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Dynamical Behavior of Euler-Bernoulli Beam Traversed by Uniform Partially Distributed Moving Masses

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Abstract: This study is concerned with dynamical behavior of Euler Bernoulli beam traversed by uniform partially distributed moving masses. The governing partial differential equation was systematically analyzed and the analytical numerical solution for classical boundary condition obtained. The deflection of the Euler-Bernoulli beam is calculated under various specified conditions and the results displayed graphically. It is found that moving force solution is not an upper bound for an accurate solution of the moving mass problem.

Key words: Dynamical, traversed, deflection, uniform, behavior, graphically

INTRODUCTION

The dynamic analysis of the behavior of an elastic beam traversed by moving loads has been an interesting problem in several fields of Engineering, Applied Physics and Applied Mathematics and continue to motivate a variety of investigations (Fryba, 1972; Esmailzadeh and Gorashi, 1992, 1993).

A comprehensive review of the subject for the vibration of structure resulting from moving loads can be found in Fryba (1972), Esmailzadeh and Gorashi (1995), Esmailzadeh and Gorashi (1992) have investigated many cases of moving load problems. Recently vibration analysis of beams traversed by uniform partially distributed moving masses was studied by Esmailzadeh and Gorashi (1995).

The research presented in this study is an extension of the work of Esmailzadeh and Gorashi (1995), Akin and Mofid (1989) in which the complementary acceleration and the centripetal acceleration, which had been neglected for many years, are now being taken into consideration.

The assumptions adopted here correspond to those of Esmailzadeh and Gorashi (1995), Akin and Mofid (1989). The analysis presented in this study is for simply supported Bernoulli-Euler beam that should be easily applied to many situations and variety of boundary conditions.

The main objectives study are to:

- Determine the response amplitude of the deflection of the moving force with respect to the mass of the load.
- See the behavior of the amplitude of the moving force with respect to time
- Determine the vibration in the lateral displacement of the moving force with respect to time, affixed length of the beam and various values of the mass of the load.
- Determine the response amplitude of the deflection of the moving mass with respect to time.
- Determine the vibration in the displacement of the moving mass with respect to time.

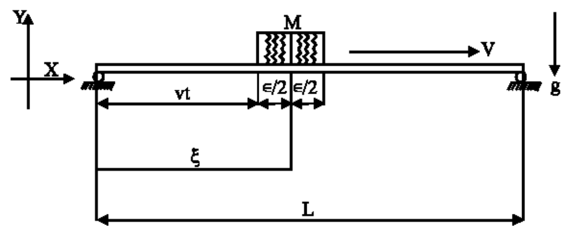

Fig. 1: Mathematical model of the problem

MATHEMATICAL MODEL OF THE PROBLEM

With reference to Fig. 1, it is assumed that we have a uniform Euler-Bernoulli beam carrying a mass M of fixed length ϵ, at time t = 0s. The load M is situated at the left hand support. The load M is partially distributed and advancing uniformly along the beam of length L with a constant velocity V.

PROBLEM FORMULATION

Based on Esmailzadeh and Gorashi (1995), Akin and Mofid (1989) the equation governing motion of a Bernoulli-Euler beam under the moving mass, m, neglecting the damping, the rotatory inertia and shearing force effects can be written as:

$$EI\frac{\partial^4 W(x,t)}{\partial x^4} + m\frac{\partial^2 W(x,t)}{\partial t^2} = F(x,t) \quad (1)$$

Where E is the modulus of elasticity, I is the second moment of area for the beam's cross-section, m is the mass per unit length of the beam, W (x, t) is the deflection of the beam, x is the spatial coordinate, t is the time and F (x,t) is the load inertia (the resultant concentrated force caused by the moving mass).

