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Performances of Some Estimators of Linear Model with Autocorrelated Error Terms in the Presence of Multicollinearity

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Abstract: Assumptions in classical linear regression model that regressors are assumed to be independent and non-stochastic in repeated sampling are often violated by economist and other social scientists. This is because their regressors are generated by stochastic process beyond their control. Consequently, in this study we examine the performances of the Ordinary Least Square (OLS) and four Generalized Least Square (GLS) estimators of linear model with autocorrelated error terms when normally distributed stochastic regressors exhibit multicollinearity. These estimators are compared by examing their finite sampling properties at various levels of autocorrelation and non-validity of the multicollinearity assumption through Monte-Carlo studies. Results show that the Maximum Likelihood (ML) and the Hildreth and LU (HILU) estimators are generally preferable in estimating all the parameters of the model at all the levels of autocorrelation and multicollinearity. Consequently, when the these two forms of correlations can not be ascertained in a data set, it is more preferable to use either the ML or HILU estimator to estimate all parameters of the model.

Key words: Stochastic regressors, linear model with autocorrelated error, OLS estimator, feasible GLS estimators, multicollinearity

INTRODUCTION

Assumption that regressors are assumed fixed (non-stochastic) in repeated sampling under the classical linear regression model is not always satisfied in business, economic and social sciences because their regressors are often generated by a stochastic process beyond their control. Situations and instances where this assumption is violated and its consequences on the Ordinary Least Square (OLS) estimator when used to estimate the model parameters have been discussed by Neter and Wasserman (1974), Fomby *et al*. (1984), Maddala (2002). Graybill (1961), Sampson (1974), Fomby *et al*. (1984) emphasized that if regressors are stochastic and independent of the error terms; the OLS estimator is not only unbiased but has minimum variance even though it is not Best Linear Unbiased Estimator (BLUE). They also pointed out that the traditional hypothesis testing is valid if the error terms are further assumed normal but modification would be required in the area of confidence interval calculated for each sample and the power of the test. The assumption of independence of regressors is also not likely to be achieved because explanatory variables like gross national product and income are most correlated over time. When regressors are dependent (correlated), that is there is multicollinearity, the OLS estimates are still unbiased as long as multicollinearity is not perfect (Johnston, 1984). However when multi-collinearity is high, only imprecise estimate may be available about the individual true regression coefficients which are often statistically insignificant because of its large standard errors (Chartterjee *et al*., 2000).

When autocorrelation of first-order exist in the OLS residuals, the OLS estimator is unbiased but inefficient. The predicted values are also inefficient and the sampling variances of the autocorrelated error terms under-estimated causing the t and F tests to be invalid (Johnston, 1984; Fomby *et al.*, 1984; Chartterjee *et al.*, 2000; Maddala, 2002). To compensate for the lost of efficiency, Cochrane and Orcutt (1949) were first to observe that the presence of autocorrelation (ρ) in linear model requires some modification of the OLS method. They suggested a transformation that uses the matrix

$$P = \begin{bmatrix} -\rho & 1 & 0 & \dots & 0 & 0 \\ 0 & -\rho & 1 & \dots & 0 & 0 \\ 0 & 0 & -\rho & \dots & 0 & 0 \\ . & . & . & \dots & . & . \\ . & . & . & \dots & . & . \\ . & . & . & \dots & . & . \\ 0 & 0 & 0 & \dots & -\rho & 1 \end{bmatrix}_{(n-1)\times n}$$

which ignores the first observation of the error terms. Paris and Winsten (1954) showed that the appropriate transformation required for the transformation is

$$Q=\begin{bmatrix}(1-\rho^2)^{\frac{1}{2}} & 0 & 0 & \dots & 0 & 0\\ -\rho & 1 & 0 & \dots & 0 & 0\\ 0 & -\rho & 1 & \dots & 0 & 0\\ . & . & . & \dots & . & .\\ . & . & . & \dots & . & .\\ . & . & . & \dots & . & .\\ 0 & 0 & 0 & \dots & -\rho & 1\end{bmatrix}_{nxn}$$

which retains the first observation. The difference in the usage of Pand Q can be negligible when is large, but in small sample investigation such as in this study, the difference may be major. The process involves the autoregressive form of the error terms to be estimated and the estimate of the autocorrelation used to obtain estimate for the regression coefficients of the linear model via the Generalized Least Square (GLS) estimator given by Aitken (1935) as

$$\hat{\beta}=(X^1\Omega^{-1}X)^{-1}X^1\Omega^{-1}Y \quad (1)$$

with the variance-covariance matrix

$$V\left(\hat{\beta}\right)=\sigma^2\left(X^1\Omega^{-1}X\right)^{-1} \quad (2)$$

where

$$\Omega^{-1}=\frac{1}{1-\rho^2}\begin{bmatrix}1 & -\rho & 0 & \dots & 0 & 0\\ -\rho & 1+\rho^2 & -\rho & \dots & 0 & 0\\ 0 & -\rho & 1+\rho^2 & \dots & 0 & 0\\ . & . & . & \dots & . & .\\ . & . & . & \dots & . & .\\ . & . & . & \dots & 1+\rho^2 & -\rho\\ 0 & 0 & 0 & \dots & -\rho & 1\end{bmatrix}_{nxn}$$

