Research Journal of Applied Sciences 2 (5): 633-637, 2007
© Medwell Journals, 2007

Comparative Study of Holt-Winter, Double Exponential and the Linear Trend Regression Models, With Application to Exchange Rates of the Naira to the Dollar

M.A. Umar
Department of Mathematical Sciences, Nasarawa State University, Keffi, Nigeria

Abstract: In this study, we examined the Holt-winter, double exponential and the linear regression trend parameter estimation techniques and compare their forecast quality via the criteria of forecast as in Gilchrist, using time's series data of exchange rates of the Naira to the dollar. Forecasts using these methods are presented and comparison statistics and statistics of errors for the methods are examined. It is found that the Holt-winter forecasting method with choice of smoothening constant a = 0.2 and b = 0.5 produced better forecasts than the rest of the methods. In general, the Holt-winter always gives outstanding forecasts than the other methods, as seen on the table of comparison statistics and statistics of error. Also, the required constraint on the smoothening constants for the Holt-winter to meet the criteria of forecast is suggested.

Key words: Times series, forecast, lead time, extrapolation trend, smoothening constant

INTRODUCTION

The need for quick, reliable, simple and medium term forecasts of various times Series is often encountered in economic and business environments. Holt-winter, double exponential and the linear regression trend provide a comprehensive, simple, accurate and applicable solution to this question. However their computational techniques differs, but their accuracy, simplicity and stability on any times series data can be compared via the behavior of their forecasts, statistics of forecast errors and comparison statistics, as shown on Table 1-3. It is observed that the Holt-winter and the double exponential parameter estimates are recursively obtained, while the linear trend regression model, have their parameters obtained via the least square method. Their accuracy and stability can be deduced from their forecast errors and other forecast criteria, referred to here as comparison statistics. However, the choice of method to use on any times series data depends on factors such, as simplicity, accuracy and stability on the times series data. Here the three computational methods considered are used to forecast exchange rate of the naira and the dollar and their forecasts are compared using forecast criteria, in Gilchrist (1976) and Fildes (1980).

COMPUTATIONAL METHODS

The computational techniques of the Holt-winter method is simply described as,

Table 1: Yearly exchange rate of the Naira to the dollar

Years	Exchange rate of the Naira to the dollar
1970	0.7142
1971	0.6944
1972	0.6579
1973	0.6579
1974	0.6293
1975	0.6158
1976	0.6266
1977	0.6466
1978	0.6067
1979	0.6027
1980	0.5469
1981	0.6052
1982	0.6731
1983	0.7506
1984	0.7672
1985	0.8924
1986	1.7323
1987	3.9691
1988	4.5367
1989	7.3651
1990	8.3469

Source: Central bank of Nigeria

$$\hat{X}_{t,h} = \hat{\mu}_t + \hat{\beta}_t h, \text{where } \hat{\mu}_t = (1-a)X_t + a(\hat{\mu}_{t-1} - \hat{\beta}_{t-1}) \quad (1)$$
$$\hat{\beta}_t = (1-b)(\hat{\mu}_t - \hat{\mu}_{t-1}) + b\hat{\beta}_{t-1}$$

Where $\hat{\mu}_t$ and $\hat{\beta}_t$ are exponentially smoothed and trend component, respectively and is $\hat{X}_{t,h}$ the forecast of, $\hat{X}_{t+h}$, at time t, h-step into the future, (Gilchrist, 1976; Fildes, 1980; Julies, 2003). Also a and b a are constant parameters defined by 0<a<1, 0<b< 1. The choice of b near zero places more emphasis on the past estimate of the trend, while the choice of b near one gives more emphasis to the current changes in level.

Table 2: Forecasts of the exchange rate of the Naira to the dollar, using, selected values of the smoothening constants

