Research Journal of Applied Sciences 2 (2): 204-209, 2007

An Advanced Three-Dimensional Inverse Computation for the Design of Turbomachinery

[1]N. Settou, [2]N. Bouzid and [1]S. Saouli
[1]Laboratoire VPRS, Université de Ouargla, Algérie
[2]Laboratoire de Physique des matériaux, Université de Ouargla, Algérie

Abstract: A practical quasi-three dimensional inverse computation model is developed for the design of turbomachine under the assumption of inviscid incompressible flow. The three-dimensional flow through impellers is decomposed into a circumferentially averaged mean flow (S2 approach) and a periodic flow according to flow periodicity (S1 approach). In this computation the blade geometry of impeller is designed according to specified blade bound circulation distribution and normal thickness distribution on a given meridional geometry. In the first step, we use the meridional stream function to define the flow field, the mass conservation is satisfied automatically. The governing equation is deduced from the relation between the azimuthal component of the vorticity and the meridional velocity. In the second step, the finite number of blades is taken into account, the inverse problem corresponding to the blade to blade flow confined in each stream sheet is analyzed. The momentum equation implies that the vorticity of the absolute velocity must be tangential to the stream sheet. The governing equation for the blade to blade flow stream function is deduced from this condition. The blade geometry is determined according to a specified blade bound circulation distribution by iterative computation of S2 and S1 surfaces.

Key words: Inverse computation, design, turbomachinery, azimuthal component, vorticity

INTRODUCTION

Most of the blading design procedures consider the velocity distribution on both sides of the blade as the initial data. They use the hodograph plane which allows to linearize the equations in case of the potential flow (Bauer *et al*., 1975). But the designer loses the control of thickness distribution (airfoil presenting a fish-tail at the trailing edge or an opened leading edge). Leonard (1990) preconizes to use singularities such as vortices to modify the velocity distribution on the blade until to obtain the desired one. But some restrictions on the required velocity are necessary to avoid problems of thickness at the leading or trailing edge. Others use the pressure or velocity distribution in physical plane with Euler equations in case of non potential flow, but the discontinuity near leading or trailing edge still exists.

For the development of computational fluid dynamics, it became possible to analyze the complex internal flow in turbomachinery by solving the 3D Euler equation or the Navier-Stokes equation. Compared with the direct problem, the inverse problem is much more difficult. A careful survey of published literature on fluid machinery indicates that there are a few real three-dimensional inverse models. Those can be classified as the Fourier Series expansion singularity method (Borges, 1994; Lin and Peng, 1998; Zangeneh *et al*., 1996), the Taylor Series expansion singularity method (Zhao *et al*., 1984), the pseudostream function method (XU and GU, 1992) and the inverse time marching method (Zanetti *et al*., 1988).

In the present research, both thickness distribution of the blades and the desired swirl $V_\theta r$ distribution are the initial data (Settou *et al*., 2001). This variation can be interpreted as a distribution of vortices which the blades must generate. Note that the total swirl variation is related to flow angle difference between upstream and downstream. This vortex distribution is represented by the function $\Delta(V_\theta r)f$ where f is given monotonous increasing regular function of streamwise coordinate. f depends on the nature of the machine (axial or radial). Zanetti (1990) makes the same approach but he handles with pressure jump between two sides of the blade instead of swirl variation.

Only incompressible non viscous flow is analyzed in this study. This method gives a physical realistic blade shape which respects the thickness distribution which could be preconized by structure analysis, has no problem of round-off of leading or trailing edge and provides the desired flow deviation. An improvement for the scheme is made by introducing an effeciency factor for each stream line in order to take into account dissipation loss as suggested by Horlock (1984). This model provides possibilities for achievement of a design of a stage which

Corresponding Author: N. Settou, Laboratoire VPRS, Université de Ouargla, Algérie

guarantee both desired total pressure jump and exit swirl distribution (rotor + stator or vice versa) by modifying the swirl jump between blade rows inside the stage. A curvilinear body fitted coordinate system is adopted and the tensor formulation is used in order to handle with all kind of geometries (axial, radial or mixed flow).

