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Dynamical Behaviour of Axial Force Rayleigh Beam Traversed by Uniform Partialy Distributed Moving Loads

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Abstract: An investigation into the dynamical behaviour of axial force Rayleigh beam traversed by uniform partially distributed moving loads is carried out. The beam is assumed to be Prismatic while the shear deformation, rotatory inertia and damping are taking into consideration. The resulting coupled partial differential equation is solved using finite difference method. Graphs were prepared for the results obtained. It was found that the response amplitude for the moving mass problem is greater than the response amplitude of the moving force problem.

Key words: Dynamical, axial, deformation, rotatory, amplitude

INTRODUCTION

The dynamic response of elastic structures subjected to a moving load is an interesting problem in several fields of Applied Mathematics, Applied Physics and Engineering. The problem has been studied by many authors and continues to motivate a variety of investigations, among them are; (Timoshenko, 1992; Inglis, 1934; Stanisic and Hardin, 1969; Gbadeyen and Oni, 1992; Ghorashi and Esmailzadeh, 1993, 1994, 1995; Leech and Tabarrok, 1970; Adetunde and Akinpelu, 2005; Adetunde, 2003; Akinpelu, 2003; Cifuentes, 1989).

Timoshenko (1992) he considered the case of a pulsating load passing over a bridge, while Inglis (1934) performed an analysis of train crossing a bridge and considered many important factors such as the effect of the moving load, the influence of damping and suspension of locomotives.

The dynamic analysis of a simply supported beam carrying a moving mass was carried out by Stanisic and Hardin (1969) is interesting enough but not easily applicable to different boundary conditions. But Gbadeyen and Oni (1992) considered vibration of beams with time dependent boundary conditions under the action of moving masses. Two prominent publications concerning the behavior of a beam carrying a moving concentrated mass under different situations are those of (Leech and Tabarrok, 1970; Cifuentes, 1989).

Ghorashi (1993, 1994, 1995) has investigated many cases of moving load problems. The vibration of an Euler Bernoulli beam traversed by uniform partially distributed moving mass has also been studied (Ghorahi, 1993).

The present research extends the scope of the previous study (Adetunde and Akinpelu, 2005) by considering the dynamical behaviour of axial Force Rayleigh beam traversed by uniform partially distributed moving loads. In the present research, the following assumptions are adopted.

- The beam is assumed to be of constant cross-section with uniform mass distribution.
- The dynamic characteristics of the beam are described by the axial force Rayleigh beam equation.
- The effect of the inertia for both the beam and moving mass are taken into consideration with the gravitational effect of load.
- The load moves with a uniform speed on the beam.
- The computations are performed for simply supported boundary conditions. Finally as to the initial conditions the beam it assumes to be free of the load.

MATHEMATICAL PROBLEM FORMULATION

Consider the Vibration of a uniform simply supported axial force Rayleigh beam carrying a mass M. The load is assumed to start entering the beam of length L, from the left hand support at t = 0, with a constant speed V. The mass is also assumed to be uniformly distributed over a fixed length of the beam (Fig. 1). Although the beam considered is simply supported at either end, the analysis and the formulation are not limited to the boundary condition. The governing differential equation describing the lateral vibration of a beam carrying the time varying force, F (x, t) per unit length is

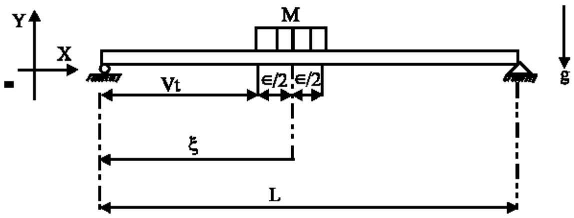


Fig. 1: Mathematical model of the problem

$$\frac{H_1\partial^4 y(x,t)}{\partial x^4} + \frac{\partial^2 y(x,t)}{\partial t^2} - \frac{H_2\partial^4 y(x,t)}{\partial x^2\partial t^2} - \frac{H_3\partial^2 y(x,t)}{\partial x^2} = F(x,t) \qquad (1)$$