In this system, when the effect of the moving load on the transverse displacement of the beam is considered, the load inertia takes the form

$$F(x,t) = \frac{1}{\in}[-Mg - M\nabla W]\left[H\left(x-\xi+\frac{\in}{2}\right) - H\left(x-\xi-\frac{\in}{2}\right)\right] \quad (2)$$

$$H(x)=\begin{cases}0 & x<0\\ 1 & x>0\end{cases} \quad (3)$$

Where
M = Mass of the load
∇ = The substantive acceleration operator

$$\text{i.e.,}\ \nabla = \frac{\partial^2}{\partial t^2} + 2V\frac{\partial^2}{\partial x\partial t} - V^2\frac{\partial^2}{\partial t^2} \quad (4)$$

$\in$ = Is the fixed length of the beam
H= The Heaviside unit function
g = Acceleration due to gravity
V =The constant velocity of the load
ζ = (V t+$\in$/2) for the limiting condition as $\in \rightarrow 0$ one obtains

$$\delta(x-\xi) = \frac{1}{\in}\left[H\left(x-\xi+\frac{\in}{2}\right) - H\left(x-\xi-\frac{\in}{2}\right)\right] \quad (5)$$

where δ (x-ζ) is the Dirac delta function.
Recall the Dirac delta integral properties:

$$\int_0^L f(x)\delta(x-x_0)dx = f(x_0)\, 0<, x_o<L \quad (6)$$

$$\int_0^L f(x)\delta(x-x_0)dx = 0\, x<0, x>L \quad (7)$$

The pertinent boundary conditions for the problem under consideration can be any of the following classical boundary conditions.

$$y(x,t) = \frac{\partial y(x,t)}{\partial x} = 0 \text{ at } x=0 \text{ or } x=L \quad (8)$$

$$y(x,t) = \frac{\partial^2 y(x,t)}{\partial x^2} = 0 \text{ at } x=0 \text{ or } x=L \quad (9)$$

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{\partial^3 y(x,t)}{\partial x^3} = 0 \text{ at } x=0 \text{ or } x=L \quad (10)$$

$$\frac{\partial y(x,t)}{\partial x} = \frac{\partial^3 y(x,t)}{\partial x^3} = 0 \text{ at } x=0 \text{ or } x=L \quad (11)$$

The initial conditions are

$$Y(x,0)=0 \quad (12)$$

$$\frac{\partial y(x,t)}{\partial x} = 0 \quad (13)$$

Substituting (4) into (2) we have

$$F(x,t) = \frac{1}{\in}\left[-Mg - M\frac{d^2W}{dt^2} - 2M\frac{V\partial^2 W}{\partial x\partial t} - MV^2\frac{\partial^2 W}{\partial x^2}\right]\left[H\left(x-\xi+\frac{\in}{2}\right) - H\left(x-\xi-\frac{\in}{2}\right)\right] \quad (14)$$

In Eq. 14, the first term in the first square bracket describes the constant gravitational force, while the second term accounts for the effect of the direction of the transverse deflection y (x, t), the third term is for the complementary acceleration and the fourth term is for the centripetal acceleration. The second square bracket describes the Heaviside functions.

Considering Eq. 5 and 14 would lead to the formation for moving point masses (Esmailzadeh, 1995; Akin and Mofid, 1989). However, in the present discussion $\in$ is not limited to be small length.

OPERATIONAL SIMPLIFICATION OF THE GOVERNING EQUATION

We assume the transverse vibration of the beam as:

$$W(x,t)=\sum_{i=1}^{\infty}\phi_i(t)X_i(x) \tag{15}$$

Where $\phi_i(t)$'s are unknown functions of time $X_i(x)$'s the normalized deflection curve for the i^{th} mode of the vibrating prismatic beam.

We further assume that the load function can be expressed as:

$$F(x,t)=\sum_{i=1}^{\infty}X_i(x)\psi_i(t) \tag{16}$$

Where Ψ_i (t)'s are unknown functions of time and $X_i(x)$ are as said earlier.

Substituting Eq. 15 and 16 into 1 and 14, respectively we have.

$$\begin{aligned}&EI\sum_{i=1}^{\infty}\phi_i(t)X_i^{iv}(x)+m\sum_{i=1}^{\infty}\ddot{\phi}_i(t)X_i(x)=\frac{1}{\in}\Bigg[-Mg-M\sum_{i=1}^{\infty}\ddot{\psi}_i(t)X_i(x)-2VM\sum_{i=1}^{\infty}\dot{\psi}_i(t)X_i'(x)\\&-MV^2\sum_{i=1}^{\infty}\psi_i(t)X_i''(x)\Bigg]\left[H(x-\xi+\frac{\in}{2})+H(x-\xi-\frac{\in}{2})\right]dx=\sum_{i=1}^{\infty}X_i\psi_i(t)\end{aligned} \tag{17}$$