However ρ and hence Ω is not always known, it is often estimated by $\hat{\Omega}$ to have what is known as Feasible GLS estimator. Many consistent estimates of $\hat{\Omega}$ can be obtained (Fomby *et al.*, 1984). Among those Feasible GLS estimators available in literature are the Cochrane and Orcutt estimator (1949), Hildreth and Lu estimator (1960), Paris and Winsten estimator (1954), Thornton estimator (1982), Durbin estimator (1960), Theil's estimator (1971), the Maximum Likelihood estimator and the Maximum Likelihood Grid estimator (Beach and Mackinnon, 1978). Some of these estimators have now been incorporated into White's SHAZAM program (White, 1978) and the new version of the time series processor (TSP, 2005).

However, all these estimators are known to be asymptotically equivalent but the question on which is to be preferred in small samples is another matter (Fomby *et al.*, 1984). Chipman (1979), Kramer (1980), Kleiber (2001) and many others have observed that the efficiency of these estimators depends on the structure of the regressors that are used. Rao and Griliches (1969) did one of the earliest Monte-Carlo investigation on this study. He examined the performances of some of these estimators with first-order autoregressive stochastic regressor. Their results show that the OLS estimator is only more efficient than any of the GLS estimators considered when; and that the performances of the GLS estimators are not far apart. Park and Mitchell (1980) observed that when regressors are trended, the estimator that retains the first observation (Paris-Winstern) is more efficient than the one that does not (Cochrane-Orcutt) and that the latter is even less efficient than the OLS estimator.

More recently, Nwabuwze (2005a) examined the performances the OLS estimator and four feasible GLS estimators namely: Cochrane-Orcutt (CORC), Hidreth and Lu (HILU), Maximum Likelihood (ML) and Maximum Likelihood Grid (MLGD) estimators with fixed regressor that is normally distributed. The estimators' performances in order of preference are HILU, ML, MLGD, OLS and COCR when the sample size is small and the autocorrelation value is large. Nwabueze (2005b) also examined the performances of these estimators with exponential independent variable. His result, among other things, shows that the OLS estimator compares favorably with the Maximum Likelihood (ML) and Maximum Likelihood Grid (MLGD) estimators for small value of but it appears to be superior to Cochrane-Orcutt (CORC) and the Hidreth and Lu (HILU) especially when is large. Some other recent works that are done with different specification of regressors include that of Iyaniwura and Nwabuwze (2005 a,b, c) and Iyaiwura and Olaomi (2006).

Consequently, this study examines the performances of some of these estimators when normally distributed stochastic regressors exhibit different degree of multicollinearity with a view of determining the estimator(s) that can be generally preferable in estimating all the model parameters at all the levels of the two types of correlation.

MATERIALS AND METHODS

Consider the GLS model with stochastic regressors and AR (1) of the form

$$y_t=\beta_0+\beta_1x_{1t}+\beta_2x_{2t}+u_t \quad (3)$$

where, $u_t = \rho_1 u_{t-1} + \varepsilon_t$ t = 1,2,…,n, $\varepsilon t \sim N(0, \sigma^2 I_n)$ and x_2 is said to have ρ_2 correlation with x, $|\rho| < I$ i = 1, 2.

Now, suppose $W_i \sim N(\mu_i, \sigma_i^2)$ $i = 1,2$. If these variables are correlated, then and can be generated with the equations

$$W_1 = \mu_1 + \sigma_1 z_1$$
$$W_2 = \mu_2 + \rho\sigma_2 z_1 + \sigma_2 z_2\sqrt{1-\rho^2} \tag{4}$$

where $Z_i \sim N(0,1)$ i = 1,2 and $|\rho|<1$ is the value of the correlation between the two variables (Ayinde and Oyejola, 2007).

Parameter estimations of model (3) can be done using the OLS and the feasible GLS estimators. Thus, the performances of the OLS estimator and the following feasible GLS estimators are studied: CORC, HILU, ML and the MLGD estimators. The CORC and HILU estimators do not retain the first observation while the ML and MLGD estimators do.

Monte Carlo experiments were performed for n = 20, a small sample size representative of many time series study (Park and Mitchell, 1980) with two replication (R) levels (R = 80, 120) and nine various degree of autocorrelation and multicollinearity ($\rho_1$and ρ_2 = -0.99,-0.75,-0.5,… 0.99, respectively) utilizing Eq. 3 and 4. At a particular choice of ρ_1, ρ_2 and R (a scenario), each replication was first obtained by generating $\varepsilon_t \sim N(0, 1)$ and hence. u_t Assuming the process start from infinite past and continue to operate, the initial value of U (i.e., u_1) was thus drawn from a normal population with mean zero and variance. $\frac{1}{1-\rho_1^2}$ Hence

$$u_1 = \frac{\varepsilon_1}{\sqrt{1-\rho_1^2}} \tag{5}$$

$$u_t = \rho_1 u_{t-1} + \varepsilon_t \; t = 2, 3,..., 20 \tag{6}$$

Furthermore $x_{1t} \sim N(0,1)$ and $x_{2t} \sim N(0,1)$ having ρ_2 correlation were generated using Eq. 4. Hence, we have

$$x_{1t} = z_{2t} \tag{7}$$

$$x_{2t} = \rho_2 x_{1t} + z_{3t}\sqrt{1-\rho_2^2} \tag{8}$$

The values of in Eq. 3 were also calculated by setting the true regression coefficients as $\beta_0 = \beta_1 = \beta_2 = 1$. This process continued until all replications in this scenario were obtained. Another scenario then started until all the scenarios were completed.

