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A Reacting Flow with Temperature Dependent Thermal Conductivity

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Abstract: This study presents a reacting system where the thermal conductivity depends linearly on the temperature of the system. A numerical solution of the resultant energy equation of the system is obtained using shooting technique with second order Runge-Kutta scheme. We show that maximum temperature decreases as thermal conductivity increases and also, the temperature of the system increases as the reaction parameter increases.

Key words: Reacting flow, thermal conductivity, temperature

INTRODUCTION

The study of the temperature field of reacting system with variable quantities has attracted the interest of many scientists. Ayeni (1982) traced the history of the temperature field of variable viscosity flows between concentric cylinders up to runaway time. He showed that in this type of flow, the velocity and temperature fields influence each other. The temperature influences the flow through convection, through dissipation and through anisotropic effect on the thermal properties. Lacey and Wake (1982) proposed that thermal conductivity varies exponentially with temperature. They showed that critical behaviour disappears when the thermal conductivity increases with temperature faster than the rate of heating. Olajuwon (2003) investigated unsteady temperature field of a power-law fluid flow with variable thermal conductivity. The heat dissipation is essential while thermal conductivity varies with time. Kim *et al.* (2003) presented an integral approach to estimate temperature dependent thermal conductivity in a transient non-linear heat conducting medium. It is assumed that the thermal conductivity is a monotonic function of the temperature and it can be expressed as a piecewise linear function. The heat conduction problem was converted to a parameter identification problem determining the unknown coefficient of thermal conductivity function that is approximated to a piece-wise linear form. The spatial temperature distribution required for the integral approach is modeled as a third order function of position in one-dimensional heat conduction domain with heated and insulated walls. The present work considers the case when thermal conductivity depends linearly on temperature.

MATHEMATICAL FORMULATION

The energy transfer of a reacting system is governed by

$$\rho c_v \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[k(T)\frac{\partial T}{\partial x}\right] + Q(T - T_0) \qquad (1)$$

with boundary conditions

$$T(0,t) = T_0, T(L,t) = T_1, T_1 > T_0 \qquad (2)$$

where,

$$k(T) = k_0 + k_1 T \qquad (3)$$

- T : Is the temperature
- T_0 : Is the initial temperature
- T_1 : Is the final temperature
- ρ : Is the density of the reactant
- t : Is the time
- c_v : Is the constant heat capacity per unit volume
- x : Is the position (direction of the flow)
- Q : Is the heat released per unit mass
- $\kappa(T)$: Is the variable thermal conductivity and the second term on the right hand side of Eq. 2 is the reaction term.

MATERIALS AND METHODS

Taking $T_1 = 1$ and $T_0 = 0$ (such that $T_1 > T_0$) and for steady case

$$\frac{\partial}{\partial t} = 0 \qquad (4)$$

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Then Eq. 1 becomes

$$\frac{d}{dx}\left[k(T)\frac{dT}{dx}\right] + Q(T) = 0 \quad (5)$$

using Eq. 3 in Eq. 2 we obtain

$$(k_0 + k_1T)\frac{d^2T}{dx^2} + k_1\left(\frac{dT}{dx}\right)^2 + QT = 0 \quad (6)$$

$$(k_0 + k_1T)T'' + k_1(T')^2 + QT = 0 \quad (7)$$

Solving Eq. 7 subject to (2) numerically when L = 1. Equation 7 is resolve into system of first order differential equations as follows:

we let

$$Z = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x \\ T \\ T' \end{pmatrix} \quad (8)$$

So,

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} 1 \\ T' \\ T'' \end{pmatrix} \quad (9)$$

now, the shooting numerical scheme for Eq. 7 becomes

$$f(Z) = \begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} 1 \\ x_3 \\ \dfrac{-k_1x_3' - Qx_2}{k_0 + k_1x_2} \end{pmatrix} \quad (10)$$

Table 1: Table of values of temperature T for various values of thermal conductivity coefficients k_0 and k_1

X	$k_0 = k_1 = 0.8$	$k_0 = k_1 = 1.0$	$k_0 = k_1 = 1.5$	$k_0 = k_1 = 2.0$
0	0	0	0	0
0.1	0.09	0.0844	0.774	0.0741
0.2	0.1734	0.163	0.1498	0.1436
0.3	0.2516	0.2369	0.2183	0.2096
0.4	0.3252	0.3068	0.2835	0.2725
0.5	0.3948	0.3732	0.3459	0.3329
0.6	0.4607	0.4364	0.4057	0.3911
0.7	0.5232	0.4966	0.4631	0.4472
0.8	0.5822	0.5539	0.5183	0.5014
0.9	0.6378	0.6084	0.5713	0.5537
1	0.6901	0.6601	0.6222	0.6042
1.1	0.7389	0.7089	0.671	0.6529
1.2	0.7842	0.7547	0.7175	0.6998
1.3	0.8258	0.7975	0.7618	0.7448
1.4	0.8635	0.8371	0.8037	0.7879
1.5	0.8973	0.8734	0.8433	0.8289
1.6	0.927	0.9063	0.8802	0.8678
1.7	0.9523	0.9356	0.9145	0.9045
1.8	0.9731	0.9611	0.946	0.9389
1.9	0.989	0.9826	0.9745	0.9707
2	1	1	1	1

Together with initial condition

$$\begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \gamma \end{pmatrix} \quad (11)$$

The constant is the initial guessed value of the temperature gradient of the system.

