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Coefficient Bounds for Certain Classes of Analytic Functions

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Abstract: In the present study we examine the coefficient bounds for certain classes of analytic functions with negative coefficient.

Key words: Coefficient bounds, certain classes, analytic function

INTRODUCTION

Let T(n) denote the class of function f(z) of the form

$$f(z) = z - \sum_{k=2}^{\infty} a_k z^k \tag{1}$$

which are analytic in the unit disk $E = \{z : |z| < 1\}$

Also, let P denote the class of functions p(z) of the form

$$p(z) = 1 + p_1 z + p_2 z^2 + \dots \tag{2}$$

which are analytic in E and satisfy the inequality Re (p(z)) >α, (0≤α<1, z ∈ E) (Srivastava and Owa, 1992; Janteng *et al*., 2006).

A function f(z) in T(n) is said to be in the class $T_n(\lambda, \alpha)$ if it satisfies the condition

$$\operatorname{Re}\left\{\frac{zf'(z)}{\lambda zf'(z) + (1-\lambda)f(z)}\right\} > \alpha \tag{3}$$

for some α(0≤α<1), λ(0≤λ<1) and all z ∈ E.

Let $C_n(\lambda,\alpha)$ denote the subclass of T (n) of all functions f (z) satisfying the inequality

$$\operatorname{Re}\left\{\frac{f'(z) + zf''(z)}{f'(z) + \lambda zf''(z)}\right\} > \alpha \tag{4}$$

for some α(0≤α<1), λ(0≤λ<1) and all z ∈ E (Darwish, 2006; Altintes and Owa, 1988).

Note here that $f(z) \in C_n(\lambda,\alpha)$ if and only if $zf'(z) \in T_n(\lambda,\alpha)$.

The aim of the present study is to derive the coefficient bounds for the functions f(z) satisfying (3) and (4), respectively.

RESULTS

Theorem 1: Let $f(z) \in T_n(\lambda,\alpha)$. Then

$$|a_2| \le \frac{2(1-\alpha)}{\lambda - 1} \tag{5}$$

$$|a_3| \le \frac{1-\alpha}{\lambda - 1} + \frac{2(\lambda+1)(1-\alpha)(\alpha-1)}{(\lambda-1)^2}$$

$$|a_4| \le \frac{2(4\lambda+3)(\alpha-1)(1-\alpha)}{3(\lambda-1)^2} + \frac{4(2\lambda+1)(\lambda+1)(1-\alpha)(\alpha-1)^2}{3(\lambda-1)^3} + \frac{2(1-\alpha)}{3(\lambda-1)}$$

and

$$|a_2 a_4 - a_3^2| \le \frac{4(1-\alpha)^2}{3(\lambda-1)^2}\left[1 + \frac{(\alpha-1)(4\lambda+3)}{2(\lambda-1)}\right] - \frac{2(1-\alpha)^2}{(\lambda-1)^2}\left[1 + \frac{4(\lambda+2)(\lambda+1)(\alpha-1)^2}{(\lambda-1)^2}\right]$$

for some α(0≤α<1), λ(0≤λ<1).

Proof: Let us define p(z) by

$$\frac{zf'(z)}{\lambda zf'(z) + (1-\lambda)f(z)} = \alpha + (1-\alpha)p(z) \tag{6}$$

On comparing the coefficient in (6) the results follow immediately.

Theorem 2: Let $f(z) \in C_n(\lambda,\alpha)$. Then,

$$|a_2| \le \frac{1-\alpha}{\lambda - 1} \tag{7}$$

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$$|a_3| \le \frac{1-\alpha}{3(\lambda-1)} + \frac{2(\lambda+1)(\alpha-1)(1-\alpha)}{3(\lambda-1)^2}$$

$$|a_4| \le \frac{(4\lambda+3)(\alpha-1)(1-\alpha)}{6(\lambda-1)^2} + \frac{(2\lambda+1)(\lambda+1)(1-\alpha)(\alpha-1)^2}{3(\lambda-1)^3} + \frac{1-\alpha}{6(\lambda-1)}$$

$$|a_2a_4 - a_3^2| \le \frac{(1-\alpha)}{18(\lambda-1)^2} + \frac{(\alpha-1)(1-\alpha)^2}{3(\lambda-1)^3}\left[\frac{4\lambda+3}{2} + \frac{(2\lambda+1)(\lambda+1)(\alpha-1)}{3(\lambda-1)}\right]$$

for some $\alpha(0\le\alpha<1)$, $\lambda(0\le\lambda<1)$ and $z \in E$

Proof: Let us define p(z) by

$$\frac{f'(z)+zf''(z)}{f'(z)+\lambda zf''(z)} = \alpha + (1-\alpha)p(z) \qquad (8)$$

On comparing the coefficient in (8) the result follows immediately.

REFERENCES

Altintas, O. and S. Owa, 1988. On subclass of univalent functions with negatine coefficients. Pusan Kyougnan Math. J., 4: 41-56.

Darwish, H.E., 2006. On generalization of Hadamard products of functions with negative coefficients. Pro. Pak. Acad. Sci., 43: 269-273.

Janteng, A, A. Suzeini and M. Darus, 2006. Coefficient inequality for a function whose derivative has a positive real part. J. Inq. In Pure and Applied. Math., 50: 1-11.

Oladipo, A.T., 2007. Generalized coefficient inequalities for certain multivalent and meromorphically multivalent analytic functions. J. Eng. Applied Sci., 2: 683-685.

Srivastava, H.M. and S. Owa, 1992. Current Topics in Analytic Function Theory, Hong Kong.