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Dynamical Behaviour of Viscously Damped Rayleigh Beam Traversed by Uniform Partially Distributed Moving Masses

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Abstract: An investigation into the dynamic behaviour of Viscously damped Rayleigh beam traversed by a uniform spatially distributed moving masses is carried out. The beam is assumed to be prismatic while the effects of two kinds of pressure are considered, that is the moving load and moving force. The pertinent governing partial differential equations, the boundary and initial conditions were analyzed by a series solution in terms of the normalized deflection curve of the beam and the unknown functions of time. This resulted into a set of coupled ordinary differential equations which are numerically solved by the finite difference scheme with the aid of a Visual Basic 6.0 programme. It was observed that the response amplitude of a Rayleigh beam due to moving mass increased as mass and length of the load increased, whilst the amplitude was found to reduce with increasing time. It was further observed that the response amplitude due to moving force was greater than the response amplitude due to moving mass.

Key words: Viscously damped rayleigh beam, moving force and moving mass

INTRODUCTION

As a matter of fact the dynamic response of elastic structures(e.g beams) subjected to moving loads is an interesting problem in several Fields of applied Mathematics, Engineering and applied Physics. This study is of considerable practical importance in the designing and construction of bridges, (on which vehicles or trains travel), vibration of turbines, hulls of ships and trolleys of overhead traveling cranes that move on their Girders (Bridge girders).

Since the middle of the last century when railway construction began, several investigations have concerned themselves with the problem of analysis of Structures due carrying moving loads. Fryba (1972), presented a comprehensive review of the subject. Vibration of Structures due to moving loads. Robes Willis (1949) was the first to considered the problem of elastic beam under the action of moving loads.He made the assumption that the mass of the beam is smaller than the mass of the load.He obtained an approximate solution. Stokes (1849) presented the problem under similar assumptions, while (Krylov, 1995) considered the mass of the load to be smaller than the mass of the beam.

Sir Charles Inglis (1934) presented detailed analysis of train crossing a bridge, in his own work he took into consideration many important factors such as the effects of moving loads,the influence of damping and spring suspension of the locomotive. Stephen Timoshenko (1927) considered the case of a pulsating load crossing over a bridge, in which he found a solution and presented an expression for the critical velocity, neglecting damping forces.

Despite the importance of the subject matter, very little had been done on dynamical behaviour of viscously damped beams.

Therefore,the real core of this study is to examine the dynamical behaviour of viscously damped Rayleigh beam traversed by uniform partially distributed moving masses.

The governing partially differential equation for the viscously damped rayleigh beam: The equation of motion that governs the behaviour of viscously damped Rayleigh beam is

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{(\partial^2 W(x,t))}{\partial x^2} + a_1\frac{\partial^3 W(x,t)}{\partial x \partial t}\right] + m(x)\frac{\partial^2 W(x,t)}{\partial t^2} + C(x)\frac{W(x,t)}{\partial t} = P(x,t) \quad (1)$$

Where

E = Young Modulus of Elasticity
I = Second Moment of Area
m = mass per unit length
a_1 = is the stiffness proportionality factor (damping complex) (residue of gyration)
W(x,t) = transverse displacement response
x = spatial coordinates
t = time
P(x,t) = The transverse loading
C(x) = External damping force per unit length

The transverse loading P(x,t) takes the form (2)

$$P(x,t) = \frac{1}{\in}[-Mg - M\Delta][H(x - \xi + \frac{\in}{2}) - H(x - \xi - \frac{\in}{2})] \quad (2)$$

$$H(x) = \begin{cases} 0 & x < 0 \\ 1 & x > 0 \end{cases} \quad (3)$$

Where

M = is the mass of the load.
Δ = the substantive acceleration operator

$$\Delta = \frac{\partial^2 W}{\partial t^2} + \frac{2V\partial^2 W}{\partial x \partial t} - \frac{V^2 \partial^2 W}{\partial x^2} \quad (4)$$

g = The acceleration due to gravity
V = The constant Velocity
H = The Heaviside Unit Function
∈ = The fixed length of the beam
ξ = Vt + ∈/2 for the limiting condition as (aparticular distance along the length of the beam). ∈>0

$$\delta(x - \xi) = \frac{1}{\in}[H(x - \xi + \frac{\in}{2}) - H(x - \xi + \in)] \quad (5)$$