Meridional flow, S2 approach: In the first step, the vortex distribution is transformed into an axisymmetrical one by spreading in the azimuthal direction, this situation is equivalent to the case where the number of blades in the rotor and in the stator is assumed to be infinite, the flow becomes also axisymmetrical and can be analyzed in a meridian plan (Wu, 1952).

The fluid is considered no viscous, the flow is supposed incompressible and permanent. The hypothesis of axisymmetric flow allows us to write: ((..)/ = 0). In the meridian flow channel, a boundary fitted coordinate system $_1$, $_3$ is created with $_2$ = . Let denote the coordinates z, and r. We have:

$$g_{ij}=\frac{\partial\zeta^m}{\partial\xi^i}\frac{\partial\zeta^n}{\partial\xi^j}(g_{mn})_\zeta \quad\text{and}\quad \sqrt{g}=\frac{D\left(\zeta^1,\zeta^2,\zeta^3\right)}{D\left(\xi^1,\xi^2,\xi^3\right)}(r)^2$$

The meridian velocity is represented by: $U = V_1e_1+V_3e_3 = W_1e_1+W_3e_3$, the continuity equation becomes:

$$\frac{1}{\sqrt{\tilde g}}\left[\frac{\partial\sqrt{\tilde g}U^1}{\partial\xi^1}+\frac{\partial\sqrt{\tilde g}U^3}{\partial\xi^3}\right]=0$$

where $\tilde g$ represents the determinant of the modified metric tensor due to the flow channel striction produced by the thickness of the blades. Therefore $\sqrt{g}$ represents the elementary volume of the cube: $(e_3\times e_1)\text{-}e_2$, in the free space $|e_2|=\sqrt{g_{22}}=r$. However in the blade row space, the thickness of the blade reduces the flow channel, if r_e denotes the thickness of measured section following the peripheral direction, N_b the number of the blading period in the rotor or stator, the modified metric tensor must be expressed by:

$$\tilde g_{22}=\left(1-\frac{N_b\delta\theta_e}{2\pi}\right)^2 r^2$$

$\tilde g$ used to simulate the flow channel striction in (1) is evaluated withe the modified $\tilde g_{22}$. Using the stream function to represent the flow field by setting:

$$U^1=\frac{1}{\rho\sqrt{\tilde g}}\frac{\partial\psi}{\partial\xi^3}\qquad\text{and}\qquad U^3=-\frac{1}{\rho\sqrt{\tilde g}}\frac{\partial\psi}{\partial\xi^1}$$

The Eq. (1) is satisfied automatically. The governing equation for is obtained by writing ×U = $_2e_2$, where $_2$ represents the peripheral component of ×V, it is deduced from the radial equilibrium condition, which writes:

$$\frac{\partial U_1}{\partial\xi^3}-\frac{\partial U_3}{\partial\xi^1}=\sqrt{g}\Omega^2$$

Where U_1 and U_3 are the covariants components of the velocity expressing from the stream function , in using the relations $U_m = g_{mn}U_n$. Are H the enthalpy and I the rothalpie given by:

$$H=\frac{p}{\rho}+\frac{V^2}{2}=\frac{p_t}{\rho}\quad\text{and}\quad I=\frac{p}{\rho}+\frac{W^2}{2}-\frac{\omega^2r^2}{2}=H+\omega\left(V_\theta r\right)$$