It is noted that Eq. 17 has two sets of unknowns viz, the ϕ_is and the ιi's. This naturally makes Eq. 17 highly uncoupled. To reduced the high degree of couple ness we would have to determine one of these sets of unknowns, we remarked, however, that we find it convenient to determine the Ψ_Is, hence multiply the right hand side of Eq. 17 by the known normalized equation over the length of the beam to obtain

$$\begin{aligned}&\int_0^L X_j(x)\Bigg\{\frac{1}{\in}\Bigg[-Mg-M\sum_{i=1}^{\infty}\ddot{\psi}_i(t)X_i(x)-2MV\sum_{i=1}^{\infty}\dot{\psi}_i(t)X_i'(x)-MV^2\sum_{i=1}^{\infty}\psi_i(t)X_i''(x)\Bigg]\\&\left[H(x-\xi+\frac{\in}{2})+H(x-\xi-\frac{\in}{2})\right]\Bigg\}dx=\sum_{i=1}^{\infty}\psi_i(t)\int_0^L X_i(x)X_j(x)dx\end{aligned} \tag{18}$$

Expanding Eq. 18 we have

$$\begin{aligned}&-\frac{Mg}{\in}\int_0^L X_j(x)\left[H(x-\xi+\frac{\in}{2})+H(x-\xi-\frac{\in}{2})\right]dx-\frac{M}{\in}\Bigg[\sum_{i=1}^{\infty}\ddot{\psi}_i(t)\int_0^L X_i(x)X_j(x)\left[H(x-\xi+\frac{\in}{2})+H(x-\xi-\frac{\in}{2})\right]dx\\&-\frac{2MV}{\in}\Bigg[\sum_{i=1}^{\infty}\dot{\psi}_i(t)\int_0^L X_i'(x)X_j(x)\left[H(x-\xi+\frac{\in}{2})+H(x-\xi-\frac{\in}{2})\right]dx-\frac{MV^2}{\in}\Bigg[\sum_{i=1}^{\infty}\psi_i(t)\int_0^L X_i''(x)X_j(x)\\&\left[H(x-\xi+\frac{\in}{2})+H(x-\xi-\frac{\in}{2})\right]dx=\sum_{i=1}^{\infty}\psi_i(t)\int_0^L X_i(x)X_j(x)dx\end{aligned} \tag{19}$$

Remarks

(i)To evaluate the left hand side of Eq. 19, we simply noted that since the normalised deflection curve X_i, $i_=1, 2, 3......$ n are orthonormal,we have

$$\sum_{i=1}^{\infty}\psi_i(t)\int_0^L X_i(x)X_j(x)dx=\begin{vmatrix}0, i\neq j\\1, i=j\end{vmatrix} \tag{20}$$

$$\int_0^L X_j(x)\left[H(x-\xi+\frac{\in}{2})-H(x-\xi-\frac{\in}{2})\right]dx = \in\left[X_i(\xi)+\frac{\in^2}{24}X_i''(\xi)+.....\right] \tag{21}$$

$$\int_0^L X_i(x)X_j(x)\left[H(x-\xi+\frac{\in}{2})-H(x-\xi-\frac{\in}{2})\right]dx = \in\left[X_i(\xi)X_j(\xi)+\frac{\in^2}{24}\left[X_i(\xi)X_j(\xi)\right]''+.....\right] \tag{22}$$

$$\int_0^L X_i'(x)X_j(x)\left[H(x-\xi+\frac{\in}{2})-H(x-\xi-\frac{\in}{2})\right]dx = \in\left[X_i'(\xi)X_j(\xi)+\frac{\in^2}{24}\left[X_i'(\xi)X_j(\xi)\right]''+.....\right] \tag{23}$$

$$\int_0^L X_i''(x)X_j(x)\left[H(x-\xi+\frac{\in}{2})-H(x-\xi-\frac{\in}{2})\right]dx = \in\left[X_i''(\xi)X_j(\xi)+\frac{\in^2}{24}\left[X_i''(\xi)X_j(\xi)\right]''+.....\right] \tag{24}$$