Where δ(x - î) is the Dirac delta function.
Recall the Dirac Delta Integral Properties

$$\int_0^L f(x)\delta(x - x_o)dx = f(x_o) \qquad 0 < x,\ x_o < L$$

$$\int_0^L f(x)\delta(x - x_o)dx = 0 \qquad x < 0,\ x > L$$

Substituting Eq. 4 into Eq. 2 we have

$$P(x,t) = \frac{1}{\in}[-Mg - \frac{M\partial^2 W}{\partial t^2} - \frac{2MV\partial^2 W}{\partial t^2} + \frac{MV^2\partial^2 W}{\partial x^2}] [H(x - \xi + \frac{\in}{2}) - H(x - \xi - \frac{\in}{2})] \quad (6)$$

In Eq. 6, the first term in the first squared bracket describes the constant gravitational force, while the second term accounts for the effect of acceleration in the direction of the transverse deflection W(x,t); the third term is for the complementary acceleration and the fourth term is for the centripetal acceleration. The second square bracket describes the Heaviside Unit function.

Considering Eq. 5 and 6 leads to the formulation of moving point masses (Craig, 1981; Plikey and Plikey, 1974; Timoshenco, *et al.*, 1974) in the operational simplification of the governing equation.

Operational simplification of the governing equation: We assume the transverse vibration of the beam as

$$W(x,t) = \sum_{i=1}^{\infty} \phi_i(x)Yi(t) \quad (7)$$

Where $\varnothing_i(x)$'s are the normalized deflection curve for the i^{th} mode of the Vibrating prismatic beam. $Y_i(t)$ are the unknown functions of time

Substituting Equ. (7) into Equ. (1) and (6), Equ. (1) becomes

$$\sum_{i=1}^{\infty} m(x)\phi_i(x)Yi(t) + \sum_{i=1}^{\infty} c(x)\phi i(x)Yi(t) + \sum_{i=1}^{\infty} \frac{d^2}{dx^2}\left[\frac{a_1 EI(x)d^2\phi i(x)}{dx^2}\right]Yi(t) + \sum_{i=1}^{\infty} \frac{d^2}{dx^2}\left[\frac{EI(x)d^2\phi i(x)}{dx^2}\right]Yi(t) = \frac{1}{\in}[-Mg - M\sum_{i=1}^{\infty} Yi(t)\phi i(x) - 2MV\sum_{i=1}^{\infty} \frac{Yid\phi i(x)}{dx} - MV^2\sum_{i=1}^{\infty} \frac{Yi(t)\ d2\phi i(x)}{dx^2}][H(\phi - \xi + \frac{\in}{2}) - H(\phi - \xi + \frac{\in}{2}) \quad (8)$$

Multiply Eq. 8 by $\varnothing_n(x)$ and integrate along the length of the beam and applying the two orthogonality relationships, to the Eq. 8

$$m\sum_{i=1}^{\infty} Y_i(t)\int_0^L \phi_i(x)\phi_n(x)dx + C(x)\sum_{i=1}^{\infty} Y_i(t)\int_0^L \phi i(x)\phi n(x)dx + \int_0^L \phi_n(x)[\sum_{i=1}^{\infty} \frac{d_2}{dx^2}[a_1 EI(x)d^2\phi_i(x)] Y_i(t)]dx + \int_0^L \phi_n(x)[\sum_{i=1}^{\infty} \frac{d^2}{dx^2}[EI(x)\frac{d^2\phi_i(x)}{dx^2}]Y_i(t)]dx = \int_0^L \phi_n(x)[\frac{1}{\in}[-Mg - M\sum_{i=1}^{\infty} Y_i(t)\phi_i(x) - 2MV\sum_{i=1}^{\infty} Y_i(t)d\phi_i(x) - MV2\sum_{i=1}^{\infty} Y_i(t)\frac{d^2\phi_i(x)}{dx^2}] [H(\phi - \xi + \frac{\in}{2}) - H(\phi - \xi - \frac{\in}{2})]]d\theta \quad (9)$$