Methods	Choice of smoothening constant	Years	Data	Forecasts	Errors
Holt-winter	a = 0.8,				
	b = 0.5	1981	0.6052	0.5819	0.0233
		1982	0.6731	0.5653	0.1077
		1983	0.7506	0.5919	0.1587
		1984	0.7672	0.6372	0.1202
		1985	0.8924	0.6898	0.2025
		1986	1.7323	0.7772	0.9551
		1987	3.9691	1.1106	2.8585
		1988	4.5367	2.1105	2.4262
		1989	7.3651	3.2666	4.0985
		1990	8.3469	5.1669	3.1799
	a = 0.2,				
	b = 0.5	1981	0.6052	0.5272	.0.0780
		1982	0.6731	0.5920	0.0811
		1983	0.7506	0.6917	0.0589
		1984	0.7672	0.7972	-.0.0300
		1985	0.8974	0.8196	0.07282
		1986	1.7323	0.9534	0.7789
		1987	3.9691	1.9636	2.006
		1988	4.5367	4.7573	-0.2206
		1989	7.3651	5.6819	1.6832
		1990	8.3469	8.8028	-0.4559
	a = 0.5,				
	b = 0.5	1981	0.6052	0.5516	0.0536
		1982	0.6731	0.5712	0.1019
		1983	0.7505	0.6404	0.1102
		1984	0.7672	0.7413	0.0259
		1985	0.8924	0.8065	0.0859
		1986	1.7323	0.9232	0.8091
		1987	3.9691	1.6038	2.3653
		1988	4.5367	3.6478	0.8889
		1989	7.2651	5.1831	2.1820
		1990	8.3419	7.9077	0.4392
Linear					
trend regression		1981	0.6052	0.6992	-0.094
model		1982	0.6731	0.6866	-0.0135
		1983	0.7506	0.6740	0.0766
		1984	0.7672	0.6614	0.1058
		1985	0.8924	0.6488	0.2436
		1986	1.7323	0.6362	1.0958
		1987	3.9691	0.6236	3.3455
		1988	4.5367	0.6110	3.9257
		1989	7.3651	0.5984	6.7657
		1990	8.3469	0.5858	7.4142
Double					
exponential	a = 0.2	1981	0.6052	0.6214	--0.0162
		1982	0.6731	0.7792	-0.1061
		1983	0.7506	1.1768	-0.4262
		1984	0.7672	0.7805	-0.0133
		1985	0.8974	1.2800	-0.3876
		1986	1.7323	5.1120	-3.3797
		1987	3.9691	13.3712	-9.4026
		1988	4.5367	13.7717	-9.2350
		1989	7.3651	27.2400	-19.8749
		1990	8.3469	23.0737	-14.7268
	a =.0.8	1981	0.6052	0.5756	0.0296
		1982	0.6731	0.6031	0.0700
		1983	0.7506	0.6644	0.0862
		1984	0.7672	0.7235	0.0437
		1985	0.8974	0.8412	0.0512
		1986	1.7323	1.4393	0.2930
		1987	3.9691	3.3249	0.6442
		1988	4.5367	5.1407	-0.6040
		1989	7.3651	8.5078	-1.1427
		1990	8.3469	11.6980	-3.3511

Table 3: Statistics of errors and comparison statistics

Data	Methods	Smoothing constant	Mean forecast error	Mean absolute forecast error	Variance of forecast error	Mean square error	Mean absolute deviation of error	Root mean square error	Mean absolute percentage error
Times series of exchange rates of the Naira to a dollar	Holt-winter	a = 0.8, b = 0.5	1.41404	1.41404	2.19794	4.19746	1.38139	0.64788	25.5006
		a = 0.2 b = 0.5	0.40521	0.54649	0.609893	0.77408	0.650412	0.278224	17.35228
		a = 0.5 b = 0.5	0.7062	0.7062	0.70397	1.2027	0.6841	0.3468	21.2464
	Linear regression on trend		2.2865	2.3080	7.6361	12.8648	2.46111	1.13423	48.3609
	Double exponential	a = 0.2	-5.7268	5.7268	46.5948	79.7353	6.0364	8.9295	120.2170
		a = 0.8	-0.3380	0.6316	1.2339	1.3419	0.7861	1.1584	14.1020

The choice of a near zero place more weight on past values of the time series, while a value of a near one gives more weight to current values of the series, (Gilchrist, 1976; Robinson, 1984; Fildes, 1980). Julies (2003) extended this definition to forecast seasonal multiplicative times series data as,

$$\hat{X}_{t,h} = (\mu_t + \beta_1 h)S_{t-S+h}$$

where s the number of periods in one cycle of the season.

$$\begin{aligned} \mu_t &= a\frac{X_t}{S_{t-s}} + (1-\alpha)(\mu_{t-1} + \beta_{t-1}) \\ \beta_t &= b(\mu_t - \mu_{t-1})(1-b)\beta_{t-1} \\ S_t &= \gamma\frac{X_t}{\mu_t} + (1-\gamma)S_{t-s} \end{aligned} \quad (2)$$

To initializing, the system we use,

$$\begin{aligned} \mu_s &= \frac{1}{S}(X_1 + X_2 + ... + X_S) \\ \beta_S &= \frac{1}{k}(\frac{X_{s+1} - X_1}{s} + \frac{X_{s+2} - X_2}{s} + ... + \frac{X_{s+k} - X_k}{s}) \end{aligned} \quad (3)$$

If the series is long enough, then a good choice is k = s. The initial seasonal indices are obtained as,

$$S_k = \frac{X_k}{\mu_s}, \quad k = 1, 2, 3, ..., s$$

Where, a, b and γ should lie in (0, 1), (Julies, 2003).