Substituting 20-24 into Eq. 19, we have

$$\begin{aligned}
&-Mg\left[X_i(\xi)+\frac{\in^2}{24}X_i''(\xi)\right]-M\sum_{i=1}^{\infty}\psi_i(t)\left[X_i(\xi)X_j(\xi)+\frac{\in^2}{24}\left[X_i''(\xi)X_j(\xi)+2X_i'(\xi)X_j'(\xi)+X_i(\xi)X_j''(\xi)\right]\right]\\
&-2MV\sum_{i=1}^{\infty}\psi_i(t)\left[X_i'(\xi)X_j(\xi)+\frac{\in^2}{24}\left[X_i'''(\xi)X_j(\xi)+2X_i''(\xi)X_j'(\xi)+X_i'(\xi)X_j''(\xi)\right]\right]+\\
&+MV^2\sum_{i=1}^{\infty}\psi_i(t)\left[X_i''(\xi)X_j(\xi)+\frac{\in^2}{24}\left[X_i^{iv}(\xi)X_j(\xi)+2X_i'''(\xi)X_j'(\xi)+X_i''(\xi)X_j''(\xi)\right]\right]\Bigg\}=\psi_i(t)
\end{aligned} \tag{25}$$

Substituting Eq. 25 into 16 and the result back into the right hand side of Eq. 9, we finally obtained the approximate governing equation of motion as follows:

$$\begin{aligned}
&EI\sum_{i=1}^{\infty}\phi_i(t)X_i^{iv}(x)+m\sum_{i=1}^{\infty}\ddot{\phi}_i(t)X_i(x)=-Mg\sum_{i=1}^{\infty}X_i(x)\left[X_i(\xi)+\frac{\in^2}{24}X_i''(\xi)\right]-M\sum_{i=1}^{\infty}\left[X_i(\xi)X_j(\xi)\sum_{i=1}^{\infty}\psi_i(t)X_j(\xi)\right]\\
&-\frac{M\in^2}{24}\sum_{i=1}^{\infty}\left[X_i(x)\sum_{i=1}^{\infty}\psi_i(t)\left[X_i''(\xi)X_j(\xi)+2X_i'(\xi)X_j'(\xi)+X_i(\xi)X_j''(\xi)\right]\right]-2MV\sum_{i=1}^{\infty}\left[X_i(x)\sum_{i=1}^{\infty}\psi_i(t)\left[X_i'(\xi)X_j(\xi)+\right.\right.\\
&\left.\left.\frac{\in^2}{24}\left[X_i'''(\xi)X_j(\xi)+2X_i''(\xi)X_j'(\xi)+X_i'(\xi)X_j''(\xi)\right]\right]\right]+MV^2\sum_{i=1}^{\infty}\left[X_i(x)\sum_{i=1}^{\infty}\psi_i(t)\left[X_i''(\xi)X_j(\xi)+\right.\right.\\
&\left.\left.\frac{\in^2}{24}\left[X_i^{iv}(\xi)X_j(\xi)+2X_i'''(\xi)X_j'(\xi)+X_i''(\xi)X_j''(\xi)\right]\right]\right]
\end{aligned} \tag{26}$$

To simplify Eq. 26 we noted that for free vibration of an Euler-Bernoulli beam,

$$X^{iv}{}_i(X)-\beta^4{}_iX_i(x)=0 \tag{27}$$

Where $\beta^4_i = \frac{mp^2_i}{EI}$ and p_i^2 is the natural frequency of the beam

$$X^{iv}{}_i(X) = \beta^4{}_i X_i = \frac{mp^2 X_i}{EI} \tag{28}$$

By putting Eq. 28 into 26, we have

$$\begin{aligned} & m\ddot{\phi}_i(t) + mp_i^2\phi_i(t) = -Mg\sum_{i=1}^{\infty}X_i(t)\left[X_i(\xi)+\frac{\in^2}{24}X_i''(\xi)\right] - M\sum_{i=1}^{\infty}\left[X_i(x)X_i(\xi)\sum_{i=1}^{\infty}\ddot{\psi}_i(t)X_j(\xi)\right] \\ & -\frac{M\in^2}{24}\sum_{i=1}^{\infty}\left[X_i(x)\sum_{i=1}^{\infty}\dot{\psi}_i(t)\left[X_i''(\xi)X_j(\xi)+2X_i'(\xi)X_j'(\xi)+X_i(\xi)X_j''(\xi)\right]\right] - \\ & 2MV\sum_{i=1}^{\infty}X_i(x)\left[\sum_{i=1}^{\infty}\dot{\psi}_i(t)\left[X_i'(\xi)X_j(\xi)+\frac{\in^2}{24}\left[X_i'''(\xi)X_j(\xi)+2X_i''(\xi)X_j'(\xi)+X_i'(\xi)X_j''(\xi)\right]\right]\right] + \\ & MV^2\sum_{i=1}^{\infty}\left[X_i(x)\sum_{i=1}^{\infty}\psi_i(t)\left[X_i''(\xi)X_j(\xi)+\frac{\in^2}{24}\left[X_i^{iv}(\xi)X_j(\xi)+2X_i'''(\xi)X_j(\xi)+X_i''(\xi)X_j''(\xi)\right]\right]\right] \end{aligned} \tag{29}$$