The momentum equation is:

$$\begin{cases}\Omega\times W=-\nabla I+\dfrac{F_b}{\rho}+\dfrac{F_d}{\rho} & \text{rotor}\\ \Omega\times V=-\nabla H+\dfrac{F_b}{\rho}+\dfrac{F_d}{\rho} & \text{stator}\end{cases}$$

where F_b/ represents the blades force. The loss scheme related to the plausible value of efficiency for each streamline of the stage is added, this scheme suggests that the dissipative force F_a/ is related to the variation of V_r via (Horlock, 1984) in writing:

$$\frac{F_d}{\rho}=-C_f\frac{|U|^2}{\cos\beta}\frac{U}{|U|}$$

$F_d = 0$ as well as $F_b = 0$ are imposed in the free space. Combining the e_2 and the e_3 components of (2), we have:

$$\sqrt{g}\Omega^2=\frac{1}{W^1}\frac{\partial I}{\partial\xi^3}-\frac{F_{d_3}}{\rho}+\frac{n_1}{n_2}\frac{\partial(V_\theta r)}{\partial\xi^3}-\frac{n_3}{n_2}\frac{\partial(V_\theta r)}{\partial\xi^1}\quad\text{rotor}$$

$$\sqrt{g}\Omega^2=\frac{1}{V^1}\frac{\partial H}{\partial\xi^3}-\frac{F_{d_3}}{\rho}+\frac{n_1}{n_2}\frac{\partial(V_\theta r)}{\partial\xi^3}-\frac{n_3}{n_2}\frac{\partial(V_\theta r)}{\partial\xi^1}\quad\text{stator}$$

$$\sqrt{g}\Omega^2=\frac{1}{V^1}\left(\frac{\partial H}{\partial\xi^3}-\frac{(V_\theta r)}{r^2}\frac{\partial(V_\theta r)}{\partial\xi^3}\right)\quad\text{free space}$$

where n_i denotes the covariant components of the normal n of the camber surface of the blade. Use has been made that V n in the stator, W n in the rotor and F_bn. The dot product of the momentum equation with V in the stator and in the free space or with W in the rotor leads to the following relations which serve to update the nodal values of H or I:

$$\frac{\partial I}{\partial m} = (1-\eta)\omega\frac{\partial(V_\theta r)}{\partial m} \qquad \text{rotor}$$

$$\frac{\partial H}{\partial m} = (\eta-1)\omega\frac{\partial(V_\theta r)}{\partial m} \qquad \text{stator}$$

$$\frac{\partial H}{\partial m} = 0 \qquad \text{free space}$$

Writing ×U = $_2e_2$, we obtain the governing equation of :

$$\frac{\partial}{\partial\xi^3}\left(\frac{g_{11}}{\rho\sqrt{\tilde g}}\frac{\partial\psi}{\partial\xi^3}\right)+\frac{\partial}{\partial\xi^1}\left(\frac{g_{33}}{\rho\sqrt{\tilde g}}\frac{\partial\psi}{\partial\xi^1}\right)-\frac{\partial}{\partial\xi^3}\left(\frac{g_{13}}{\rho\sqrt{\tilde g}}\frac{\partial\psi}{\partial\xi^1}\right)-\frac{\partial}{\partial\xi^1}\left(\frac{g_{31}}{\rho\sqrt{\tilde g}}\frac{\partial\psi}{\partial\xi^3}\right)=\sqrt{g}\Omega^2$$

For the inverse problem, the distribution of V_r is assigned, $_2$ is updated iteratively. Let the form of the blade camber be defined by = $_2(_1, _3)$ if the coordinate lines are updated to the streamlines iteratively, $_2$ can be computed using the slip condition:

$$\xi^2 = \xi^2_{le} + \int_{\xi^1_{le}}^{\xi^1}\frac{U^2}{U^1}d\xi^1$$

Blade surface pressure evaluation: Usually the S2 approach leads to the determination of the mean velocity on both faces of the blade:

$$\begin{cases}\text{rotor} \quad W \\ \quad = \left[g_{11}V^1V^1 + 2g_{13}V^1V^3 + g_{33}V^3V^3 + \begin{matrix} g_{22}\left(V_\theta r+\omega r^2\right)^2 \\ g_{22}\left(V_\theta r\right)^2\end{matrix}\right]^{1/2} \\ \text{stator} \quad V\end{cases} \tag{5}$$