An additive Holt-winter seasonally forecasting method is defined by,

$$\hat{X}_{t,h} = (\mu_t + \beta_t h)S_{t-S+h}$$

(Julies, 2003). Where s is as defined above. The initial starting values, μ_S and β_S are obtained as in the multiplicative model and the initial seasonal indices can be taken as, $S_k = X_k - \mu_S$, K = 1, 2,...s. The exponentially smoothed and trend components are obtained as,

$$\begin{aligned} \mu_t &= a(X_t - S_{t-1}) + (1-a)(\mu_t + S_{t-1}) \\ \beta_t &= b(X_t - \mu_{t-1}) + (1-b)\beta_{t-1} \\ S_t &= \gamma(X_t - \mu_t) + (1-\gamma)S_{t-s} \end{aligned} \quad (4)$$

The model parameters, a, b and γ are defined as in the multiplicative case.

However, in double exponential method the parameters are expressed as functions of single and double smoothed series,

$$\bar{X}_t \text{ and } \bar{\bar{X}}_t,$$

(Robinson, 1984; Gilchrist, 1976) and defined as:

$$\bar{X}_t = (1-a)X_t + a\bar{X}_{t-1} \text{ and } \bar{\bar{X}}_t = (1-a)\bar{X}_t + a\bar{\bar{X}}_{t-1}$$

with forecasts described as,

$$\hat{X}_{t,h} = \hat{\mu}t + \hat{\beta}_t h' \quad (5)$$

Where,

$$\hat{\mu}_t = 2\bar{X}_t - \bar{\bar{X}}_t \text{ and } \hat{\beta}_t = \frac{(1-a)}{a}(\bar{X}_t - \bar{\bar{X}}_t) \text{ and } 0 < a < 1$$

The first and second smoothening values, is obtained as in Robinson (1984) and the initial starting value for

$$\bar{X}_0 \text{ and } \bar{\bar{X}}_0$$

are obtained as,

$$\bar{X}_0 = \hat{\mu}_0 - \frac{a}{1-a}\hat{B}_0$$

and

$$\bar{\bar{X}} = \hat{\mu}_0 - \frac{2a}{1-a}\hat{B}_0$$

While the initial estimate μ_0 and $\hat{B}_0$ are obtained as coefficients of the linear trend model fitted to the time series data. The mean and slope are computed recursively as,

$$\mu_t = 2\overline{X_t} - \bar{\bar{X}}_t, \hat{B}_t = \frac{(1-a)}{a}(\overline{X}_t - \bar{\bar{X}}_t), 0 < a < 1 \quad (6)$$

However, this is an approximation valid for large t, for small t, these equations can still be used, as a separate method, provided we have enough prior information to enable us obtain starting values for

$$\overline{X}_t \text{ and } \bar{\bar{X}}_t$$

An interested property of both methods is that, the coefficients of the double exponential method is related to that of the Holt-winter trend method, as observed in Gilchrist (1976) using the forecast error term defined by

$$X_t - (\hat{\mu}_{t-1} - \hat{\beta}_{t-1})$$

the parameters in the Holt-winter (1) reduces to,

$$\hat{\mu}_t = \hat{\mu}_{t-1}\hat{\beta}_{t-1} + (1-a_n)R_t, \hat{\beta}_t = \hat{\beta}_{t-1} + C_n R_t, C_n = (1-b)(1-a)$$

Where a and b are constant coefficients defined by o<a<1, 0<b<1. While in the double exponential method, the expression for the parameters in terms of the forecast errors are,

$$\hat{\mu}_t = \hat{\mu}_{t-1} + \hat{\beta}_{t-1} + (1-a^2)R_t \hat{\beta}_t \hat{\beta}_{t-1} + (1-a)^2 R_t \quad (7)$$

A comparison of both methods shows that the estimates of the parameters are identical if $a_n = a^2, c_n = (1-a)^2$.