Equation 29 is a set of coupled linear second order ordinary differential Equations, by solving Eq. 16 the values of $\Psi_i(t)$'s can be determined. When substituting these values into Eq. 21 the desired solution for the vibration of the beam, under different boundary conditions and with any number of model shapes can be determined.

Proper consideration of Eq. 24 will result into two interesting special cases of the problem which may be analyzed as follows:

- If $\in$ tends to zero, then the model would revert to the problem of a single point mass traveling on a suspension bridge which has been fully treated by some scholars (Esmailzadeh and Gorashi, 1995; Akin and Mofid, 1989).
- If the inertia effect of the load is ignored then Eq. 24 becomes uncoupled. This can then be verified by replacing Mg by P and M by zero in Eq 24. Therefore, it only the load inertia effect which results in the coupling of Eq. 24. It can be concluded that the moving force solution is a special case of the more general form of the moving mass one.
- Remark; to solve the above set of coupled Eq. 16, we need to know the exact form of the normalized deflection $X_j(x)$.

As a matter of fact there exist various forms of $X_j(x)$ depending on the vibration configuration of the beam.

SIMPLY SUPPORTED BEAM

For the illustration of the result of the foregoing analysis, we considered the case of a simply supported beam. The normalized deflection curves $X_i(x)$ for a simply supported beam are

$$X_i(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{i\pi x}{L}\right), i = 1,2,3... \tag{30}$$

We obtained the set of exact governing differential Equation for the vibration of the beam by deriving exact governing Eq. 30 and evaluating the exact values of the integral in Eq. 11a-e, we have

$$\begin{aligned} & m\ddot{\phi}_i(t) + mp_i^2\phi_i(t) = -\frac{Mg}{i\pi\in}\sqrt{8L}\sin\left(\frac{i\pi\xi}{L}\right)\sin\left(\frac{i\pi\in}{2L}\right) - \frac{2M}{\in\pi}\sum_{i=1}^{\infty}\ddot{\phi}_i(t)\left[\frac{1}{(i-j)}\cos\frac{\pi\xi}{L}(i-j)\sin\frac{\pi\xi}{2L}(i-j)\right] \\ & +\frac{2M}{\in\pi}\sum_{i=1}^{\infty}\ddot{\phi}_j(t)\left[\frac{1}{(i+j)}\cos\frac{\pi\xi}{L}(i+j)\sin\frac{\pi\xi}{2L}(i+j)\right] - \frac{2MV}{\in}\sum_{i=1}^{\infty}\dot{\phi}_i(t)\left[\sqrt{\frac{2}{L}}\frac{1}{(i+j)}\sin\frac{\pi\in}{2L}(i+j)\sin\frac{\pi\xi}{L}(i+j)\right] - \\ & \frac{2MV}{\in}\sum_{i=1}^{\infty}\dot{\phi}_i(t)\left[\sqrt{\frac{2}{L}}\frac{1}{(i-j)}\sin\frac{\pi\in}{2L}(i-j)\sin\frac{\pi\xi}{L}(i-j)\right] - \frac{MV^2}{\in}\left(\frac{i\pi}{L}\right)\sum_{i=1}^{\infty}\phi_i(t)\left[\sqrt{\frac{2}{L}}\frac{1}{(i-j)}\sin\frac{\pi\in}{2L}(i-j)\sin\frac{\pi\xi}{L}(i-j)\right] \\ & +\frac{MV^2}{\in}\left(\frac{i\pi}{L}\right)\sum_{i=1}^{\infty}\phi_i(t)\left[\sqrt{\frac{2}{L}}\frac{1}{(i+j)}\sin\frac{\pi\in}{2L}(i+j)\cos\frac{\pi\xi}{L}(i+j)\right] + \frac{MV^2}{\in}\left(\frac{i\pi}{L}\right)\sum_{i=1}^{\infty}\phi_i(t)\left[\sqrt{\frac{2}{L}}\frac{1}{(i+j)}\cos\frac{\pi\in}{2L}(i+j)\cos\frac{\pi\xi}{L}(i+j)\right] \\ & i = 1,2,3... \end{aligned} \quad (31)$$

For the case of of $i = j$, the expression involved should be replaced by $\frac{\pi\varepsilon}{2L}$

We solved the system of Eq. 26 by numerical procedure called Finite difference method.

