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Delection Analysis of Clamped Rectangular Plates of Variable Thickness on Elastic Foundation by the Galerkin Method

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Abstract: Deflection of rectangular plates of variable thickness resting on an elastic foundation is studied. The fourth order differential equation is solved by using the Garlerkin method. The thickness of plate is assumed to vary linearly along the x axis and this variation is taken to be symmetric with respect to the middle surface. The center deflection has been computed for different values of the taper parameter, aspect ratios and foundation modulus. Simplicity and quick convergence are the advantages of the present method in comparison with more laborious numerical methods requiring extensive computer facilities.

Key words: Delection, rectangular plates, variable thickness, elastic foundation, garlerkin method

INTRODUCTION

Plates of variable thickness are widely used in engineering structures, such as bunkers, reinforced concrete breast walls, rectangular reservoir, buttress dams reinforced concrete pavement airport runways and foundation slabs of buildings. Moreover, in ship design, the ship bottom is frequently considered as a complex plate of variable thickness. The deflections of plates of variable thickness are generally small in comparison with the plate thickness and the middle plane of the plate remains as a neutral surface during bending. The normal stresses in the transverse direction are neglected as well as shear strain. Conway and Ithaca (1951) Conway (1953, 1958) was the first to solve the problem of bending of circular and rectangular plates of variable thickness. Bastin *et al*. (1972) proposed a solution in closed from to calculate the moments and tensions of rectangular plates of variable thickness subjected to hydrostatic pressure. Anon-linear iteration procedure was used by Soong (1972) to approach the problem of deflection of plates of variable thickness. Petriana and Conway (1972) reported some deflection and moment data of plates with variable thickness. Chen (1976) attempted the problem of bending of plates variable thickness in general manner. Banerjee (1979) investigated and obtained limited deflection data for skews plates of variable thickness. In the study reported herein, an attempt is made to obtain numerical results for bending of lamped rectangular plates with variable thickness on elastic foundations using the Galerkin method.

GOVERNING DIFFERENTIAL EQUATION

For a clamped rectangular plate of variable thickness with 2a and 2b as the dimension s in x and y coordinates and subjected to a uniform distributed load q, the governing differential equation in terms of the displacement w of point in the middle plate of variable thickness can be expressed as:

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+2\frac{\partial D}{\partial x}\left(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 w}{\partial x\,\partial y^2}\right)+\frac{\partial^2 D}{\partial x^2}\left(\frac{\partial^2 w}{\partial x^2}+\upsilon\frac{\partial^2 w}{\partial y^2}\right)=q \qquad (1)$$

By using the dimensionless ratio, Eq. 1 can be transformed into dimensionless from as:

$$(1+c\xi)^3\left(\frac{\partial^4 W}{\partial \xi^4}+2R^2\frac{\partial^4 W}{\partial \xi^2\partial \eta^2}+R^4\frac{\partial^4 W}{\partial \eta^4}\right)+6(1+c\xi)^2\left(\frac{\partial^3 W}{\partial \xi^3}+R^2\frac{\partial^3 W}{\partial \xi\partial \eta^2}\right)+6(1+c\xi)\left(\frac{\partial^2 W}{\partial \xi^2}+\upsilon R^2\frac{\partial^2 W}{\partial \eta^2}\right)=\frac{qa^4}{h_0 D_0} \qquad (2)$$

Where,

- E = Modulus of elasticity of the plate material.
- υ = Poisson's ratio.
- h_0 = Plate thickness at x = 0.
- h = Plate thickness.
- c = Taper parameter.
- D_0 = Flexural rigidity of the plate at x = 0.

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$$D_0 = \frac{Eh_0^3}{12(1-\upsilon^2)}$$

q = Lateral load per unit area.
R = Aspect ratio of plate (a/b).
ξ,η = Dimensionless parameters in directional coordinates for rectangular plate of variable thickness $\xi = x/a$, $\eta = y/b$.
W = Lateral displacement $W = w/h_0$.

In the analysis of plates of variable thickness on elastic foundations it is assumed that the restoring pressure is everywhere proportional to the deflection. i.e, a Winkler type foundation. Thus the governing differential equation can be easily obtained by adding KW to the left hand side of Eq. (2), where

$$K = \frac{ka^4}{h_0 D_0}$$

and k is the modulus of elastic support reaction per unit area per unit deflection.