Evaluation and comparison of estimators were examined using the finite sampling properties of estimators which include Bias (B), Absolute Bias (AB) and Variance (Var) and the more importantly the Mean Squared Error (MSE) criteria. Mathematically, for any estimator $\hat{\beta}1$ of $\beta 2$ of model (3)

$$\bar{\hat{\beta}}_i = \frac{1}{R}\sum_{j=1}^{R}\hat{\beta}_{ij} \tag{9}$$

$$B\left(\hat{\beta}_i\right) = \frac{1}{R}\sum_{j=1}^{R}\left(\hat{\beta}_{ij} - \beta_i\right) = \bar{\hat{\beta}}_i - \beta_i \tag{10}$$

$$AB\left(\hat{\beta}_i\right) = \frac{1}{R}\sum_{j=1}^{R}\left|\hat{\beta}_{ij} - \beta_i\right| \quad 11)$$

$$Var\left(\hat{\beta}_i\right) = \frac{1}{R}\sum_{j=1}^{R}\left(\hat{\beta}_{ij} - \bar{\hat{\beta}}_i\right)^2 \tag{12}$$

$$MSE\left(\hat{\beta}_i\right) = \frac{1}{R}\sum_{j=1}^{R}\left(\hat{\beta}_{ij} - \beta_i\right)^2 = Var\left(\hat{\beta}_i\right) + \left[B\left(\hat{\beta}_i\right)\right]^2 \tag{13}$$

for i = 0, 1, 2 and j= 1,2,…,R.

For each of the estimators, a computer program was written using TSP software to estimate all the model parameters and to evaluate the criteria. The effect of autocorrelation and multicollinearity on the performances of the methods (estimators) is examined via the Analysis of Variance (ANOVA) of the criteria of each of the model parameters. Consequently, the highest order significant interaction effect which has method has a factor is discussed. However, since at least one of the estimators (CORC) is biased in small samples (Rao and Griliches, 1969) and that the mean squared error is known to replace the absolute bias (Kruthkoff, 1970) and also comprises variance and bias; therefore a further test using the LSD technique on the highest order significant interaction effect which has method as a factor was performed on the basis of the mean squared error criterion. The LSD test of the estimated marginal mean was done at a particular combination of levels of the two correlations. At a particular combination of levels, estimators were preferred if their estimated marginal means are not significantly different from the most preferred one. An estimator is most preferred if its estimated marginal mean is the smallest. Estimators that are preferred at all the levels of various combinations of autocorrelation and multicollinearity in estimating all the model parameters are simply said to be generally preferable.

Table 1: Summary of the ANOVA TABLE showing the sum of squares of the model parameters based on the criteria.

			Type III Sum of squares			
Parameter	Source	d.f	Bias	Absolute bias	Variance	Mean squared error
β_0	R2	8	.000	.000	.000	.000
	R1	8	2.868***	2322.860***	184234.265***	184551.437***
	M	4	4.372E-02***	.378***	14.365***	14.250***
	R1*R2	64	.000	.000	.000	.000
	R2*M	32	.000	.000	.000	.000
	R1*M	32	.168***	2.052***	94.450***	93.423***
	R1*R2*M	256	.000	.000	.000	.000
	Error	405	0.213	0.949	41.856	42.188
	Total	809	3.293	2326.238	184384.9	184701.3
β_1	R2	8	21.881***	224.649***	7780.196***	8251.934***
	R1	8	8.72E-02	17.982***	7764.727***	8110.818***
	M	4	2.57E-02	23.876***	4743.151***	4993.031***
	R1*R2	64	4.896***	17.681***	21820.640***	22846.583***
	R2*M	32	3.822***	22.867***	13268.350***	14004.106***
	R1*M	32	0.172	91.899***	31977.982***	33387.006***
	R1*R2*M	256	7.428***	91.792***	89928.208***	94089.641***
	Error	405	3.998	0.284	1911.193	1334.4
	Total	809	42.311	491.029	179194.4	187017.5
β_2	R2	8	11.650***	223.493***	7777.521***	8250.190***
	R1	8	2.722***	18.268***	8038.759***	8418.937***
	M	4	2.124***	23.783***	4890.440***	5162.846***
	R1*R2	64	2.607***	17.497***	21786.194***	22816.533***
	R2*M	32	2.035***	22.779***	13253.799***	13992.058***
	R1*M	32	4.129***	94.407***	33129.855***	34672.740***
	R1*R2*M	256	3.955***	90.421***	89786.688***	93968.132***
	Error	405	4.259	0.268	1964.408	1351.296
	Total	809	33.482	490.916	180627.7	188632.7