Where $H_1 = \frac{EI}{m}$ = EI-Flexural rigidity of the beam.
E-Young modulus of elasticity
I(x)-the variable moment of inertia
m = mass per unit length of the beam

$H_2 = \frac{b^2}{m}$ = b = radius of gyration
m = as defined above

H_3 = Prestressed constant term
y(x, t) = the transverse displacement
t = time
x = spatial coordinate
F(x, t) = moving force

while the time varying force F(x, t) per unit length acting on the beam is defined as

$$F(x,t)=\frac{1}{\in}[-Mg - M\Delta][H(x-\xi+\frac{\in}{2})-Hex-\xi-\frac{\in}{2})] \qquad (2)$$

$$H(x)=\begin{cases}0 & x<0\\ 1 & x>0\end{cases} \qquad (3)$$

Where

M = Mass of the moving load;
∈ = is the fixed length of the load.
ξ = is a particular distance along the beam.
H = Heaviside Unit function
g = acceleration due to gravity

$$\Delta = \frac{\partial^2 y}{\partial t^2} + \frac{\partial^2 y}{\partial x\partial t} + \frac{V^2\partial^2 y}{\partial x^2} \qquad (4)$$

Using Eq. 2 and 4 in 1 resulted to the governing equation of motion

$$\frac{H_1\partial^4 y(x,t)}{\partial x^4} + \frac{\partial^2 y}{\partial t^2} - \frac{H_2\partial^4 y(x,t)}{\partial x^2\partial t^2} - \frac{H_3\partial^2 y(x,t)}{\partial x^2} = \frac{1}{\in}\left[P + \frac{G_1\partial^2 y(x,t)}{\partial x^2} + \frac{G_2\partial^2 y(x,t)}{\partial t\partial x} + \frac{G_3\partial^2 y(x,t)}{\partial x^2}\right]\left[H(x-\xi+\frac{\in}{2})-H(x-\xi-\frac{\in}{2})\right] \qquad (5)$$

Where

$$G_1=\frac{M}{m},\ G_2=2V_2G_1,\ G_3=V_2{}^2G_1,\ P=\frac{Mg}{m}$$

From Eq. 5, the first term in the square brackets of Eq. 5 describes the constant gravitational force, while the other terms accounts for the effect of the moving mass.

Boundary conditions: Equation 5 is subject to the following end support at the end x = 0 one of these holds.

$$\left.\begin{matrix} y(x,t)=0=y'(x,t); & y(x,t)=0=y''(x,t) \\ y'''(x,t)=0=y''(x,t); & y(x,t)=0=y'''(x,t); \end{matrix}\right\} \qquad (6a)$$

and the initial conditions are

$$y(x,0)=\frac{\partial y}{\partial t}(x,0)=0 \qquad (6b)$$

Operational simplification of the equations: To solve the problems, an approximate solution describing the transverse vibration of the beam is used,

$$y(x,t)=\sum_{i=1}^{\infty}\varnothing_i(t)X_i(x) \qquad (7)$$

Where $\varnothing_i(t)$'s are the unknown functions of time
$X_i(x)$ is the known normalized deflection curves
Introducing Eq. 7 into Eq. 1 we have

$$\sum_{i=1}^{\infty}\phi_i(t)X_i^{iv}(x) + \sum_{i=1}^{\infty}\ddot{\phi}_i(t)X_i(x) - H_2\sum_{i=1}^{\infty}\ddot{\phi}_i(t)X_i''(x) - H_3\sum_{i=1}^{\infty}\phi_i(t)X''_i(x) = \frac{1}{\in}\left[-P - G_1\sum_{i=1}^{\infty}\ddot{\phi}_i(t)X_i(x) - G_2\sum_{i=1}^{\infty}\dot{\phi}_i(t)X_i'(x) - G_3\sum_{i=1}^{\infty}\phi_i(t)X_i''(x)\right]\left[H\left(x-\xi+\frac{\in}{2}\right)-H\left(x-\xi-\frac{\in}{2}\right)\right] \qquad (8)$$