BOUNDARY CONDITIONS AND APPROXIMATING FUNCTIONS

The plate is considered thin and clamped all round. A sketch of the plate with the coordinate system is shown in Fig. 1. The thickness of the plate varies linearly along the x axis and this variation is taken to be symmetric with respect to the middle surface. In the Galerkin method, a linear combination of independent functions must be chosen for the displacement functions satisfying all boundary conditions. Moreover, these functions must have the same order of derivatives as called for by the differential equation. The boundary conditions for the clamped plates of variable thickness are at

$$\frac{\partial W}{\partial \xi} = W = 0 \quad \xi = \pm 1 \text{ and } \frac{\partial W}{\partial \eta} = W = 0$$

at $\eta = \pm 1$. In order to satisfy the boundary conditions, the following expression is chosen:

$$W = (1-\xi^2)^2(1-\eta^2)^2(C_1 + C_2\xi^2 + C_3\eta^2 + C_4\xi^2\eta^2 + C_5\xi^6 + C_6\eta^6)$$

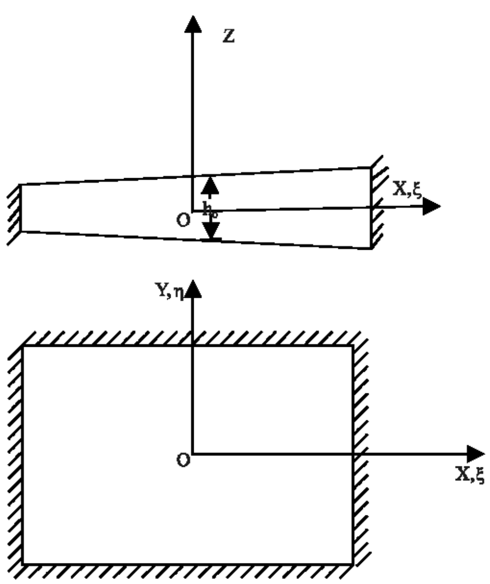

Fig. 1: Rectangular plate of variable thickness and the coordinate system ($h=h_0(1+c\xi)$)

METHODS OF SOLUTION

The Galerkin method is obtain a solution for the differential Eq. 2. The governing differential equations for the deflection analysis of plates of variable thickness are similar the problem of the form:

$$\ell(w) - p = 0 \tag{3}$$

Where, ℓ is the differential operator, p is the external force. The equilibrium of the structural system will be obtained by integrating the differential equation over the domain A. By denoting δw as virtual displacement and noting that the virtual work of the internal and external forces must vanish

$$\delta W_i + \delta W_e = 0 \tag{4}$$

and therefore

$$\int\int \ell(w) - p)\delta w dA = 0 \tag{5}$$

Where, A indicates the integration domain.

Equation 5 is true only if the displacement function w is the exact solution of the problem under investigation; this function is written in the form;

$$w = \sum_{i=1}^{n} C_i f_i(x,y) \tag{6}$$

Where, f_i (x, y) are functions satisfying all boundary conditions of the problem and C_i are undetermined coefficients. The function f_i (x,y) are considered linearly independent in the region.

The variation of small displacements will be expressed by:

$$\partial w = \sum_{i=1}^{n} f_i(x,y) \partial C_i \tag{7}$$

Substituting Eq. 7 into 5

$$\sum_{i=1}^{n}\delta C_i \iint (\ell(w) - p) f_i(x, y) dA = 0 \quad (8)$$

Equation 8 have to be satisfied for any small variation δw_i. Thus, the variation of δC_i are arbitrary: Therefore, we arrive at the system of equations:

$$\iint (\ell(w) - p) f_i(x, y) dA = 0 \quad (9)$$

Where, dA=dxdy
Equation 9 can be written in the form;

$$\iint (\ell(w) - p) f_i(x, y) dxdy = 0 \quad (10)$$

These are n Galerkin equations with n unknowns. These coefficients can be determined by integrating the function over the entire domain.

Throughout the present study, numerical integration was carried out by a program coded in fortran 77 using the trapezoidal rule, 2a and 2b are taken as the characteristic lengths of the plates in x and y directions, respectively.

RESULTS AND DISCUSSION

For the purpose of demonstrating the accuracy and the convergence of the present method, results for deflection analysis of clamped rectangular plates of both uniform and variable thickness are shown in Table 1. Results are good agreement with those obtained in an early work by Ng and Chan (1977) and by Banerjee (1979). Also the present method shows excellent convergence even for high taper parameters.