* $\Rightarrow$Computed F value is significant at $\alpha = 0.05$. ** Computed F value is significant at $\alpha = 0.01$. *** $\Rightarrow$Computed F value is significant at á = 0.001. $=\rho_1$ R1 $\Rightarrow$ Autocorrelation levels. ρ_2= R2 $\Rightarrow$ Multicollinearity levels. M $\Rightarrow$ Methods

Table 2: Summary of the ANOVA TABLE of the reduced model showing the sum of squares of parameters based on the criteria

			Type III Sum of squares			
Parameter	Source	d.f	Bias	Absolute bias	Variance	Mean squared error
β_0	R1	8	.319***	258.096***	20470.474***	20505.716***
	M	4	4.858E-03	4.201E-02**	1.596**	1.583*
	R1*M	32	1.862E-02	.228***	10.494***	10.380***
	Error	45	2.367E-02	.105	4.651	4.688
	Total	89	.366	258.471	20487.216	20522.367

RESULTS AND DISCUSSION

The summary of our findings on the performances of the estimators based on the criteria for each model parameters in the two replication groups is given in Table 1.

From Table 1 it is observed that in estimating, β_0 the main effect of multicollinearity and any of its interaction effects are completely insignificant. Thus, multicollinearity does not have effect in estimating β_0 in the regression model. The analysis of variance of the reduced model is therefore run in the absence of multicollinearity (i.e., run when ρ_2= 0). The resulting analysis of variance is given in Table 2.

From Table 2 the interaction effect of autocorrelation by method is significant except under bias criterion. Thus, the performances of the estimators are affected by autocorrelation in all the criteria except bias. The estimated marginal means on the basis of the mean squared error is shown in appendix. From appendix, it is observed that the estimated marginal mean of all the estimators increase as ρ_1 increases. However, the estimated marginal means of the OLS estimator decreases as ρ_1 increases in the interval $-1<\rho_1\leq-0.5$. Furthermore, as at each level of autocorrelation the most preferred estimate is not significantly different from others except when $\rho_1\to1$ where the ML and HILU estimators exhibit significant difference from others. Thus, the ML and HILU estimators are preferred in all the levels of autocorrelation.

In estimating $\beta_1$and β_2, the highest order interaction effect of autocorrelation by multicollinearity by method is significant in all the criteria. Thus, the performances of the estimators are affected by the joint contribution of autocorrelation and multicollinearity. Their estimated marginal means based on the mean squared error of the estimated parameters are also given in appendix. From appendix, it is observed that the performances of the estimators in estimating β_2 are symmetric over the levels of multicollnearity. At each level of multicollinearity, the estimated marginal means of the GLS estimators decrease while that of the OLS estimator increases as autocorrelation increases in its absolute form ($|\rho_1|$). Also at each level of autocorrelation, the estimated marginal means of all the estimators increase as multicollinearity increases in its absolute form ($|\rho_2|$). Furthermore from the appendix, it is observed that all the estimators do not exhibit significant difference from the most preferred one except $|\rho_1| \to 1$ and $|\rho_2| < 1$ and when autocorrelation is negatively high (ρ_1 = -0.75) and $|\rho_1| \to 1$. In these exceptions, only the OLS estimator is significantly different from the most preferred one. Therefore, the GLS estimators are preferred in all the levels of autocorrelation and multicollinearity However, the OLS estimator competes favourably with the GLS estimators when $|\rho_1| \leq 0.5$ and $|\rho_2| \leq 0.5$ and in estimating $\beta_1$and β_2.

Summarily, the two GLS methods namely, ML and HILU can therefore be generally preferable to estimate all the parameters of the model at all the levels of autocorrelation and multicollinearity. Consequently, when the these two forms of correlations can not be ascertained in a data set, it is more preferable to use either the ML or HILU estimator to estimate all parameters of the model.

Appendix 1: Summary of the results on the LSD test of the estimated marginal means based on mean squared error of the estimated parameter with significant interaction effects