Remark: Since the r.h.s of equation is

$$F(x,t)=\sum_{i=1}^{\infty}\phi_i(t)X_i(x)$$

multiply through the r.h.s of Eq. 8 by $x_i(x)$ and integrate along the length of the beam, we have;

$$\sum_{i=1}^{\infty}\phi_i(t)\int_0^L X_i(x)X_j(x)dx=-\frac{p}{\in}\int_0^L X_i(x)\left[H\left(x-\xi+\frac{\in}{2}\right)-H\left(x-\xi-\frac{\in}{2}\right)\right]$$
$$dx-\frac{G_1}{\in}\sum_{i=1}^{\infty}\ddot{\phi}_i(t)\int_0^L X_i(x)X_j(x)dx-\frac{G_2}{\in}\sum_{i=1}^{\infty}\dot{\phi}_i(t)\int_0^L X_i'(x)X_j(x)\left[H\left(x-\xi+\frac{\in}{2}\right)-H\left(x-\xi-\frac{\in}{2}\right)\right]dx-\tag{9}$$
$$\frac{G_3}{\in}\sum_{i=1}^{\infty}\dot{\phi}_i(t)\int_0^L X_i''(x)X_j(x)\left[H\left(x-\xi+\frac{\in}{2}\right)-H\left(x-\xi-\frac{\in}{2}\right)\right]dx$$

By orthogonality property

$$\int_0^L X_i(x)X_j(x)dx=1$$

for i = j then follow reference (Ghorashi and Esmailzadeh, 1994, 1995; Adetunde, 2003; Akinpelu, 2003) we have Eq. 9 becomes;

$$\phi_i=\frac{1}{\in}\left[-P\left[X_i(\xi)+\frac{\in^2}{24}X_i''-G_1\sum_{i=1}^{\infty}\ddot{\phi}_i(t)\left[X_i(\xi)X_j(\xi)+\frac{\in^2}{24}\left[X_i''(\xi)X_j(\xi)+2X_i'(\xi)X_j(\xi)\ +\ X_i(\xi)X_j''(\xi)\right]\right]\right.\right.$$
$$-G_2\sum_{i=1}^{\infty}\ddot{\phi}_i(t)\left[X_i'(\xi)X_j(\xi)+\frac{\in^2}{24}\left[X_i'''(\xi)X_j(\xi)+2X_i''(\xi)X_j'(\xi)\ +\ X_i'(\xi)X_j''(\xi)\right]\right]-\tag{10}$$
$$G_3\sum_{i=1}^{\infty}\ddot{\phi}_i(t)\left[X_i''(\xi)X_j(\xi)+\frac{\in^2}{24}\left[X_i^{iv}(\xi)X_j(\xi)+2X_i''(\xi)X_j''(\xi)\ +\ X_i''(\xi)X_j''(\xi)\right]\right]$$

Hence Eq. 10 can be written as

$$F(x,t)=\sum_{i=1}^{\infty}\phi_i(t)X_i(x)=\sum_{i=1}^{\infty}X_i(x)\left\{\frac{1}{\in}\left[\left[-P\left[X_i(\xi)+\frac{\in^2}{24}X_i''(\xi)\right]\right]-\right.\right.$$
$$G_1\sum_{i=1}^{\infty}\ddot{\phi}_i(t)\left[X_i(\xi)X_j(\xi)+\frac{\in^2}{24}[X_i''(\xi)X_j(\xi)+2X_i'(\xi)X_j'(\xi)+X_i(\xi)X_j''(\xi)]\right]$$
$$-G_2\sum_{i=1}^{\infty}\dot{\phi}_i(t)\left[X_i'(\xi)X_j(\xi)+\frac{\in^2}{24}[{}'''X_i(\xi)X_j(\xi)+2X_i''(\xi)X_j'(\xi)+X_i'(\xi)X_j''(\xi)]\right]\tag{11}$$
$$\left.\left.-G_3\sum_{i=1}^{\infty}\phi_i(t)\left[X_i''(\xi)X_j(\xi)+\frac{\in^2}{24}\left[X_i^{iv}(\xi)X_j(\xi)+2X_i''(\xi)X_j''(\xi)\ +\ X_i''(\xi)X_j''(\xi)\right]\right]\right]\right\}$$