In Table 2, numerical comparison is shown for centre deflection of clamped rectangular plate of uniform thickness on a continuous foundation between the present method and those obtained by Ng and Chan (1977). For the case of clamped plates of variable thickness on elastic foundations no comparison of results can be made as no data are as yet available in the technical literature.

Figure 2 and 3 shows the centre deflection for clamped rectangular plates of variable thickness for various taper parameters. It is interesting to observe thickness that the centre deflection decreases with an increase of both the taper parameter c and the plate aspect ratio R. This can be attributed to the fact that both increases in aspect ratio and parameter increase significantly the stiffness of the plate.

Table 1: Comparison of centre deflection á of rectangular plates of variable thickness for R=a/b=1 $W = \alpha \frac{qa^4}{D_0}$

	Taper parameter C				
	0.0	0.2	0.4	0.6	0.8
1 term solution	0.02082	0.02055	0.01866	0.01618	0.01365
2 terms solution	0.02073	0.02010	0.01848	0.01640	0.01428
3 terms solution	0.02020	0.01957	0.01798	0.01593	0.01383
4 terms solution	0.02024	0.01960	0.01800	0.01594	0.01384
5 terms solution	0.02424	0.01960	0.01800	0.01597	0.01397
6 terms solution	0.02025	0.01961	0.01801	0.01600	0.01398
(Ng and Chan, 1977)	0.020201				
(Banerjee, 1979)	0.02062	0.01986	0.01785	0.01530	0.01274

Table 2: Variation of max small deflection coeff. α of clamped rectangular plates of variable With dimensionless foundation modulus K for taper parameter c = 0 $W = \alpha \frac{qa^4}{D_0}$

	R=a/b=1		R=a/b=0.75		R=a/b=0.5	
K	Present	(Ng and Chan, 1977)	Present	(Ng and Chan, 1977)	Present	Ref[9]
0	0.020250	0.020201	0.031480	0.031412	0.040570	0.040545
20	0.016090	0.016033	0.022230	0.022201	0.025530	0.025525
40	0.013260	0.013245	0.017050	0.017036	0.018420	0.018411
60	0.011260	0.011251	0.013750	0.013741	0.014310	0.014307
80	0.009760	0.009756	0.011470	0.011463	0.011650	0.011656
100	0.008600	0.008594	0.009800	0.009797	0.009800	0.009812
120	0.007670	0.007665	0.008540	0.008530	0.008400	0.008459
140	0.006910	0.006903	0.007540	0.007534	0.007410	0.007427
160	0.006280	0.006377	0.006740	0.006734	0.006590	0.006616
180	0.005750	0.005745	0.006090	0.006076	0.005900	0.005963
200	0.005290	0.005291	0.005540	0.005528	0.005400	0.005425

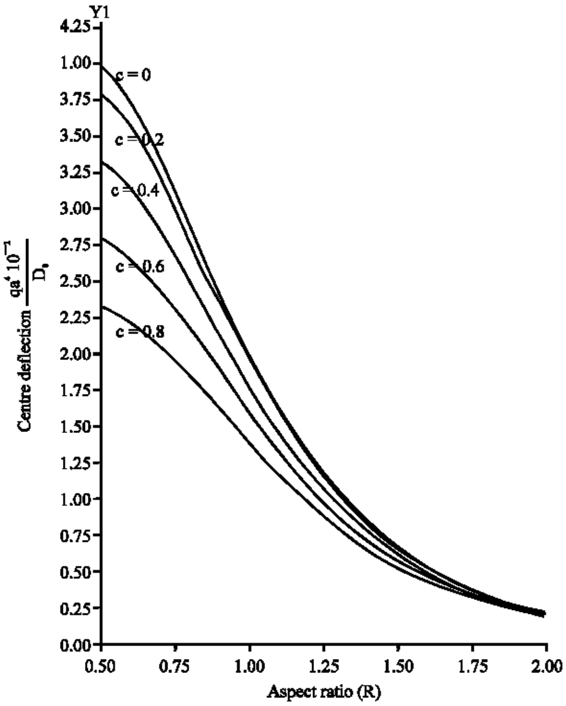

Fig. 2: Variation of centeral deflection with aspect ration of clamped rectangular plates of variable thickness for various taper parameter c