			ρ_2 = -0.99		ρ_2 = -0.75	
ρ_1	M	β_0	β_1	β_2	β_1	β_0
-.99	OLS	0.467	221.834+	216.510+	9.899+	9.848+
	COCR	1.50E-02	2.23	2.285	9.88E-02	0.104
	HILU	1.47E-02	2.181	2.236	9.71E-02	0.102
	ML	1.49E-02	2.174	2.226	9.77E-02	0.101
	MLGD	1.51E-02	2.135	2.17	0.1	9.87E-02
-.75	OLS	3.89E-02	8.279+	8.560+	0.33	0.389
	COCR	1.96E-02	2.699	2.813	0.113	0.128
	HILU	1.97E-02	2.683	2.8	0.112	0.127
	ML	1.93E-02	2.616	2.678	0.117	0.122
	MLGD	1.93E-02	2.599	2.66	0.116	0.121
-.5	OLS	3.35E-02	4.592	4.774	0.18	0.217
	COCR	2.67E-02	3.1	3.238	0.129	0.147
	HILU	2.66E-02	3.115	3.252	0.13	0.148
	ML	2.56E-02	3.028	3.119	0.13	0.142
	MLGD	2.55E-02	3.026	3.118	0.129	0.142
-.25	OLS	3.83E-02	3.379	3.489	0.133	0.159
	COCR	3.86E-02	3.486	3.621	0.145	0.165
	HILU	3.85E-02	3.495	3.622	0.146	0.165
	ML	3.61E-02	3.325	3.449	0.135	0.157
	MLGD	3.59E-02	3.329	3.449	0.136	0.157
0	OLS	5.19E-02	2.769	2.841	0.113	0.129
	COCR	5.86E-02	3.571	3.639	0.158	0.166
	HILU	5.88E-02	3.57	3.639	0.158	0.166
	ML	5.30E-02	3.146	3.24	0.135	0.147
	MLGD	5.31E-02	3.129	3.218	0.135	0.146
.25	OLS	8.33E-02	2.51	2.568	0.112	0.117
	COCR	9.56E-02	3.231	3.248	0.148	0.148
	HILU	9.65E-02	3.262	3.275	0.149	0.149
	ML	8.51E-02	2.789	2.83	0.126	0.129
	MLGD	8.52E-02	2.815	2.858	0.127	0.13
.5	OLS	0.169	2.811	2.849	0.143	0.13
	COCR	0.204	2.795	2.713	0.132	0.123
	HILU	0.214	2.819	2.735	0.133	0.124
	ML	0.169	2.433	2.372	0.116	0.108
	MLGD	0.168	2.417	2.356	0.115	0.107
.75	OLS	0.571	4.797	4.764	0.254	0.217
	COCR	0.936	2.398	2.317	0.113	0.105
	HILU	0.861	2.382	2.3	0.113	0.105
	ML	0.549	2.116	2.046	0.102	9.31E-02
	MLGD	0.546	2.097	2.028	0.101	9.23E-02
.99	OLS	48.841+	10.076+	10.175+	0.452	0.463
	COCR	49.897+	2.093	2.008	0.101	9.13E-02
	HILU	46.888	2.073	1.994	9.87E-02	9.07E-02
	ML	47.537	1.957	1.888	9.37E-02	8.59E-02
	MLGD	47.727+	1.95	1.882	9.36E-02	8.56E-02