Substitution this Eq. 11 back into Eq. 1 we have;

$$H_i\sum_{i=1}^{\infty}\phi_i(t)X_i^{iv}(x)+\sum_{i=1}^{\infty}\phi_i(t)X_i(x)-H_2\sum_{i=1}^{\infty}\ddot{\phi}_i(t)X_i''(x)-H_3\sum_{i=1}^{\infty}\phi_i(t)X_i''(x)$$
$$=\sum_{i=1}^{\infty}X_i(x)\left\{\frac{-1}{\in}\left[\left[P\left[X_i(\xi)-\frac{\in^2}{2}X_i''(\xi)\right]\right]\right.\right.$$
$$-G_1\sum_{i=1}^{\infty}\ddot{\phi}_i(t)\left[X_i(\xi)X_j(\xi)+\frac{\in^2}{24}\left[X_i''(\xi)X_j(\xi)+2X_i'(\xi)X_j'(\xi)+X_i(\xi)X_j''(\xi)\right]\right] \tag{12}$$
$$-G_2\sum_{i=1}^{\infty}\dot{\phi}_i(t)\left[X_i'(\xi)X_j(\xi)+\frac{\in^2}{24}\left[X_i'''(\xi)X_j(\xi)+2X_i''(\xi)X_j'(\xi)+X_i'(\xi)X_j''(\xi)\right]\right]-$$
$$\left.\left.G_3\sum_{i=1}^{\infty}\phi_i(t)\left[X_i''(\xi)X_j(\xi)+\frac{\in^2}{24}\left[X_i^{iv}(\xi)X_j(\xi)+2X_i''(\xi)X_j''(\xi)+X_i''(\xi)X_j''(\xi)\right]\right]\right]\right\}$$

Free vibration system: For free Vibration system see (Ghorahi, 1994, 1995; Adetunde and Akinpelu, 2005), we have

$$X^{IV}(x)-B_i^4X_i(x)=0 \tag{13}$$

Where

$$B_i^4=\frac{m\lambda^2{}_i}{\in I}$$

$$\Rightarrow\quad X_i^{iv}(x)=B_i^4X_i(x)=\frac{m\lambda^2{}_iX_i(x)}{\in I} \tag{14}$$

Where λ_I is the i[th] Natural frequency of the beam, putting Eq. 14 into Eq. 12 we have

$$\sum_{i=1}^{\infty}\phi_i(t)X_i(x)\lambda_i^2+H_1\sum_{i=1}^{\infty}\ddot{\phi}_i(t)X_i(x)-H_2\sum_{i=1}^{\infty}\ddot{\phi}_i(t)X_i''(x)$$
$$-H_3\sum_{i=1}^{\infty}\phi_i(t)X_i''(x)=\sum_{i=1}^{\infty}X_i(x)\left\{\frac{1}{\in}\left[\left[-P\left[X_i(x)-\frac{\in^2}{24}X_i''(x)\right]\right]\right.\right.$$
$$-G_1\sum_{i=1}^{\infty}\ddot{\phi}_i(t)\left[X_i(x)X_j(x)+\frac{\in^2}{24}\left[X_i''(x)X_j(x)+2X_i'(x)X_j'(x)+X_i(x)X_j''(x)\right]\right] \tag{15}$$
$$G_2\sum_{i=1}^{\infty}\dot{\phi}_i(t)\left[X_i'(x)X_j(x)+\frac{\in^2}{24}\left[X_i'''(x)X_j(x)+2X_i''(x)X_j'(x)+X_i'(x)X_j''(x)\right]\right]$$
$$\left.\left.-G_3\sum_{i=1}^{\infty}\phi_i(t)[X_i''(x)X_j(x)+\frac{\in^2}{24}\left[X_i^{iv}(x)X_j(x)+2X_i'''(x)X_j'(x)+X_i''(x)X_j''(x)\right]\right]\right]\right\}$$

Equation 15 is now the foundamental equation of our problem, which is a set of coupled linear second order differential equations. In solving Eq. 13 the values of $\phi_i(t)$'s can be determined which will help in solving Eq. 7 i.e., the derived solution for the Vibration of the beam.