However, Julies (2003) described the double exponential forecasting method as,

$$\hat{X}_{t,h} = D_t + T_t h, \quad (8)$$

This is consistent with the definition given by Gilchrist (1976). Where $T_t = \gamma C_t + (1-\gamma) T_{t-1}$, $0 \le \gamma \le 1$, is the smoothening constant for the trend and C_t A_t-A_{t-1}. Where $A_t = \alpha D_t + (1-\alpha) X_{t-1}$. D_t is the demand in the current time period and $0 \le \alpha \le 1$. This system is initialized by assuming the following, $X_t = D_1$, $C_1 = T_1 = D_2 - D_1$, $A_t = D_t - T_t$ (Julies, 2003).

However, the linear trend model is a hypothetical curve that shows the direction of movement of a time series over a period of time. It is simply a linear function of time, described by Fildes (1980) as,

$$X_t = B_0 + B_1 t + R_t \quad (9)$$

Where B_0, B_1 are constants and R_t is an error term.
The Mean Square Error of forecast, MSE is defined by,

$$MSE = \sum_{r=1}^{t} R_t^2 = \sum_{r=1}^{t} (X_r - \hat{B}_0 - B_1 r)^2$$

Where R_t is the forecast error for period t, with the associated normal equation as:

$$\sum_{r=1}^{t} X_r - \hat{B}_0 t - \hat{B}_1 \sum_{r=1}^{t} r = 0 \; \sum_{r=1}^{t} X_r - \hat{B}_0 \sum_{r=1}^{t} r - \hat{B}_1 \sum_{r=1}^{t} r^2 = 0$$

The estimates of the trend parameters are obtained as,

$$\hat{B}_0 = \frac{1}{t}\sum_{r=1}^{t} X_r \text{ and } \hat{B}_1 = \sum_{r=1}^{t} rX_r \Big/ \sum_{r=1}^{t} r^2$$

The estimates of the forecasts are obtained as,

$$\hat{X}_{t,h} = \hat{B}_0 + \hat{B}_1 h,$$

where h is the lead time into the future, (Gilchrist, 1976). However, linear least square regression methods suffers from inefficiency in utilizing the Autocorrelation structure of the time series and may likely become unstable in forecasting outside the region of observation, as the underlying condition may change which may render the model useless (Fildes, 1980). This means that the trend model is not a good extrapolative model and not sufficient to forecast long time into the future but can only serve as one of a battery of technique to be employed in forecasting.

APPLICATION TO EXCHANGE RATE OF THE NAIRA TO THE DOLLAR

The data used in this research are times series data of the exchange rate of the Naira to the dollar, from 1970 - 1991, collected from central bank of Nigeria economic and financial review, (CBN, 1992). The results of forecasts using the methods above on the times series data of the exchange rate of the naira to the dollar from 1970 to 1991 for various choice of the smoothening constant are showed on Table 1-3.

DISCUSSION

The table of forecasts and error comparison clearly revealed that the optimal choice for the smoothening constants for the Holt-winter and the double exponential methods on this times series data are, a = 0.2, b = 0.5 and, a = 0.8, respectively. This can be seen on Table 1-3. These choices best satisfy the criteria of forecasts, in Gilchrist (1976). Also the stable behavior of the model on the times series data can be observed on Table 2. However, the linear trend regression model present a contrary behavior on this times series data, as forecasts of the time series data becomes unstable, when long time forecasts of the time series are made. This observation is in agreement with Filde (1980). Thus, this method is not stable and not appropriate for forecasting long time into the future. The most outstanding amongst the methods on this times series data is the Holt-winter method with a choice of smoothening constants, a = 0.2 and b = 0.5, as observed on Table 3. In general, for stability and good forecasts of the series data in line with forecast criteria on Table 3 we must have, $a<b<1$, for the Holt-winter forecasting model.

REFERENCES

CBN, 1992. Central Bank of Nigeria Economic and Financial Review, Lagos: CBN Press and sons, pp: 2-3.

Fildes, 1980. Quantitative forecasting, Great Britain J. Operational Res. Soc., 20: 705-715.

Gilchrist, W., 1976. Statistical forecasting, New York: John Wiley and Sons, pp: 221-273.

Julies, M. Hayes, 2003. J. Stat. Edu. 11: 15-22.

Robinson, 1984. Business forecasting and Economic approach, Thomas Nelson and Sons London, pp: 663-673.