Remark:

$$\omega^2_n \; m(x)\phi_n(x)=\frac{d^2}{dx^2}[EI(x)\frac{d^2\phi n(x)}{dx^2}] \quad (9a)$$

$$\int_{i-1}^{L} \phi_n \frac{d^2}{dx^2}[EI(x)\frac{d^2\phi_i(x)}{dx^2}]dx = \omega^2_n \int_{i-1}^{L} \phi_n{}^2(x)m(x)dx \quad (9b)$$

$$\int_{i-1}^{L} \phi_n(x)\frac{d^2}{dx^2}[EI(x)\frac{d^2\phi_i(x)}{dx^2}]dx=0 \quad \text{when } i \neq {}^1 n, \quad (9c)$$

$$\int_{i-1}^{L} \phi_n(x)[\, H(\phi(x)-\xi+\frac{\in}{2})-H(\phi(x)-\xi-\frac{\in}{2}\,)]d\phi=\in[\phi i\,(\xi)+\frac{\in}{24}2\phi_i''(\xi)]\;\;]\text{-}(9d)$$

$$\int_{i-1}^{L} \phi_n(x)\phi_i(x)[\, H(\phi(x) - \xi+\frac{\in}{2})-H(\phi(x)-\xi-\frac{\in}{2})]d\phi=\frac{\in}{2}[\phi i(\xi)\phi n(\xi) + \frac{\in}{24}^2 [\phi_i''(\xi)\;\phi_n(\xi)+2\phi_i'(\xi)\phi_n'(\xi)+\phi_i''(\xi)\phi_n(\xi)] \quad (9e)$$

$$\int_{i-1}^{L} \phi_i'(x)\phi_n(x)\;[\, H(\phi(x)-\xi+\frac{\in}{2})-H(\phi(x)-\xi-\frac{\in}{2})]d\phi=\in[\phi_i'(\xi)\phi_n(\xi) +\frac{\in}{24}^2 [\phi_i'''(\xi)\;\phi_n(\xi)+{}^2\phi_i''(\xi)\phi_n'(\xi)+\phi_i'(\xi)\phi_n''(\xi)] \quad (9f)$$

$$\int_{i-1}^{L} \phi_i''(x)\phi_n(x)[H(\phi(x)-\xi+\frac{\in}{2})\text{- } H(\phi(x)-\xi-\frac{\in}{2})]d\phi=\frac{\in}{2}[\phi_i''(\xi)\phi_n(\xi) +\frac{\in}{24}^2 [\phi_i^{iv}(\xi)\phi_n(\xi)+2\phi_i'''(\xi)\;\phi_n'(\xi)+\phi_i''(\xi)\phi_n''(\xi)] \quad (9g)$$

Substituting Eq. 9a-9g into Eq. 9, we have.

$$M_nY_n(t)+\sum_{i=1}^{\infty} Y_i(t)\int_0^L \phi n(x)[C(x)\phi i(x)+\frac{d^2}{dx^2}[a1EI(x)\frac{d^2\phi_i(x)]dx}{dx^2}+w^0M_nY_n(t)=$$
$$\sum_{i=1}^{\infty} \phi_n(x)\;[-Mg[\phi_i(\xi)+\frac{\in}{24}^2 \phi i'']\text{-}M\sum_{i=1}^{\infty} Y_i(t)[\phi_i(\xi)\phi_n(\xi)+\frac{\in}{24}^2 [\phi_i''(\xi)\phi_n(\xi)+2\phi_i'(\xi)\phi_n'(\xi)+$$
$$\phi_i''(\xi)\phi_n(\xi)]]\text{-}2MV\sum_{i=1}^{\infty} Y_i(t)[\phi_i'(\xi)\phi_n(\xi)+\frac{\in}{24}^2 [\phi_i'''(\xi)\phi_n(\xi)+{}^2\phi_i''\,(\xi)\phi_n'(\xi)+\phi_i'(\xi)\phi_n''(\xi)]]\text{-}$$
$$MV^2\sum_{i=1}^{\infty} Y_i(t)[\phi_i''(\xi)\phi_n(\xi)+\frac{\in}{24}^2 [\phi_i^{iv}(\xi)\phi_n(\xi)+{}^2\phi_i'''\,(\xi)\phi_n'(\xi)+\phi_i''(\xi)\phi_n''(\xi)]] \quad (10)$$