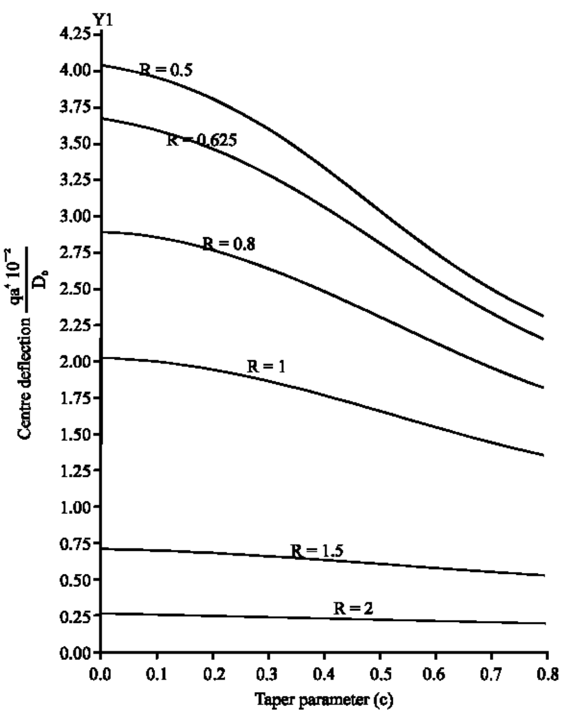

Fig. 3: Variation of centeral deflection with taper parameter (c) of clamped rectangular plates of variable thickness for various various aspect ratio

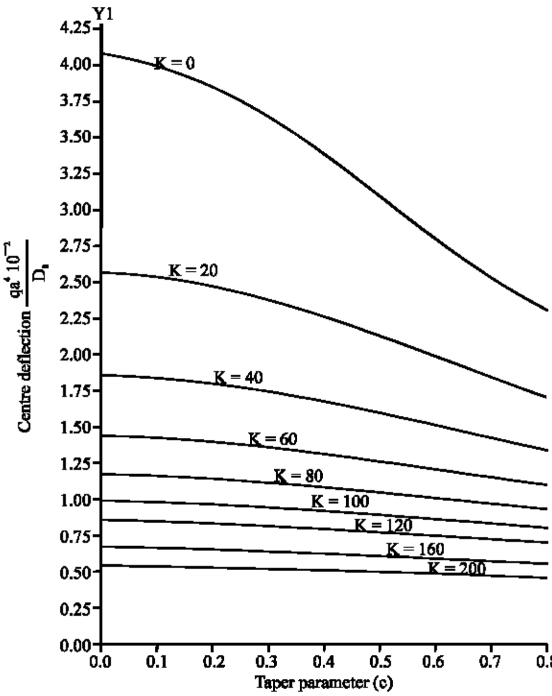

Fig. 4: Variation of centeral deflection with taper parameter (c) and foundation modules for rectangular plates of variable thickness for various aspect ratio R = 0.5

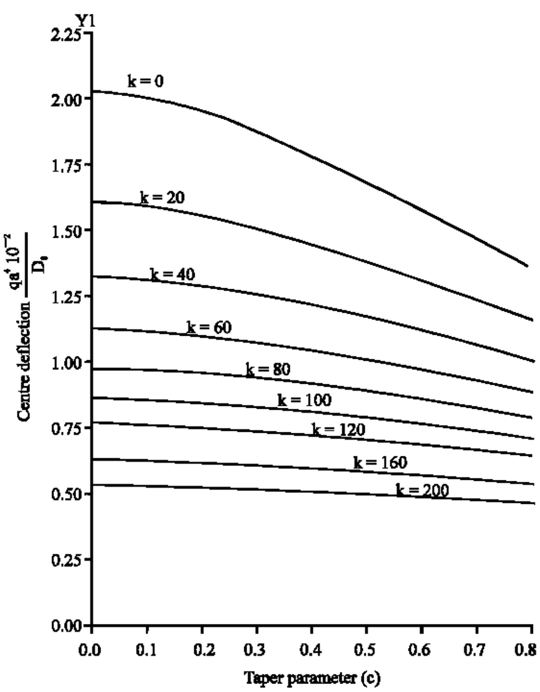

Fig. 5: Variation of centeral deflection with taper parameter (c) and foundation modules for rectangular plates of variable thickness for various aspect ratio R = 1