Since our elastic system has simply support at the edges x = 0 and x = L, we choose the normalized deflection

$$X_i(x)=\sqrt{\frac{2}{L}}\,\mathrm{Sin}\,\frac{iJIXi}{L} = 1,2,3... \tag{16}$$

Direct substitution of (16) into (15) will yield the desired governing equation which is however an approximate one. It is remarked that for the configuration under discussion an exact differential governing equation can be defined by going through arguments similar to these used in obtaining system (15). Hence by substitution (16) into (15) we finally have

$$\sqrt{\frac{2}{L}}\sin\frac{i\pi x}{L}\sum_{i=1}^{\infty}\phi_i(t)\lambda_i^2 + H_1\sum_{i=1}^{\infty}\ddot{\phi}_i(t) + H_2\sum_{i=1}^{\infty}\ddot{\phi}_i(t)\left(\frac{i\pi x}{L}\right)^2 + H_3\sum_{i=1}^{\infty}\phi_i(t)\left(\frac{i\pi x}{L}\right)^2 =$$

$$\sqrt{\frac{2}{L}}\sin\frac{i\pi x}{L}\Bigg\{\frac{-P}{i\pi\in}\sqrt{8}\sin\frac{i\pi}{L}\sin\frac{i\pi\in}{L} - \frac{2G_1}{\in\pi}\sum_{i=1}^{\infty}\ddot{\phi}_i(t)\frac{1}{(i-j)}\left[\sin\frac{\pi\in(i-j)}{2L}\cos\frac{\pi\xi(i-j)}{L}\right]$$

$$+2G_1\sum_{i=1}^{\infty}\ddot{\phi}_i(t)\frac{1}{(i+j)}\left[\cos\frac{\pi\xi(i+j)}{L}\sin\frac{\pi\in(i+j)}{2L}\right] - \frac{2G_2(i\pi)}{\in L}\sum_{i=1}^{\infty}\dot{\phi}_i(t)\Bigg[\Bigg(\sqrt{\frac{2}{L}}\Bigg)i\Bigg[\Bigg(\sin\frac{\pi\xi(i+j)}{L}\Bigg)\sin\frac{\pi\in}{2L}(i+j)\Bigg)+ \tag{17}$$

$$\Bigg(\sin\frac{\pi\xi(i-j)}{L}\sin\frac{\pi\in(i-j)}{2L}\Bigg) + \frac{2G_3(i\pi)}{\in}\sum_{i=1}^{\infty}\phi_i(t)\sqrt{\frac{2}{L}}\frac{1}{(i+j)}\Bigg[\Bigg(\sin\frac{\pi\xi(i+j)}{L}\Bigg)\cos\frac{\pi\in}{2L}(i+j)\Bigg)+$$

$$\cos\frac{\pi\xi}{L}(I=i+j)\sin\frac{\pi\in}{2L}(i+j)\Bigg)\Bigg]\Bigg]\Bigg\}\; i=1,2,3,...$$

$$i\neq j$$

The above Eq. 17 is the exact governing equation of a simply supported Beam. We now use the finite difference method in order to solve the above question Numerically. To obtain the results, we made use of approximate central difference formula.

RESULTS AND DISCUSSION

To solve Eq. 17, we made use of approximate Central difference formula for the derivatives in (17). Thus for N modal shapes, Eq. 17 are transformed to a set of N linear algebraic equations which are to be solved for each interval of time.

As an illustration, the uniform Axial force Rayleigh beam is taken to be 10 m long, (L = 10 m), $V = 12\ km\ h^{-1}$. E = 2.07x10, $M = 70\ kg\ m^{-1}$, $m = 7.04\ kg\ m^{-1}$, time = 0.01sec b = 0.5, $I = 1.04 \times 10^{-6} m^4$, h = 0.1

The analysis was carried out separately for both moving force case and moving mass case. The results are shown on the various graphs.