Remarks: Because all terms in the series in the third term of Eq. 9 go to zero except for i=n the modes are obviously uncoupled as far as the stiffness proportional damping is concerned. Coupling will be present, however, due to c(x), unless it takes on a form allowing only the term with i = n to remain in the series. This is indeed the case for mass-proportional damping, that is, if we let $c(x) = a_0m(x) = a_0\,m$, in which the proportionality constant a_0 has a dimension of t^{-1}, then we have

$$M_nY_n(t)+(aoM_n+a1M_n\omega^2)Y_n(t)+\omega^2M_nY_n(t)= \sum_{i=1}^{\infty} \phi_n(\xi)\{\text{-}Mg[\phi_i(\xi)+\frac{\in}{24}^2 \phi_i''(\xi)]\text{-}M\sum_{i=1}^{\infty} Y_i(t)[\phi_i(\xi)\phi_n(\xi)+$$
$$\frac{\in}{24}^2 [\phi_i''(\xi)\phi_n(\xi)+2\phi_i'(\xi)\phi_n'(\xi)+\phi_i''(\xi)\phi_n(\xi)]]\text{-}2MV\sum_{i=1}^{\infty} Y_i(t)[\phi_i'(\xi)\phi_n(\xi)+\frac{\in}{24}^2 [\phi_i'''(\xi)\phi_n(\xi)+2\phi i''(\xi)\phi_n'(\xi)+$$
$$\phi_i'(\xi)\phi_n''(\xi)]]\text{-}MV^2\sum_{i=1}^{\infty} Y_i(t)[\phi_i''(\xi)\phi_n(\xi)+\frac{\in}{24}^2 [\phi_i^{iv}(?)\phi_n(\xi)+2\phi i'''\,(\xi)\phi_n'(\xi)+\phi_i''(\xi)\phi_n''(\xi)]]] \quad (11)$$

By introducing the damping ratio for the n^{th} mode, we have

$$\xi_n = \frac{Cn}{2M_n\omega_n} = \frac{ao}{2\omega n} = \frac{a_1\omega_n}{2} \quad (12)$$

Hence Eq. 11 now becomes

$$Y_n(t)+2\xi_n\omega_n Y_n(t)+\omega^2{}_n Y_n(t)=M\sum_{i-1}^{\infty}\phi_i(x)\{-g[\phi i(x)+ \frac{\in^2}{24}\phi i''(x)-\sum_{i-1}^{\infty}Y_i(t)[\phi_i(x)\phi_n(x)+\frac{\in^2}{24}[\phi_i''(x)\phi_n(x)+ 2\phi_i'(x)\phi_n'(x)+\phi_i''(x)\phi_n(x)]]-2V\sum_{i-1}^{\infty}Y_i(t)[\phi_i'(x)\phi_n(x)+ \frac{\in^2}{24}[\phi_i'''(x)\phi_n(x)+2\phi_i''(x)\phi_n'(x)+\phi_i'(x)\phi_n''(x)]]- V2\sum_{i-1}^{\infty}Y_i(t)[\phi_i''(x)\phi_n(x)+\frac{\in^2}{24}[\phi_i^{iv}(x)\phi_n(x)+ 2\phi_i'''(x)\phi_n'(x)+\phi_i''(x)\phi_n''(x)]]\} \quad (13)$$

SIMULATIONS

Simply supported beam: For the case of simply supported beam, the normalized deflection curves

$$\phi_i(x)=\sqrt{(2/L)}\ \mathrm{Sin}\ \frac{n\pi x}{L} \qquad n = 1,2,3.......... \quad (14)$$

$$\omega_n=n^2\pi^2\sqrt{\frac{(EI)}{MnL4}} \quad n= 1,2,3........ \quad (14a)$$

By substituting Eq. 14 into the r.h.s. of Eq. 13, after a lot of simplification had been done, we finally have

$$Y_n(t)+2\xi_n\omega_n Y_n(t)+\omega^2{}_n Y_i(t)=M[-1\ \sqrt{8}_L\ \mathrm{Sin}\ i\pi\xi$$

$$\mathrm{Sin}\ n\pi\xi-2\sum_{i-1}^{\infty}Y_i(t)[1\ \mathrm{Cos}\ \pi\xi(i-n)\ \mathrm{Sin}\ \pi(i-n)]$$