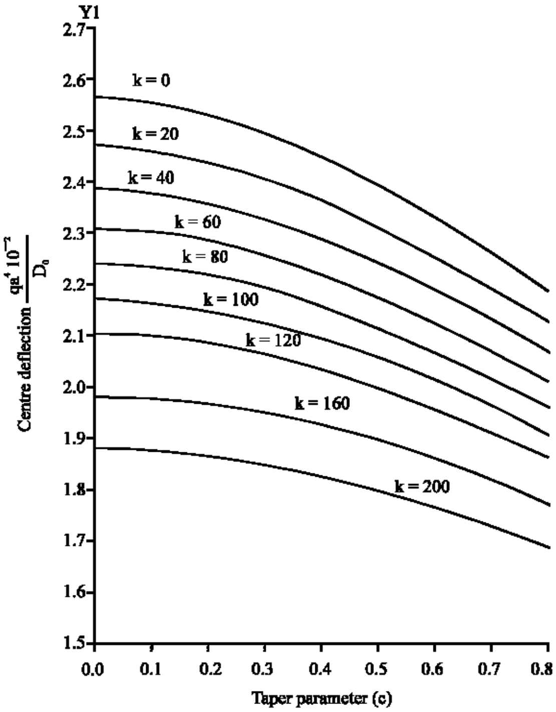

Fig. 6: Variation of centeral deflection with taper parameter (c) and foundation modules for rectangular plates of variable thickness for various aspect ratio R = 2

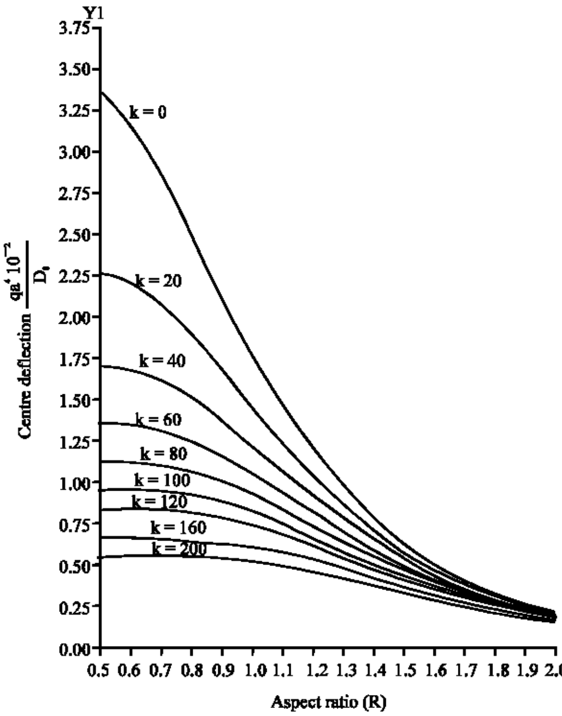

Fig. 8: Variation of centeral deflection with aspect ratio and foundation modules for rectangular plates of variable thickness with taper parameter C = 0.4

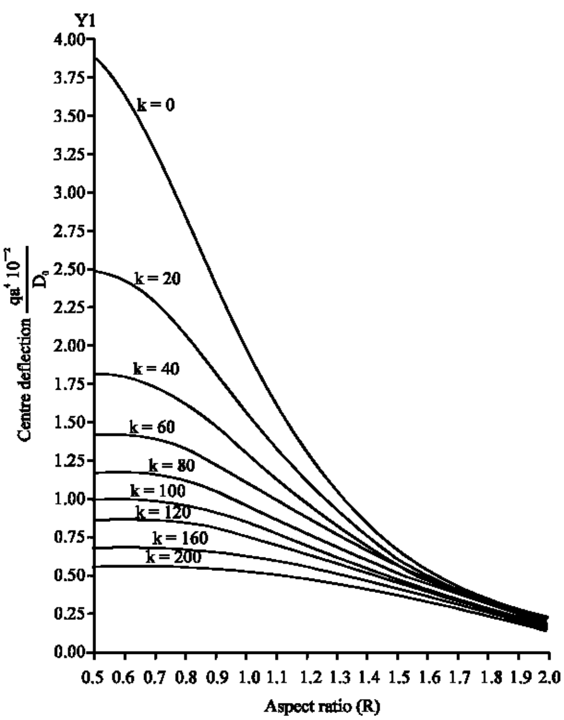

Fig. 7: Variation of centeral deflection with aspect ratio and foundation modules for rectangular plates of variable thickness with taper parameter C = 0.2