Remarks

In the remark, we considered the solutions of two cases viz:

- The moving force Euler- Bernoulli Beam case
- The moving mass Euler-Bernoulli Beam case

Case I

Moving force euler-bernoulli beam problem: We mean the case in which only the force effects are taken into consideration. For the particular case the governing equations obtained by neglecting all terms appearing after the first term on the right hand side of the Eq. 31.

Case II

Moving mass euler- bernoulli beam problem: This is the case in which both the inertia effects and the force effects are retained i.e., the whole Eq. 31 is the moving mass problem.

RESULTS AND DISCUSSION

To solve the two cases in our problem, we employed numerical method. (i.e., approximate central difference formulae have been utilized for the derivative in Eq. 31, which are latter transformed to a set of N linear algebraic equations, which are to be solved for each interval of time. Regarding the degree of approximations involved, in order to ensure the stability and convergence of the solution, sufficiently small time steps have been utilized. The package MATLAB was used for the following numerical data which are the same as those chosen by (Esmailzadeh and Gorshi, 1995). $E = 2.07\times10^{11}$ N m^{-2}, $I = 1.04\times10^{-6}m^4$, $V = 12$ km h^{-1}, $M = 7.04$ kg m^{-1} $g = 9.81$ m s^{-2}, $m = 7.0$ kg m^{-1}, $t = 0.1$ s and 1.0 s, $\varepsilon = 0.1$ m and 1.0 m, $L = 10$ m. $h = 0.01$ m.

Figure 2 illustrates the displacement response of the simply supported Bernoulli-Euler beam for both cases of moving force and moving mass for fixed value of $\in$ and t. Clearly the result shows that the response amplitude due to the moving mass is greater than that due to the moving force. As a result of this the moving force is not an upper bound for the accurate solution for the moving mass problem. Figure 3 and 4 depict the deflection of the simply supported Bernoulli-Euler beam for both the moving force and moving mass, respectively. In the graphs the deflection Y (x, t) is plotted against various values of x for various time t. It is noted that, as time t increases, the amplitude of deflection increases for both the moving force and moving mass problem.

The deflection profiles of the elastic Euler-Bernoulli beam for various values of $\in$ for a fixed length of the beam for both moving force and moving mass problem are displayed in Fig. 5 and 6, respectively. It is observed that $\in$ increases with an increase in the amplitude of deflection for both cases of our consideration.

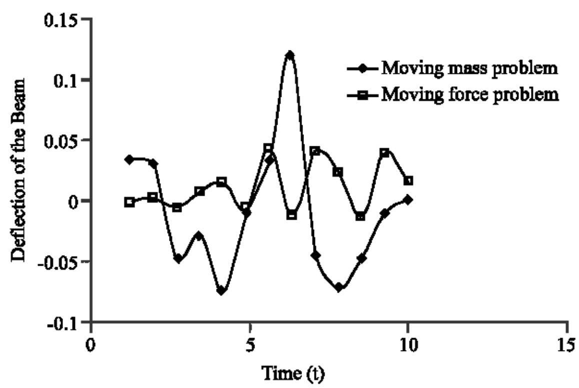

Fig. 2: Displacement responce of the simply supported Euler-Bernoulli beam for both moving mass and moving force

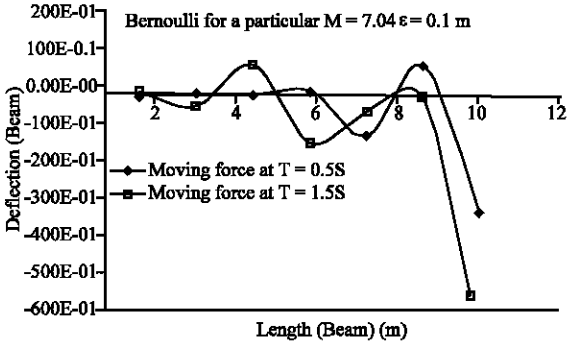

Fig. 3: Deflection of the simply support Bernoulli Euler for the moving force at diffirent time T. (I.e., T = 0.5s and 1.0s) for a particular m = 7.04 kg when ε = 0.1 m