$$M_n \quad n\pi\xi \quad 2L \quad L\in p\ i=1 \quad (i-n) \quad L \quad 2L$$

$$\frac{+2}{\in\pi}\sum_{i=1}^{\infty}Y_i(t)[\ 1\ \mathrm{Cos}\ \frac{\pi\xi}{L}(i+n)\ \mathrm{Sin}\ \frac{\pi\in}{2L}(i+n)]$$

$$\sum_{i=1}^{\infty}\frac{-2V}{\in}Yi(t)[\frac{\sqrt{2}\ \ 1}{L\ \ (i+n)}\ \mathrm{Sin}\pi\xi\ (i+n)\ \mathrm{Sin}\ \frac{\in\pi}{L}(i+n)]$$

$$\frac{-2V}{\in}\sum_{i=1}^{\infty}Yi(t)[\frac{\sqrt{2}\ \ 1}{L\ \ (i-n)}\ \mathrm{Sin}\frac{\in\pi}{2L}(i-n)$$

$$\mathrm{Sin}\ \frac{\pi\xi}{L}(i-n)]+\frac{V2}{\in}\frac{(i\pi)}{L}\sum_{i=1}^{\infty}Y_i(t)[\frac{\sqrt{2}\ 1}{L\ \ (i+n)}$$

$$\frac{\mathrm{Sin}\in\xi\ (i+n)}{2L}\ \mathrm{Sin}\frac{\pi\xi}{L}(i+n)] \quad (15)$$

The above Eq. 15 is the exact governing Equation of a simply supported viscously damped Rayleigh beam. We now use the finite difference method to solve the above Eq. 15 numerically. To obtain the results, we make use of central difference formula, which finally resulted into 9 system of equations which was in turn solved by a Visual Basic programme.

RESULTS AND DISCUSSION

The numerical analysis is divided into two. The first one concerns a viscously damped Rayleigh beam traversed by a uniform partially distributed moving mass whilst the second concerns a Rayleigh beam traversed by a uniform partially distributed moving force.

For the purpose of numerical analysis, the following data was used.

m (Mass per unit length of beam) = 70 Kg m^{-1}, M(Mass of the load) = 7.04, 8,10 Kg m^{-1}, g=9.81m s^{-2}

π =22/7, L = 10 m, $\xi = vt+\varepsilon/2$, ε = 0.001 m, 0.01 m, 0.1 m, v = 3.33 m/s, t = 0.5 s, t = 1.0 s and t = 1.5 s

h = 0.01, I = 1.04 x $10^{-6}m^4$, E = 2.07 X $10^{11}N/m^2$, $\omega_n = n^2\pi^2\sqrt{(\frac{EI}{M_nL^4})}$ ξ_n=1

Hence, we have the followings.

Table 1, 3 and Fig. 1 to 3 show the variation of response amplitude of deflection of the beam acted upon by a moving mass.

We see from the graph that for a uniform Rayleigh beam traversed by a moving load, the response amplitude increases with increasing mass of the load.

It is observed that the response amplitude of the beam increases with increasing span of load.

It is observed that the response amplitude of the beam increases as time elapses.

Table 1: Variation of response amplitude of Rayleigh beam traversed by moving load of various Masses at t = 0.5s and length of load = 0.01m

	Amplitude		
X m	M = 7.04Kg m^{-1}	M = 8.0Kg m^{-1}	M = 10Kg m^{-1}
1	-0.00165	-0.00187	-0.00234
2	-0.00303	-0.00344	-0.00431
3	-0.00397	-0.00451	-0.00564
4	-0.0044	-0.005	-0.00626
5	-0.00439	-0.00498	-0.00623
6	-0.004	-0.00454	-0.00568
7	-0.00331	-0.00376	-0.0047
8	-0.00237	-0.0027	-0.00337
9	-0.00124	-0.00141	-0.00176
10	5.09E-06	5.78E-06	7.22E-06

Table 2: Variation of response amplitude of rayleigh beam traversed by moving load of various length of load at t = 0.5s and mass of load = 7.04 Kg m^{-1}