Appendix continue

ρ_1	M	ρ_2 = -0.5 β_1	ρ_2 = -0.5 β_2	ρ_2 = -0.25 β_1	ρ_2 = -0.25 β_2	ρ_2 =0 β_1	ρ_2 =0 β_2	ρ_2 = 0.25 β_1	ρ_2 = 0.25 β_2
-.99	OLS	5.297+	5.745+	3.862+	4.596+	3.343+	4.309+	3.399+	4.596+
	COCR	6.02E-02	6.06E-02	5.08E-02	4.85E-02	4.99E-02	4.55E-02	5.51E-02	4.85E-02
	HILU	5.95E-02	5.93E-02	5.04E-02	4.75E-02	4.96E-02	4.45E-02	5.48E-02	4.75E-02
	ML	6.01E-02	5.91E-02	5.09E-02	4.73E-02	5.01E-02	4.43E-02	5.52E-02	4.73E-02
	MLGD	6.27E-02	5.76E-02	5.31E-02	4.61E-02	5.18E-02	4.32E-02	5.63E-02	4.61E-02
-.75	OLS	0.189	0.227	0.156	0.182	0.154	0.17	0.175	0.182
	COCR	6.87E-02	7.46E-02	5.91E-02	5.97E-02	5.97E-02	5.60E-02	6.78E-02	5.97E-02
	HILU	6.85E-02	7.43E-02	5.91E-02	5.95E-02	5.99E-02	5.57E-02	6.81E-02	5.95E-02
	ML	7.12E-02	7.11E-02	6.00E-02	5.69E-02	5.90E-02	5.33E-02	6.50E-02	5.69E-02
	MLGD	7.12E-02	7.06E-02	6.01E-02	5.65E-02	5.90E-02	5.29E-02	6.50E-02	5.65E-02
-.5	OLS	0.103	0.127	8.57E-02	0.101	8.57E-02	9.50E-02	9.84E-02	0.101
	COCR	7.93E-02	8.59E-02	6.87E-02	6.87E-02	6.97E-02	6.44E-02	7.92E-02	6.87E-02
	HILU	7.93E-02	8.63E-02	6.85E-02	6.90E-02	6.94E-02	6.47E-02	7.89E-02	6.90E-02
	ML	7.80E-02	8.28E-02	6.57E-02	6.62E-02	6.50E-02	6.21E-02	7.25E-02	6.62E-02
	MLGD	7.73E-02	8.27E-02	6.51E-02	6.62E-02	6.44E-02	6.21E-02	7.20E-02	6.62E-02
-.25	OLS	7.51E-02	9.26E-02	6.12E-02	7.41E-02	6.03E-02	6.94E-02	6.86E-02	7.41E-02
	COCR	8.75E-02	9.61E-02	7.45E-02	7.69E-02	7.48E-02	7.21E-02	8.47E-02	7.69E-02
	HILU	8.76E-02	9.61E-02	7.43E-02	7.69E-02	7.43E-02	7.21E-02	8.39E-02	7.69E-02
	ML	7.94E-02	9.15E-02	6.66E-02	7.32E-02	6.65E-02	6.86E-02	7.56E-02	7.32E-02
	MLGD	7.96E-02	9.15E-02	6.67E-02	7.32E-02	6.64E-02	6.86E-02	7.53E-02	7.32E-02
0	OLS	6.46E-02	7.54E-02	5.26E-02	6.03E-02	5.13E-02	5.65E-02	5.75E-02	6.03E-02
	COCR	9.44E-02	9.66E-02	7.83E-02	7.72E-02	7.61E-02	7.24E-02	8.35E-02	7.72E-02
	HILU	9.44E-02	9.66E-02	7.84E-02	7.72E-02	7.62E-02	7.24E-02	8.37E-02	7.72E-02
	ML	8.11E-02	8.60E-02	6.84E-02	6.88E-02	6.77E-02	6.45E-02	7.55E-02	6.88E-02
	MLGD	8.10E-02	8.54E-02	6.82E-02	6.83E-02	6.73E-02	6.41E-02	7.50E-02	6.83E-02
.25	OLS	6.84E-02	6.81E-02	5.76E-02	5.45E-02	5.65E-02	5.11E-02	6.22E-02	5.45E-02
	COCR	8.78E-02	8.62E-02	7.15E-02	6.89E-02	6.79E-02	6.46E-02	7.29E-02	6.89E-02
	HILU	8.84E-02	8.69E-02	7.18E-02	6.95E-02	6.81E-02	6.52E-02	7.30E-02	6.95E-02
	ML	7.60E-02	7.51E-02	6.31E-02	6.01E-02	6.11E-02	5.63E-02	6.65E-02	6.01E-02
	MLGD	7.69E-02	7.58E-02	6.39E-02	6.07E-02	6.18E-02	5.69E-02	6.74E-02	6.07E-02
.5	OLS	9.40E-02	7.56E-02	8.17E-02	6.05E-02	8.02E-02	5.67E-02	8.62E-02	6.05E-02
	COCR	7.35E-02	7.20E-02	5.52E-02	5.76E-02	4.84E-02	5.40E-02	4.88E-02	5.76E-02
	HILU	7.40E-02	7.26E-02	5.54E-02	5.81E-02	4.85E-02	5.44E-02	4.88E-02	5.81E-02
	ML	6.65E-02	6.29E-02	5.10E-02	5.03E-02	4.56E-02	4.72E-02	4.64E-02	5.03E-02
	MLGD	6.55E-02	6.25E-02	5.01E-02	5.00E-02	4.46E-02	4.69E-02	4.54E-02	5.00E-02
.75	OLS	0.164	0.126	0.139	0.101	0.133	9.48E-02	0.139	0.101
	COCR	6.22E-02	6.15E-02	4.60E-02	4.92E-02	3.97E-02	4.61E-02	3.96E-02	4.92E-02
	HILU	6.23E-02	6.10E-02	4.60E-02	4.88E-02	3.98E-02	4.58E-02	3.96E-02	4.88E-02
	ML	5.76E-02	5.43E-02	4.34E-02	4.34E-02	3.80E-02	4.07E-02	3.80E-02	4.34E-02
	MLGD	5.69E-02	5.38E-02	4.29E-02	4.31E-02	3.75E-02	4.04E-02	3.76E-02	4.31E-02
.99	OLS	0.267	0.27	0.218	0.216	0.208	0.202	0.226	0.216
	COCR	5.55E-02	5.33E-02	4.07E-02	4.26E-02	3.47E-02	4.00E-02	3.40E-02	4.26E-02
	HILU	5.42E-02	5.29E-02	3.97E-02	4.23E-02	3.40E-02	3.97E-02	3.35E-02	4.23E-02
	ML	5.22E-02	5.01E-02	3.89E-02	4.01E-02	3.37E-02	3.76E-02	3.36E-02	4.01E-02
	MLGD	5.23E-02	4.99E-02	3.90E-02	4.00E-02	3.39E-02	3.75E-02	3.38E-02	4.00E-02