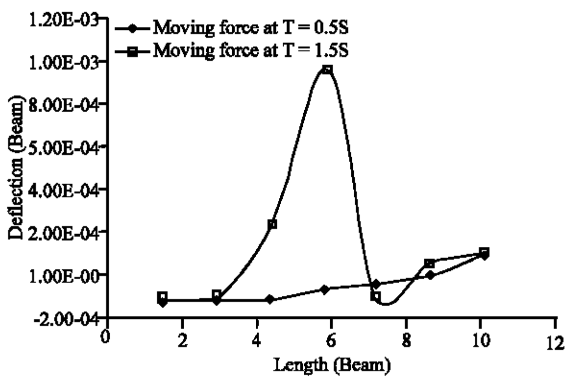

Fig. 4: Deflection of the simply support Bernoulli Euler for the moving force at diffirent time T. (I.e., T = 0.5s and 1.0s) for a particular m = 7.04 kg when ε = 0.1 m

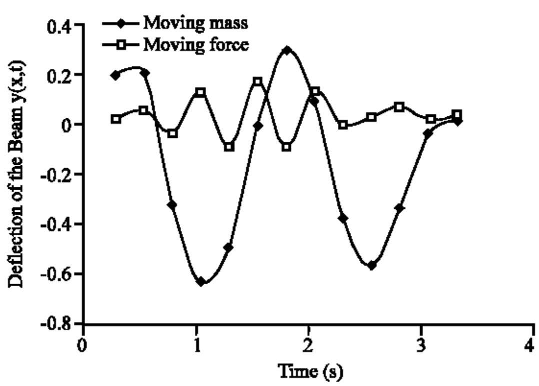

Fig. 5: Deflection profile of Euler-Bernoulli Beam for both moving mass and moving force, when ε = 0.1 m with in 3 min for a fixed length of the beam (L = 10 m)

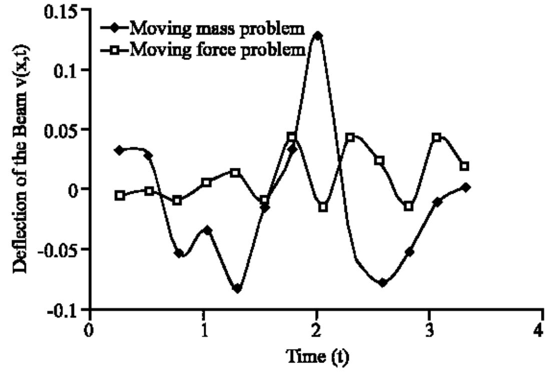

Fig. 6: Deflection profile of Euler-Bernoulli Beam for both moving mass and moving force, when ε= 0.1 m with in 3 min for a fixed length of the beam (L = 10 cm)

CONCLUSION

This study presented dynamical behaviour of Euler-Bernoulli beam traversed by uniform partially distributed moving masses. The theory based on orthogonal functions and inertial effect of the load and the results indicate that the governing differential Equations of motion can be transformed into coupled ordinary differential equations. Hence, ignoring this effect (inertial effect) result in solving a set of uncoupled linear second order differential equations which is the solution for the corresponding moving distributed force and not the moving distributed mass problem. In solving the governing differential equations the technique of central difference expansions was employed.

It was observed that the length of the distributed moving mass affects the dynamic response considerably, this brings about the response amplitude due to the moving mass to be greater than the moving force amplitude which indicate that the moving force is not the upper bound for the accurate solution for the moving mass problem.

Furthermore, a comparison of the moving mass and moving force (Fryba, 1972) results, indicated an at least 80% difference between the two results and thus shows the importance of including mass in real design conditions where the velocity is high.

So also, in designing structures such as railway and suspension bridges, the moving mass inertial effects is very important and these effects should be borne in mind with utmost care, because of the fact that moving loads have a great effect on dynamic stresses in such bodies and structures and cause them to vibrate intensively, especially at high velocity.

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