Let U denote the difference of the absolute velocities V_+-V_- or the relative velocity W_+-W_- on the two faces of the blade, when the number of blades is finite, this difference is related to the local density of bound vortex generated by the blade. In the S2 scheme, consider the blade section cut by a $_3$ = cnst surface, the flux of bound vortices generated by the element $_1$ of the blade is determined by the flux of through the elementary surface $(\delta S)_3 e^3 = \sqrt{g}\delta\xi^1\delta\xi^2 e^3$, where $_2$ should be equal to 2/N_b. Using the Stokes relation that implies the circulation produced by U is equal to the flux of the bound vortices, we get the following relation:

$$(\Delta U)_{i,k} = \frac{2\pi}{N_b}\frac{\cos\beta}{\sqrt{g_{11}}}\left|\left[V_\theta r\right]_{i-1/2,k}^{i+1/2,k}\right|$$

where denotes the local angle of the blade camber line with respect to the meridian plane. The relation (5) is used to compute surface velocity on both faces of the blades, then the pressure distribution by the S2 approach can be deduced.

Blade to blade flow, S1 approach: The blade to blade flow confined in each axisymmetrical stream sheet is analyzed in order to define the final geometry for each section of the blade and to obtain the pressure distribution. At the beginning, the contour of the blade is created from the camber line obtained from the S2 step with the assigned thickness distribution. The conformal mapping (m,)(x_1,x_2):

$$\begin{cases} x^1 = r_0\displaystyle\int_{m_0}^{m}\frac{dm}{r} \\ x^2 = r_0(\theta-\theta_0)\end{cases}$$

transforms the blade to blade flow confined in an axisymmetrical stream sheet into 2D cascade flow in the (x_1, x_2) plane. The body fitted coordinate system constitued by the equipotential lines $_1$ = cnst and the streamlines $_2$ = cnst of a 2D flow around the cascade is created using the method of singularities (Seettou et al., 2001). In this system, the continuity equation becomes:

$$\frac{1}{\sqrt{g}}\left[\frac{\partial}{\partial\xi^1}\left(\rho\sqrt{g}U^1\right)+\frac{\partial}{\partial\xi^2}\left(\rho\sqrt{g}U^2\right)\right]=0 \tag{6}$$

where U_1 represent the contravariant components of the absolute velocity V for the stator and relative velocity W for the rotor and

$$\sqrt{g} = \frac{D\left(x^1,x^2\right)}{D\left(\xi^1,\xi^2\right)}\left(\frac{r}{r_0}\right)^2\tau$$

where represents the local thickness of the stream sheet. Introducing the stream function with:

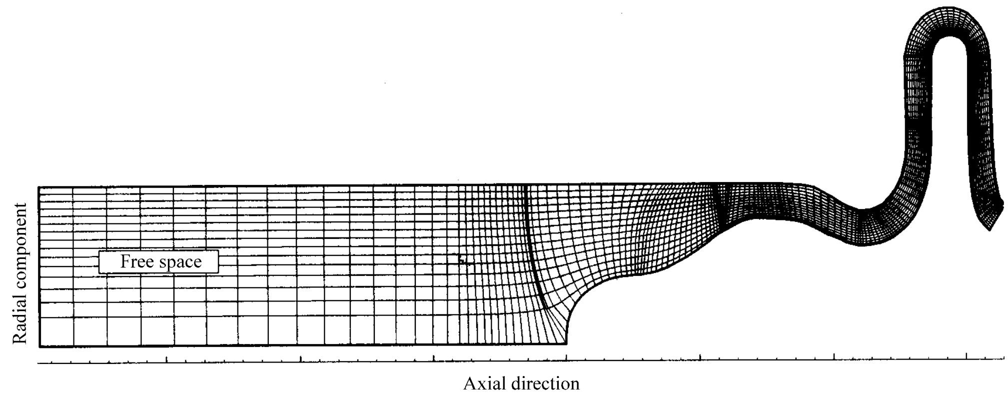

Fig. 1: Meridional plan of a multistage turbopump

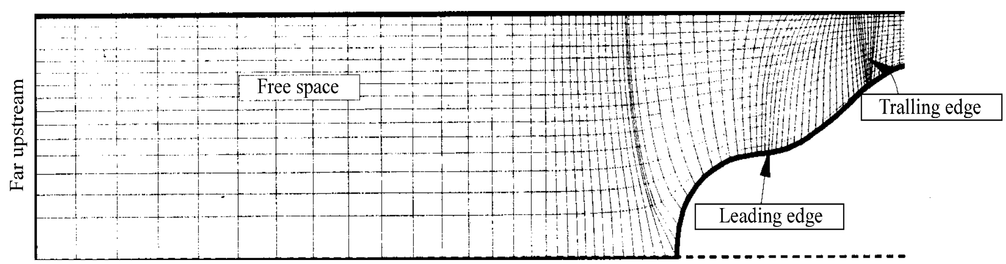

Fig. 2: Geometry of flow passage and computational mesh

$$\begin{cases} U^1 = \dfrac{1}{\rho\sqrt{g}}\dfrac{\partial\psi}{\partial\xi^2} \\ U^2 = -\dfrac{1}{\rho\sqrt{g}}\dfrac{\partial\psi}{\partial\xi^1} \end{cases}$$

The relation (6) is satisfied. From the momentum equation, we can show that the free vortex must be tangential to the axisymmetrical stream sheet, the governing equation of the blade to blade flow stream function is deduced from this condition: for the relative flow around the blades of the rotor, we have:

$$-\left[\frac{\partial}{\partial\xi^1}\left(\frac{g_{22}}{\rho\sqrt{g}}\frac{\partial\psi}{\partial\xi^1}\right)+\frac{\partial}{\partial\xi^2}\left(\frac{g_{11}}{\rho\sqrt{g}}\frac{\partial\psi}{\partial\xi^2}\right)\right]=$$
$$-\frac{\partial g_{21}W^1}{\partial\xi^1}+\frac{\partial g_{12}W^2}{\partial\xi^2}+2\sqrt{g}\,\frac{\omega r}{\tau}\frac{d\log r}{dm}$$

Boundary conditions for the inverse problem:

$$\begin{cases} \text{Penetrating flux conservation:} & [\psi]_-^+ = 0 \\ \text{Bound vorticity assigned:} & \left[W_1 d\xi^1 - \omega r^2 d\theta\right]_-^+ = \Gamma df \end{cases}$$

The solution of the inverse problem leads to the determination of flux penetration on the blade contour, the camber line inclination correction $_9$ is given by:

$$\delta\vartheta = 0.5\left[\tan^{-1}\left(\frac{\sqrt{g}}{\tau}\frac{W^2}{W^1}\right)^+ + \tan^{-1}\left(\frac{\sqrt{g}}{\tau}\frac{W^2}{W^1}\right)^-\right]$$

The design of a turbopump is presented in this study. Figure 1 shows the network in the meridional plan for the multistage turbopump. Figure 2 shows the network in the meridional plan obtained by the computation for the 2D incompressible potential flow (Settou *et al*., 2001). Figure 3 shows the blading obtained by the S2-S1 inverse solution. Figure 4 shows the comparison of the camber lines of the impeller obtained from the S2 approach and rectified by the S1 approach. Figure 5 shows the pressure distribution obtained from the S2-S1 calculation for the upper and the lower sides of blade. For the case of the centrifugal impeller, the loading is optimised to avoid the cavitation. The results from the S2 and S1 computations are similar, but not identical, the need of the S1 computation to obtain the final geometry definition of the blades is confirmed. The meridional velocity distributions along averaged mean flow streamlines are shown in Fig. 6.