In the numerical analysis, the analysis was carried out separately for both cases of moving force and moving mass problems.

They dynamic response of the axial force Rayleigh beam to a moving force is displayed in (Fig. 2), it is evident that the response amplitude for the moving mass problem is greater than the response amplitude for the moving force problem.

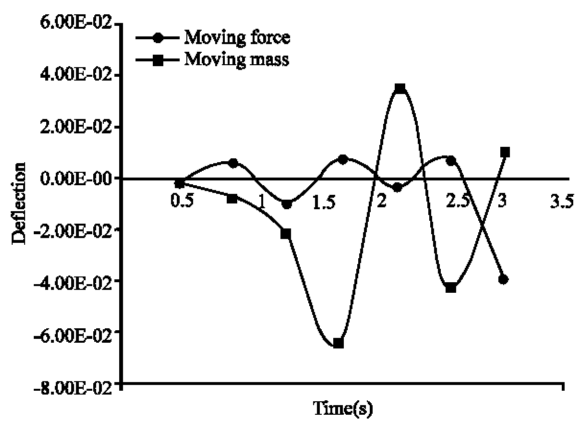


Fig 2: The response amplitude of the axial force rayleigh beam for the moving force and moving mass

Figure 3, shows that deflection of the axial force Rayleigh beam moving force for a particular value of the mass of the load (that is M = 7.04 kg with constant $\in = 0.1$ m at different values of time t (t = 0.5, 1, 1.5 sec) It was observed that as t increases the deflection decreases.

Figure 4, illustrates the response of the axial force Rayleigh beam moving mass for a particular value of the

Fig. 3: The deflection of the axial force rayleigh beam for moving force, for a particular value of the mass of the load m = 7.04 kg with a constant ϵ = 0.1m at different values of time t (t = 0.5s, t = 1.05s and t = 1.55s)

Length of the beam X(m)	When t = 0.5 sec.	When t = 1.0 sec.	When t = 1.5 sec.
1.429	-3.40E-08	-3.50E-08	-6.00E-08
2.88	2.59E-07	2.41E-07	4.06E-07
4.306	-3.10E-07	3.02E-07	-5.08E-07
5.726	1.32E-07	1.32E-07	2.20E-07
7.145	8.39E-07	8.44E-07	1.42E-06
8.564	-1.79E-06	-1.83E-06	-3.08E-06
9.983	2.66E-06	2.74E-06	4.61E-06

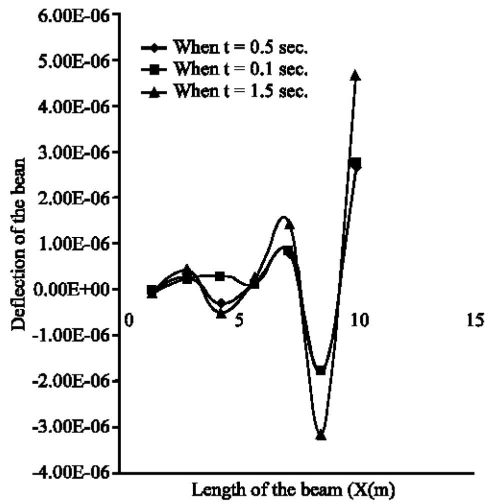


Fig 4: The deflection of the axial force rayleigh beam for a particular m = 7.04 kg m^{-1} with a constant value of ϵ = 0.1 m at different values of time t(i.e., t = 0.5, 1.5 and 1.0 sec)

Length of the beam X(m)	When t = 0.5 sec.	When t = 1.0 sec	When t = 1.5 sec.
1.429	-2.96E-06	-5.71E-05	-8.72E-05
2.88	-8.19E-05	-1.52E-04	-2.30E-04
4.306	-3.47E-05	-5.83E-05	-8.45E-05
5.726	8.45E-05	1.60E-04	2.42E-04
7.145	2.40E-05	3.78E-05	5.38E-05
8.564	-2.77E-04	-5.11E-05	-7.68E-04
9.983	-4.90E-04	-8.81E-04	-1.31E-03