	Amplitude		
X	Eps = 0.01 m	Eps = 0.1 m	Eps = 1.0 m
1	-1.6856E-03	-1.6489E-03	-2.0087E-03
2	-3.0310E-03	-3.0999E-03	-3.7163E-03
3	-3.9670E-03	-4.0600E-03	-4.9093E-03
4	-4.4031E-03	-4.5098E-03	-5.5033E-03
5	-4.3852E-03	-4.4942E-03	-5.5248E-03
6	-3.9969E-03	-4.0974E-03	-5.0558E-03
7	-3.3093E-03	-3.3923E-03	-4.1846E-03
8	-2.3721E-03	-2.4309E-03	-2.9902E-03
9	-1.2394E-03	-1.2697E-03	-1.5570E-03
10	5.0860E-06	5.2099E-06	6.3806E-06

Table 3: Variation of response amplitude of rayleigh beam traversed by moving load at various times for mass of load.7.04Kg m^{-1} and length of 0.01m

	Amplitude		
Xm	t = 0.5s	t = 1.0s	t = 1.5s
1	-0.001649	-0.00259	-0.002687
2	-0.003031	-0.004873	-0.005165
3	-0.003967	-0.006589	-0.007204
4	-0.004403	-0.007571	-0.008566
5	-0.004385	-0.007756	-0.009061
6	-0.003997	-0.00718	-0.008609
7	-0.003309	-0.005957	-0.007272
8	-0.002372	-0.004239	-0.00523
9	-0.001239	-0.002195	-0.002722
10	5.086E-06	8.971E-06	1.114E-05

Table 4: Variation of response amplitude of rayleigh beam traversed by moving force load for various masses of load.

	Amplitude		
X m	M = 7.04Kg m^{-1}	M = 8.0Kg m^{-1}	M = 10Kg m^{-1}
1	-0.00174	-0.00198	-0.00247
2	-0.00313	-0.00356	-0.00444
3	-0.00395	-0.00449	-0.00561
4	-0.00417	-0.00474	-0.00593
5	-0.00392	-0.00445	-0.00556
6	-0.00336	-0.00382	-0.00477
7	-0.00263	-0.00299	-0.00374
8	-0.00181	-0.00206	-0.00257
9	-0.00092	-0.00105	-0.00131
10	3.75E-06	4.27E-06	5.33E-06

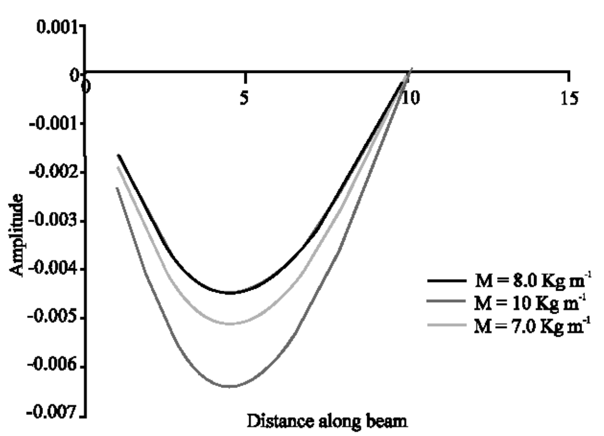

Fig. 1: Variation of amplitude of beam traversed by moving load of various masses

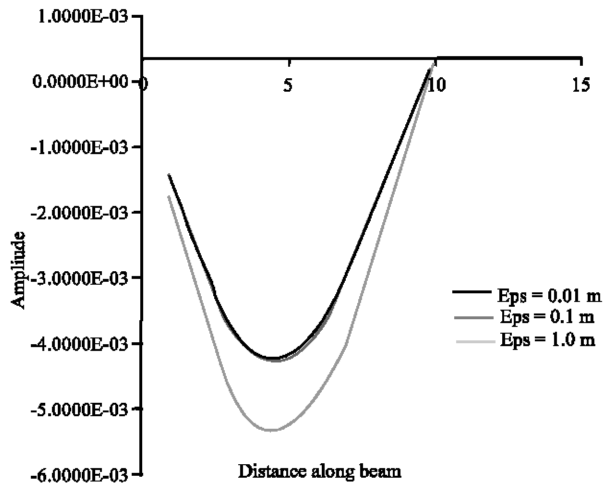

Fig. 2: Effect of length of load on amplitude of deflection of beam traversed by moving load