ρ_1	M	ρ_2 = 0.5 β_1	ρ_2 = 0.5 β_2	ρ_2 = 0.75 β_1	ρ_2 = 0.75 β_2	ρ_2 = 0.99 β	ρ_2 = 0.99 β_2
	OLS	4.262+	5.745+	7.867+	9.848+	209.255+	216.510+
	COCR	6.99E-02	6.06E-02	1.18E-01	1.04E-01	2.35E+00	2.29E+00
q-.99	HILU	6.94E-02	5.93E-02	1.17E-01	1.02E-01	2.30E+00	2.24E+00
	ML	6.97E-02	5.91E-02	1.17E-01	1.01E-01	2.29E+00	2.23E+00
	MLGD	6.97E-02	5.76E-02	1.14E-01	9.87E-02	2.22E+00	2.17E+00
	OLS	0.233	0.227	0.416	0.389	8.808+	8.560+
	COCR	8.81E-02	7.46E-02	1.51E-01	1.28E-01	2.93E+00	2.81E+00
-.75	HILU	8.85E-02	7.43E-02	1.51E-01	1.27E-01	2.93E+00	2.80E+00
	ML	8.23E-02	7.11E-02	1.38E-01	1.22E-01	2.75E+00	2.68E+00
	MLGD	8.21E-02	7.06E-02	1.38E-01	1.21E-01	2.73E+00	2.66E+00
	OLS	0.132	0.127	2.36E-01	0.217	4.94E+00	4.77E+00
	COCR	1.03E-01	8.59E-02	1.76E-01	1.47E-01	3.39E+00	3.24E+00
-.5	HILU	1.03E-01	8.63E-02	1.76E-01	1.48E-01	3.40E+00	3.25E+00
	ML	9.33E-02	8.28E-02	1.60E-01	1.42E-01	3.22E+00	3.12E+00
	MLGD	9.28E-02	8.27E-02	1.59E-01	1.42E-01	3.21E+00	3.12E+00
	OLS	9.17E-02	9.26E-02	1.66E-01	1.59E-01	3.58E+00	3.49E+00
	COCR	1.10E-01	9.61E-02	1.90E-01	1.65E-01	3.76E+00	3.62E+00
-.25	HILU	1.09E-01	9.61E-02	1.88E-01	1.65E-01	3.75E+00	3.62E+00

Appendix continue

	ML	9.94E-02	9.15E-02	1.74E-01	1.57E-01	3.57E+00	3.45E+00
	MLGD	9.90E-02	9.15E-02	1.74E-01	1.57E-01	3.57E+00	3.45E+00
	OLS	7.57E-02	7.54E-02	1.35E-01	1.29E-01	2.90E+00	2.84E+00
	COCR	1.06E-01	9.66E-02	1.81E-01	1.66E-01	3.71E+00	3.64E+00
0	HILU	1.06E-01	9.66E-02	1.81E-01	1.66E-01	3.72E+00	3.64E+00
	ML	9.72E-02	8.60E-02	1.66E-01	1.47E-01	3.34E+00	3.24E+00
	MLGD	9.63E-02	8.54E-02	1.65E-01	1.46E-01	3.32E+00	3.22E+00
	OLS	7.87E-02	6.81E-02	1.32E-01	1.17E-01	2.64E+00	2.57E+00
	COCR	9.10E-02	8.62E-02	1.54E-01	1.48E-01	3.27E+00	3.25E+00
.25	HILU	9.12E-02	8.69E-02	1.55E-01	1.49E-01	3.30E+00	3.28E+00
	ML	8.37E-02	7.51E-02	1.41E-01	1.29E-01	2.88E+00	2.83E+00
	MLGD	8.47E-02	7.58E-02	1.43E-01	1.30E-01	2.91E+00	2.86E+00
	OLS	1.04E-01	7.56E-02	1.63E-01	1.30E-01	2.94E+00	2.85E+00
	COCR	5.92E-02	7.20E-02	1.04E-01	1.23E-01	2.62E+00	2.71E+00
.5	HILU	5.92E-02	7.26E-02	1.04E-01	1.24E-01	2.64E+00	2.74E+00
	ML	5.61E-02	6.29E-02	9.60E-02	1.08E-01	2.31E+00	2.37E+00
	MLGD	5.50E-02	6.25E-02	9.46E-02	1.07E-01	2.29E+00	2.36E+00
	OLS	0.165	0.126	0.255	0.217	4.806	4.76E+00
	COCR	4.79E-02	6.15E-02	8.49E-02	1.05E-01	2.22E+00	2.32E+00
.75	HILU	4.78E-02	6.10E-02	8.44E-02	1.05E-01	2.21E+00	2.30E+00
	ML	4.55E-02	5.43E-02	7.85E-02	9.31E-02	1.97E+00	2.05E+00
	MLGD	4.51E-02	5.38E-02	7.78E-02	9.23E-02	1.95E+00	2.03E+00
	OLS	0.285	0.27	0.486	0.463	10.285+	10.175+
	COCR	4.05E-02	5.33E-02	7.13E-02	9.13E-02	1.91E+00	2.01E+00
.99	HILU	4.02E-02	5.29E-02	7.13E-02	9.07E-02	1.90E+00	1.99E+00
	ML	4.03E-02	5.01E-02	7.04E-02	8.59E-02	1.81E+00	1.89E+00
	MLGD	4.05E-02	4.99E-02	7.06E-02	8.56E-02	1.81E+00	1.88E+00

CONCLUSION

Although, the OLS estimator competes favourably with the GLS estimators when the levels of autocorrelation and multicollinearity are moderately low, however, this study has revealed that the two GLS estimators namely, ML and HILU are generally preferable in estimating all the parameters of the model in all the levels of autocorrelation and multicollinearity. Thus when these correlations are present in a data set but can not be ascertained, it is more preferable to use either the ML or HILU estimator in estimating the model parameters.