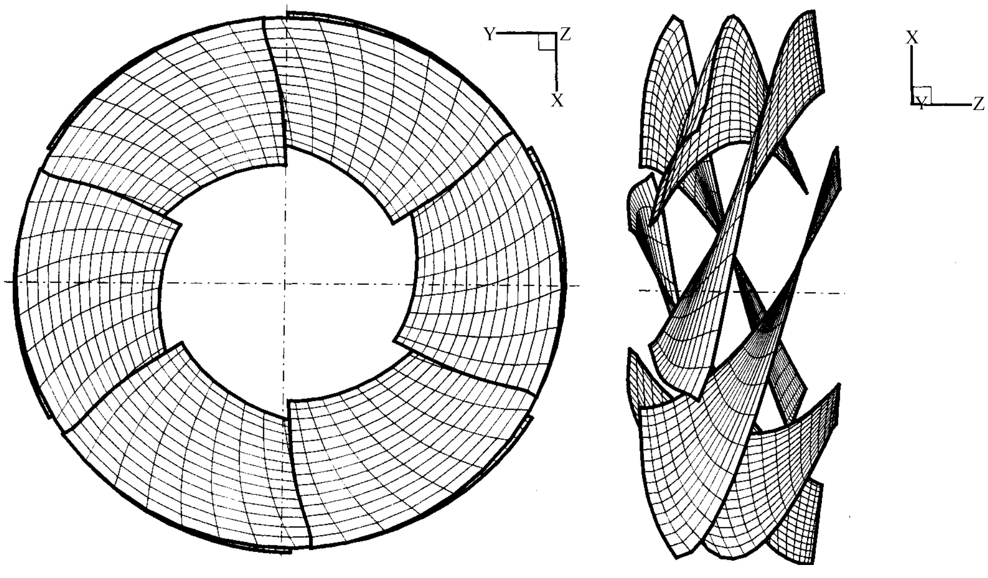

Fig. 3: The blading obtained by the S2-S1 inverse solution

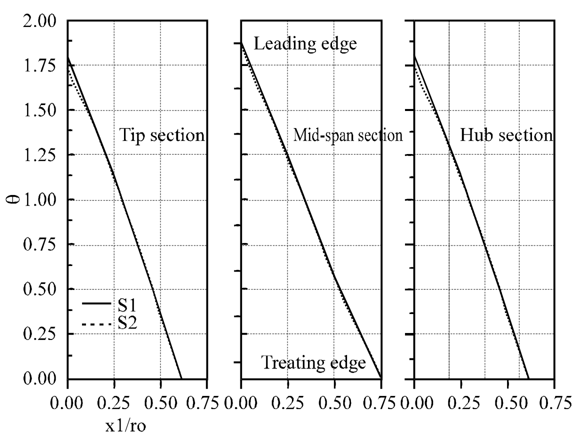

Fig. 4: Camber line of the impeller obtained from S2-S1 approaches

Fig. 5: Pressure distribution for Upper and Lower sides of the impeller from S2-S1 approaches

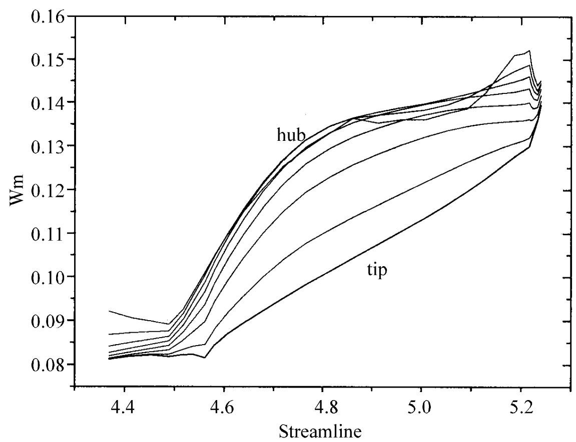

Fig. 6: Meridional velocity distribution along streamlines

CONCLUSION

A design procedure more suitable for engineering use is proposed in this study. This design approach accepts the prescription of the thickness distribution of the blade and the loading distribution as the initial data. It is relatively easy to obtain the optimized design by some adjustement of the initial data. The representation of the blades by the vortex distribution enables the formulation of the well-posed inverse problem and which leads to design the blading of a turbomachine.