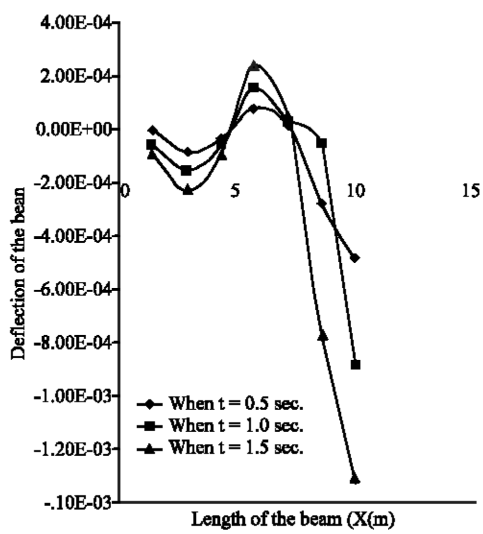
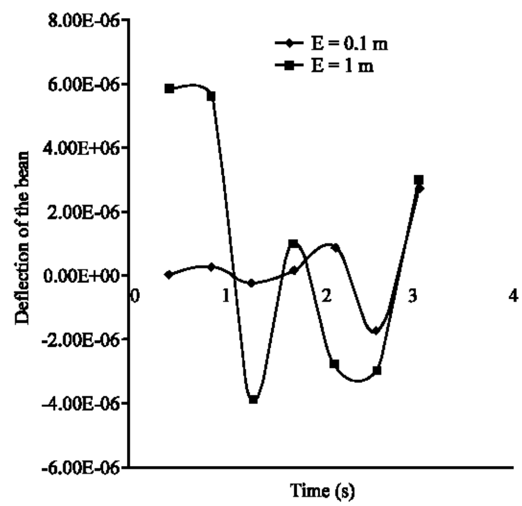


Fig. 5: The displacement of axial force rayleigh beam for the moving force for different values of ϵ = 0.1 and 1 m

Length of the beam X(m)	When ϵ = 0.1 m	When ϵ = 1.0 m
0.43	-3.40E-08	5.80E-06
0.86	2.59E-07	5.55E-06
1.29	-3.10E-07	-3.96E-06
1.72	1.32E-07	9.65E-07
2.15	8.39E-07	-2.85E-06
2.58	-1.79E-06	-3.01E-06
3.01	2.66E-06	2.93E-06



mass the load (that is M = 7.04 kg with constant ϵ = 0.1 m at different values of time t (t = 0.5, 1, 1.5sec) It was observed that as t increases the deflection decreases which bring about increased in the amplitude.

Figure 5, shows the displacement of axial force Rayleigh beam moving force for different values of ϵ(ϵ = 0.1 and 1 m) it was observed that as ϵ increases, the deflection of the beam increases.

CONCLUSION

The dynamical behaviour of axial force Rayleigh beam traversed by uniform partially distributed moving loads. The theory based on orthogonal functions and inertia effect of load and the results indicate that the governing differential equation of motion can be transformed into a coupled ordinary differential equation.

Numerical analysis was carried out and the work exhibits the following interesting features.

- Ignoring this effect (inertial effect) result in solving a set of uncoupled linear second order differential equations which is the solution of the corresponding moving distributed force and not the moving mass problem.

- In solving the governing differential equations (moving distributed mass problem) the technique of Central difference expansion was employed. It was observed that the length of the distributed moving mass affect the dynamic response considerably.
- The response amplitude for the moving mass problem is greater than the response amplitude of the moving force problem.
- If the rotatory inertial term is zero then the resulting problem leads to Euler Bernoulli problem.
- It was also observed that as time t increases for both cases of our consideration the deflection decreases.
- Finally, a comparison of the moving mass and moving force (Fryba, 1971) results indicate and at least 80% difference between two results and thus shows the importance of moving load having a grate effect on dynamics stress of the bodies or structures of consideration and causes them to vibrate intensively, especially when the velocity is high.

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