynamics of Sexually Transmitted Diseases. The Behaviour Components in Sexual Behaviour and Networking: Anthropological and Social-Cultural Studies on the Transmission of HIV, pp: 23-48.

Bashiru K.A., 2005. Stochastic modeling of specific mode of HIV/AIDS epidemic with reference to Nigeria. A project for M. Tech. in Industrial Mathematics submitted in the department of Mathematical Sciences. Federal University of Technology Akure, Nigeria.

Gumel, A.B., 2003. Using Mathematics to Understand HIV Pathogenesis and Epidemiology. Book of Abstract - African Mathematics Conference.

Luboobi, L.S., 1991. Mathematical Models for the Dynamics of the AIDS Epidemic in Biometric. ICIPE Science Press, Nairobi, pp: 76-83.

Luboobi, L.S., 1994. A Three Age-group Model for HIV/AIDS Epidemic and Effects of Medical/Social Interventions. Math. Comput. Modeling, 19: 91-105.

Olowofeso, O.E., O. Olabode and A.O. Fasoranbaku, 1997. On Mathematical Modeling of Sexually Transmitted Diseases. Bull. Sci. Assoc. Nigeria, 21: 117-119.

Waema, R., 2002. HIV Epidemic-Mother To Child Transmission (MTCT) Model. University of Nairobi, Kenya, pp: 1-97.

Waema, R. and O.E. Olowofeso, 2005. Mathematical Modelling for Human Immunodeficiency Virus (HIV) Transmission using Generating Function Approach. Kragujevae J. Sci., 27: 115-130.