REFERENCES

Aitken, A.C., 1935. On Least Squares and Linear combinations of observations. Proc. Royal Stat. Soc. Edinburgh, 55: 42-48.

Ayinde, K. and B.A. Oyejola, 2007. A Comparative study of the Peformances of the OLS and Some GLS estimators when regressors are correlated with error terms. Res. J. Applied Sci., 2: 215-220.

Beach, C.M. and J.S. Mackinnon, 1978. A Maximum Likelihood Procedure for Regression with autocorrelated errors. Econometrica, 46: 51-57.

Chatterjee, S., A.S. Hadi and B. Price, 2000. Regression Analysis by Example. (3rd Edn.), A Wiley-Interscience Pubication. John Wiley and Sons.

Chipman, J.S., 1979. Efficiency of Least Squares Estimation of Linear Trend when residuals are autocorrelated. Econometrica 47: 115-127.

Cochrane, D. and G.H. Orcutt, 1949. Application of Least Square to Relationship Containing Autocorrelated Error Terms. J. Am. Stat. Assoc., 44: 32-61.

Durbin, J., 1960. Estimation of Parameters in Time-Series Regression Models. J. Royal Stat. Soc. B., 22: 139-153.

Fomby, T.B., R.C. Hill and S.R. Johnson, 1984. Advanced Econometric Methods. Springer-Verlag, New York Berlin Heidelberg London Paris Tokyo.

Graybill, F.A., 1961. An introduction to Linear Statistical Models. New York. McGraw-Hill.

Hildreth, C. and J.Y. Lu, 1960. Demand Relationships with Autocorrelated Disturbances. Michigan State University. Agricultural Experiment Statistical Bulletin 276, East Lansing, Michigan.

Iyaniwura, J.O. and J.C. Nwabueze, 2004a. Estimators of Linear Model with Autocorrelated Error Terms and Trended Independent Variable. J. Nig. Stat. Assoc., 17: 20-28.

Iyaniwura, J.O. and J.C. Nwabueze, 2004b. Estimating the Autocorrelated Error Model with GNP Data. J. Nig. Stat. Assoc., 17:29.

Iyaniwura, J.O. and J.O. Olaomi, 2006. Efficiency of GLS estimators in Linear Regression Model with Autocorrelated Error Terms which are Also correlated with Regressor. An Int. J. Biol. Physic. Sci. (Science Focus), 11: 129-133

Johnston, J., 1984. Econometric Methods, (3rd Edn.), New York, McGraw Hill.

Kleiber, C., 2001. Finite Sample Efficiency of OLS in Linear Regression Models with Long-Memory Disturbances. Econ. Lett., 72: 131-136.

Kramer, W., 1980. Finite Sample Efficiency of Ordinary Least Squares in the Linear Regression Model with Autocorrelated Errors. J. Am. Stat. Assoc., 81: 150-154.

Krutchkoff, R.G., 1970. Probability and Statistical Inference. Gordon and B reach, Science Publishers Ltd. 12 Bloomsbury Way London W.C.I.

Maddala G.S., 2002. Introduction to Econometrics. John Wiley and sons Lt, England. (3rd Edn.)

Neil, H.T., 1975. Multivariate Analysis with application in Education and Psychology. A Division of Wadsworth Publishing Company, Inc., pp: 73-75.

Neter, J. and W. Wasserman, 1974. Applied Linear Model. Richard D. Irwin, Inc.

Nwabueze, J.C., 2005a. Performances of estimators of linear model with auto-correlated error terms when independent variable is normal. J. Nig. Assoc. Math. Phys., 9: 379-384.

Nwabueze J.C., 2005b. Performances of estimators of linear auto-correlated model with exponential independent variable. J. Nig. Assoc. Math. Phys., 9: 385-388.

Nwabueze, J.C., 2005c. Performances of estimators of linear models with auto-correlated error terms when the independent variable is autoregressive. Global J. Pure and Applied Sci., 11: 131-135.

Park, R.E. and B.M. Mitchell, 1980. Estimating the Autocorrelated Error Model with Trended Data. J. Econom., 13: 185-201.

Paris, S.J. and C.B. Winstein, 1954. Trend Estimators and Serial Correlation. Unpublished Cowles Commission, Discussion Paper, Chicago.

Rao, P. and Z. Griliches, 1969. Small Sample Properties of Several Two-Stage Regression Methods in the context of Autocorrelation Errors. J. Am. Stat. Assoc., 64: 251-272.

Sampson, A.P., 1974. A tale of two regressions. J. Am. Stat. Assoc., 69: 682-689.

Theil, H., 1971. Principle of Econometrics. New York, John Wiley and sons.

Thornton, D.L., 1982. The appropriate autocorrelation transformation when the autocorrelation process has a finite past. Federal Reserve Bank of St. Louis, pp: 82-002.

TSP, 2005. Users Guide and Reference Manual. Time Series Processor. New York.

White, K.J., 1978. A General Computer Program for Econometric Methods-SHAZAM. Econometrica, 46: 239-240.