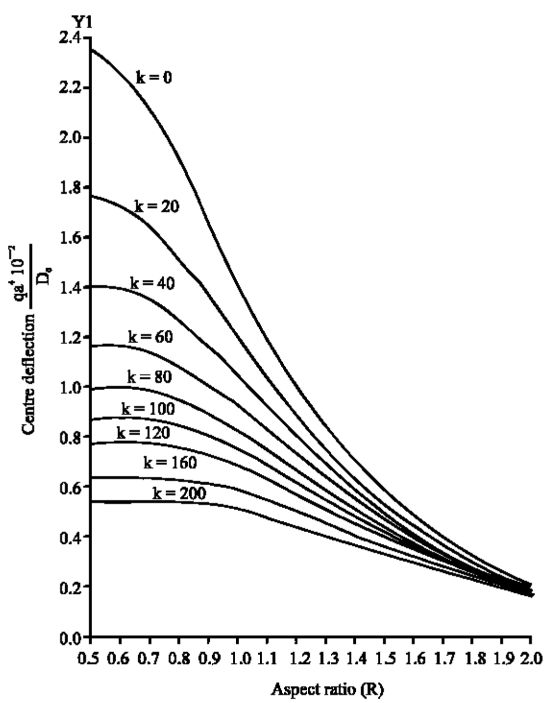

Fig. 9: Variation of centeral deflection with aspect ratio and foundation modules for rectangular plates of variable thickness with taper parameter C = 0.8

The centre deflections of clamped rectangular plates of variable thickness on a continuous elastic foundation are shown graphically in Fig. 4-9. Figure 4-6 show the centre deflection of plates of variable thickness on elastic foundations for various aspect ratios. Figure 7-9 show the centre deflection of clamped of variable thickness on elastic foundation for various taper parameters. As can be expected the centre deflection decreases with an increase in the foundation modulus. It is also investigating to note that the curves tend to become linear with an increase in the modulus of the elastic foundation.

CONCLUSION

From the present study it can be concluded the Galerkin's method is both effective and very simple to apply. The method yields very accurate results for deflection analysis of clamped rectangular plates of variable thickness and the requirements for computer facilities are very modest in comparison with other numerical methods.

REFERENCES

Banerjee, M.M., 1979. On The Vibration of Skew Plates of Variable Thickness. J. Sound and Vibrat., 63: 377-383.

Bastin, H., M. Delchanbre and P. Janssens, 1972. Bending of Rectangular Plates of Linearly Varying thickness. ZAM, 23: 35-42.

Chen, S.S.H., 1976. Bending and Vibration of Plates of Variable Thickness. J. Eng. Indust. Trans. ASME, Series B, 98: 166-170.

Conway, H.D. and N.Y. Ithaca, 1951. Axilly Symmetrical Plates with Linearly Varying Thickness. J. Applied Mechanics, pp: 140-142.

Conway, H.D., 1953. Closed From Solution for Plates of variable thickness. J. Applied Mechanics, Trans. ASME, Series E, 20: 564-564.

Conway, H.D., 1958. A levy type Solution for Rectangular Plate of Variable Thickness. J. Applied Mechanics, Trans. ASME, 80: 297-298.

Ng, S.F. and M.Y. Chan, 1977. Solution of Somme Boundary Value Problems in Applied Mechanics by Collocation least Square Method. Computer Methods In Applied Mechanics and Ing, 11: 137-150, 1-3.

Petrina, P. and H.D. Conway, 1972. Deflection and Moment Data For Rectangular Plates of Variable Thickness J. Applied Mechanics, Series E, 39: 814-815.

Sladek, J., V. Sladek and H.A. Mang, 2002. Meshless local boundary integral equation method for simply supported clamped plates resting on elastic foundation. Comput. Methods Applied Mech. Engreg., 191: 5943-5959.

Soong, T.C., 1972. A Non linear Method Applied to Variable Thickness Plates. AIAA J., 10: 148-154.

Szilard, R., 1974. Theory and Analysis of Plates, Classical and Numerical Method. Prentice Hall, Inc. Englewood Cliff, New Jersey.