NOMENCLATURE

V	=	Resultant velocity
W	=	Relative velocity
	=	Curl of the velocity
	=	Stream function
x, y, z	=	Cartesian coordinate system
z, , r	=	Cylindrical coordinate system
^{1}, ^{2}, ^{3},	=	Body fitted curvilinear coordinates
e_1, e_2, e_3	=	Covariant base vectors
e^1, e^2, e^3	=	Contravariant base vectors
	=	Propeller rotational speed, radians per unit time.
U_1, U_2, U_3	=	Covariant components of the absolute or relative velocity
^{1}, ^{2}, ^{3}	=	Covariant components of the absolute vorticity
p	=	Static pressure
	=	Mass density of fluid
H	=	Total enthalpy, p_t/
I	=	Rothalpy, H+ (V_r)
F_b	=	Vector force on blade surface
F_d	=	Dissipative force
C_f	=	Friction coefficient
g_{ij}	=	Metric tensor elements
g	=	Determinant of the metric tensor
	=	Propeller efficiency
Q	=	Propeller torque
K_Q	=	Mean torque coefficient, $Q/(n^2D^5)$
T	=	Propeller thrust
K_T	=	Mean thrust coefficient, $T/(n^2D^4)$
D	=	Propeller diameter
R	=	Propeller radius
n	=	Propeller rotational speed, revolutions per unit time
n	=	Vector normal to blade camber surface
N_b	=	Number of blades
	=	Circulation around blade section

REFERENCES

Bauer, F., P. Garabedian and D. Korn, 1975. Supercritical wings sections, II. Springer-Verlag, New York.

Borges, J.E., 1994. A three dimensional inverse method for turbomachinery: Part I Theory. ASME J. Turbomach., 112: 346-354.

Horlock, J.H., 1984. An entropy production in adiabatic flow in Turbomachines. J. Basic Eng. Tr. ASME serie D.

Leonard, O., 1990. Subsonic and Transonic cascade design, AGARD-VKI Special course, on Inverse method in airfoil design for aeronautical and Turbomachinery applications, Rhode-Saint-Genèse, Belgium.

Lin, R.C. and G.Y. Peng, 1998. An advanced three-dimensional inverse model for the design of hydraulic machinery runner. ASME Proc., FED-Vol. 245, FEDSM98-4867, pp: 1-7.

Settou, N., A. Atif and S. Benmansour, 2001. Turbomachine blading designed by solving the inverse problem. 5th Maghrebine Conference for Mechanical Studies, Sabha University, pp: 58-67.

Wu, C.H., 1952. A general theory of three dimensional flow in subsonic and supersonic turbomachines of axial, radial and mixed flow type, NACA TN 2604.

Xu, J.Z. and W. Gu, 1992. A numerical procedure of three-dimensional design problem in turbomachinery, ASME, J. Turbomach., 114: 548-582.

Zanegeneh M., A. Goto and T. Takemura, 1996. Suppression of application of three-dimensional inverse design method part i-design and numerical validation. ASME J. Turbomach., 118: 536-543.

Zanetti, L., 1990. Inverse methods for 3D internal flows, AGARD-VKI Special course on Inverse method in airfoil design for aeronautical and turbomachinery applications, Rhode-Saint-Genèse, Belgium.

Zanetti, L., F. Larocca and R. Marsibio, 1988. Euler solver for 3D inverse problem problems. Proc. Adv. Applic. Comput. Fluid Dyn., ASME FED, 66: 71-79.

Zhao, X.L., C.L. Sun and C.H. Wu, 1984. A simple method for solving three-dimensional inverse problems of turbomachine flow and annular constraint condition. ASME pp: 84-GT-198.