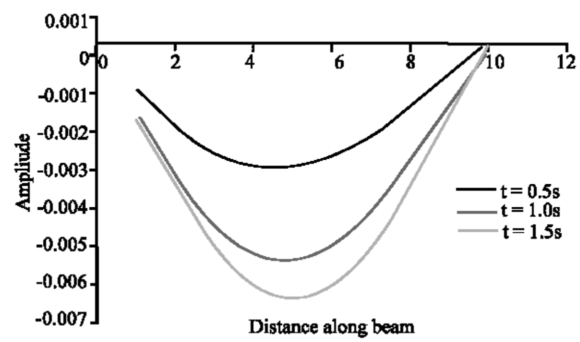

Fig. 3: Effect of time of load on amplitude of deflection of beam traversed by moving load

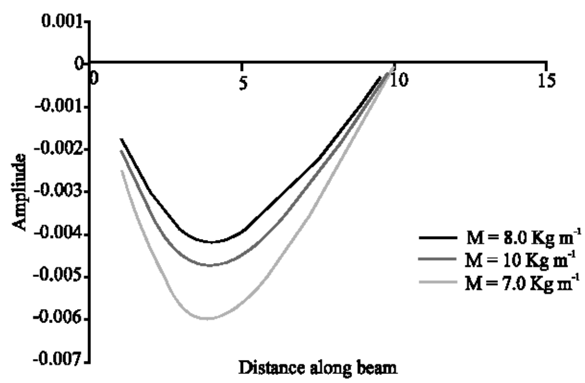

Fig. 4: Variation of amplitude of beam traversed by moving force for various masses of load

In the second part of the analysis, w proceed to study the dynamical behavior of a uniform simply supported Rayleigh beam traversed by a uniform

Table 5: Variation of response amplitude of Rayleigh beam traversed by moving force of various length (Eps) of load at t = 0.5s and mass of load = 7.04 Kg m^{-1}

	Amplitude		
X m	Eps = 0.01 m	Eps = 0.1 m	Eps = 1.0 m
1	-1.7392E-03	-1.7785E-03	-2.1297E-03
2	-3.1286E-03	-3.2009E-03	-3.8540E-03
3	-3.9496E-03	-4.0436E-03	-4.9087E-03
4	-4.1724E-03	-4.2751E-03	-5.2383E-03
5	-3.9176E-03	-4.0167E-03	-4.9618E-03
6	-3.3581E-03	-3.4443E-03	-4.2735E-03
7	-2.6316E-03	-2.6989E-03	-3.3471E-03
8	-1.8098E-03	-1.8553E-03	-2.2913E-03
9	-9.2244E-04	-9.4522E-04	-1.1619E-03
10	3.7549E-06	3.8470E-06	4.7193E-06

Table 6: Variation of response amplitude of Rayleigh beam traversed by moving force load at various times for mass of load.7.04 Kg m^{-1} and length of 0.01 m

	Amplitude		
x	t = 0.5s	t = 1.0s	t = 1.5s
1	-0.001739	-0.002795	-0.003
2	-0.003129	-0.005135	-0.005618
3	-0.00395	-0.006686	-0.007528
4	-0.004172	-0.007316	-0.008518
5	-0.003918	-0.007085	-0.008531
6	-0.003358	-0.006186	-0.007669
7	-0.002632	-0.004858	-0.006154
8	-0.00181	-0.003305	-0.004242
9	-0.000922	-0.001662	-0.002146
10	3.755E-06	6.723E-06	8.7E-06

Table 7: Variation of response amplitude of beam acted upon by moving mass and moving force for fixed parameters of beam and load

	Amplitude	
X m	Moving mass	Moving force
1	-0.00165	-0.00174
2	-0.00303	-0.00313
3	-0.00397	-0.00395
4	-0.0044	-0.00417
5	-0.00439	-0.00392
6	-0.004	-0.00336
7	-0.00331	-0.00263
8	-0.00237	-0.00181
9	-0.00124	-0.00092
10	5.09E-06	3.75E-06

spartially distributed moving for. The values of the parameters remain as was used in the analysis for moving mass. The results are presented in Table 4 and Fig. 4.

We see from the graph that for a uniform Rayleigh beam traversed by a moving force, the response amplitude increases with increasing mass of the load.

We see from the graph that for a uniform Rayleigh beam traversed by a moving force, the response amplitude increases with increasing length of the load.

It is observed that the response amplitude of the beam increases as time elapses. The results are represented in Fig. 5-7 and Table 5-7.