RESULTS AND DISCUSSION

Results: The iterative scheme of Eq. 10 is computed and the results for various values of thermal conductivity

Table 2: Table of value of temperature T for various values of reaction parameter Q

X	Q = 0.25	Q = 0.50	Q = 0.75	Q = 1.0
0	0	0	0	0
0.1	0.0763	0.0788	0.0832	0.09
0.2	0.1473	0.1522	0.1605	0.1734
0.3	0.214	0.2211	0.2331	0.2516
0.4	0.277	0.2862	0.3015	0.3252
0.5	0.3368	0.348	0.3664	0.3948
0.6	0.3938	0.4068	0.4281	0.4607
0.7	0.4484	0.4629	0.4867	0.5232
0.8	0.5007	0.5166	0.5425	0.5822
0.9	0.5509	0.5679	0.5956	0.6378
1	0.5993	0.6171	0.646	0.6901
1.1	0.6459	0.6641	0.6937	0.7389
1.2	0.6909	0.7092	0.7388	0.7842
1.3	0.7343	0.7522	0.7813	0.8258
1.4	0.7762	0.7933	0.8211	0.8635
1.5	0.8167	0.8325	0.8581	0.8973
1.6	0.8559	0.8698	0.8924	0.927
1.7	0.8938	0.9052	0.9238	0.9523
1.8	0.9304	0.9387	0.9523	0.9731
1.9	0.9658	0.9703	0.9777	0.989
2	1	1	1	1

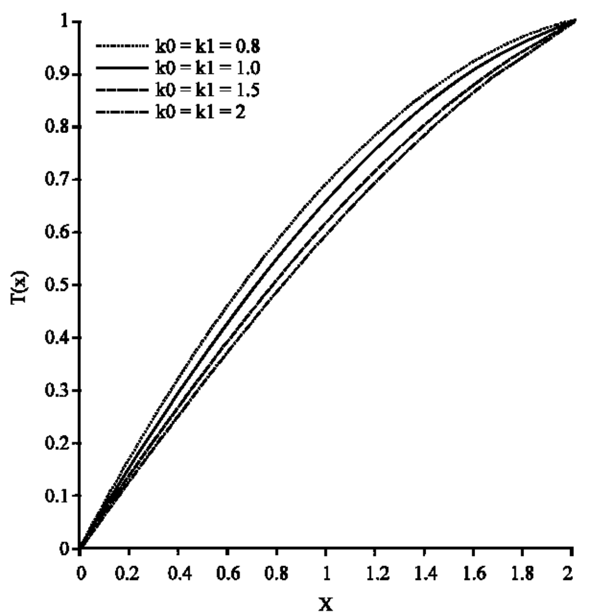

Fig. 1: Graph of temperature T(x) against position x when Q = 1 at various values of k_0 and k_1

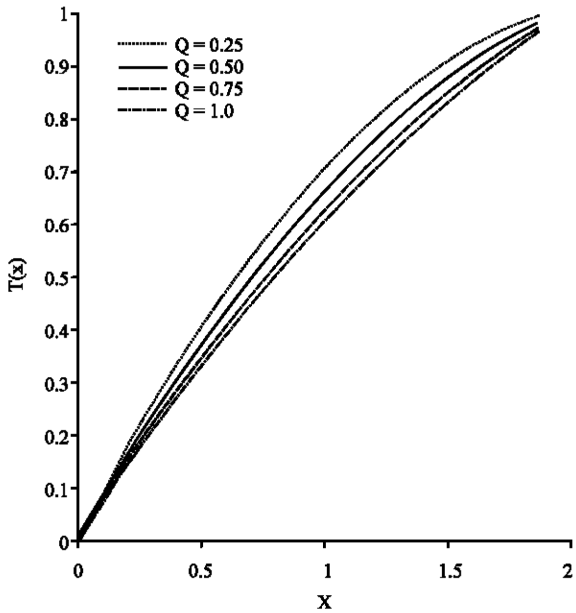

Fig. 2: Graph of temperature T for various values of the reaction parameter Q at fixed thermal conductivity k 0 and k 1 equal to 0.8

coefficients k_0 and k_1 is as presented in Table 1 when Q = 1. Also, the numerical solution of Eq. 10 is computed for various values of the reaction parameter Q at fixed values of thermal conductivity coefficients k_0 and k_1 and displayed in Table 2.

DISCUSSION

The Fig. 1 above shows the steady profile for $k(T) = k_0 + k_1 T$. The temperature profile was studied for the different values of k_0 and k_1. It was shown that as thermal conductivity coefficients of k (T) increases, maximum temperature of the system decreases. While Fig. 2 shows that as the values of reaction parameter Q increases, the maximum temperature also increases.

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