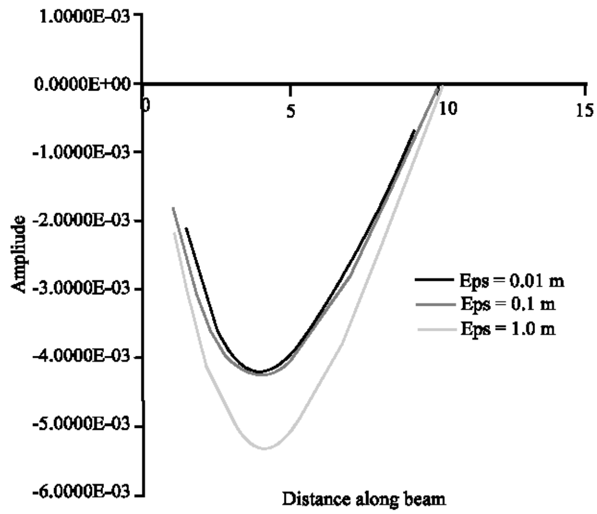

Fig. 5: Effect of length of load on amplitude of deflcetion of beam under moving force

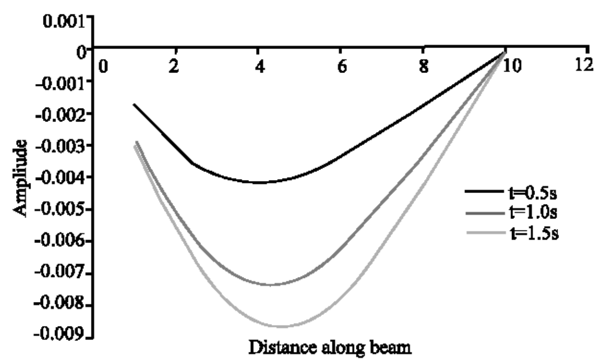

Fig. 6: Effect of time of load on amplitude of deflcetion of beam traversed moving force

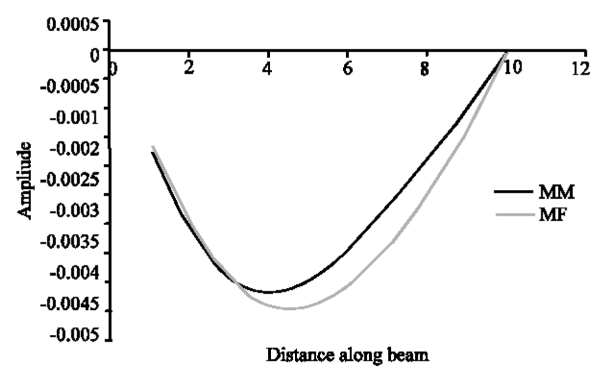

Fig. 7: Variation of amplitude of de flection of beam under moving mass and force for same parameters of system

Summary of results: An analysis has been carried out for calculating the response of

- A uniform simply supported Rayleigh beam traversed by a uniform spartially distributed moving load.
- A uniform Rayleigh beam simply supported and traversed by uniform spartially distributed moving force.

The results obtained can be summarized as follows.

- The response amplitude of a viscously damped Rayleigh beam simply supported and traversed by a uniform spartially distributed moving mass or moving force increases with increasing mass of load
- The response amplitude of a viscously damped Rayleigh beam simply supported and traversed by a uniform spartially distributed moving mass or moving force increases with increasing span of load
- The response amplitude of a viscously damped Rayleigh beam simply supported and traversed by a uniform spartially distributed moving mass or moving force increases with increasing duration of motion.
- The response amplitude of beam due to moving force is greater than the response due to moving mass.

CONCLUSION

The problem of investigating the dynamic behavior of viscously damped Rayleigh beam traversed by uniform partially distributed moving mass is studied. The theory is based on orthogonal functions and the results indicate that the governing differential equation can be transformed into a series of coupled ordinary differential equations which is the solution for the corresponding moving distributed force and not the mass problem. The resulting governing differential equation is solve by numerical approach (Finite central difference method). A comparison of the results indicates a very good agreement between the two solutions.

The moving mass inertial effects was observed to be even more important for large load distributions. Increasing the span of the load results in sharp increase in the slope of the deflection curves.

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