Research Journal of Applied Sciences 2 (6): 773-779, 2007

Statistical Consideration of the Structure of Atmospheric Refractive-Index

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Abstract: In relation to the irregular vertical distribution of atmospheric refractive index, the mean effective-earth-radius-factor, k, was determined for Akure (7.15°N, 5.12°E) from experimental data and also, some variation characteristics of air refractive index with space and time were discussed. The air temperature, the water-vapour pressure and the barometric pressure were measured at various heights above the earth's surface from which the distribution of the refractive index were obtained. Also, the refractive index was statistically considered and it was found that the average refractivity gradient of the atmospheric layer of thickness ΔH (m) approximately follows the normal distribution. Hence, by introducing ΔH, the height range of the propagation path and defining the mean gradient of M-profile in this range, the conception of the effective earth radius factor, k, was brought out.

Key words: Refractive index, refractivity gradient, atmospheric layer, normal distribution, k-factor

INTRODUCTION

One of the most significant parameters in the influence of the troposphere on radio wave propagation is the large-scale variation of the refractive index with height. A consequence of this variation is that, radio-waves propagating through the atmosphere progressively curve towards the earth. Furthermore, the range of the radio-waves is determined by the height dependence of the refractivity (Bean and Dutton, 1968). Therefore, the refractivity of the atmosphere will affect not only the curvature of the ray path (expressed by a *k*-factor) but will also give some insight into the fading phenomenon. The 'reference atmosphere' model proposes that the refractivity profile N(h) decreases exponentially with height under normal atmospheric condition (Hall, 1979). But atmospheric anomalies disturb radio-wave propagation and hence the 'reference atmosphere' model cannot explain the behavior of the radio-waves under real atmospheric conditions and hence, a more sophisticated model like that of the layered atmosphere would be more appropriate.

In the previous studies, approximate value of the earth's equivalent radius coefficient (k-factor) for a given climatic region was derived by approximating ΔN (the difference between the refractivity at 1km above the ground level N and the surface refractivity N_s). However, ΔN or the equivalent earth radius coefficient, *k* are not appropriate to describe the refractivity profile within a layered atmosphere (Bonkoungou and Low, 1993) and so by introducing ΔH, the height range of the propagation path and defining the mean gradient of M-profile in this range, the conception of the effective earth radius factor can be appropriately defined.

In order to obtain the vertical distribution of atmospheric refractive index, the air temperature, the water vapour pressure and the atmospheric pressure were measured at different heights above the earth's surface to determine the refractive index at these levels.

In this study, study was made on refractive index characteristics in the lower atmosphere, the irregular vertical distribution of the atmospheric refractive index and the earth's equivalent radius coefficient, k, was determined for this locality.

Theory: For an ideal condition of the atmosphere, the atmosphere is uniformly stratified and the vertical gradient of the refractive index is assumed constant and defined by (Barclay, 2003)

$$k = \frac{1}{1 + a\dfrac{dn}{dh}} = \frac{1}{1 + a\dfrac{dN}{dh}10^{-6}} = \frac{10^6}{a\dfrac{dM}{dh}} \qquad (1)$$

Where n(h) is the refractive index at a height h(m) and the value is given with the formula:

$$n(h) - 1 = \frac{79}{T}(P + \frac{4800e}{T}).10^{-6} \equiv N.10^{-6} \qquad (2)$$

P: Barometric pressure (mb) at a height h(m) above the earth's surface.
T: Air temperature (K)

e : Vapour pressure (mb)
a : Average radius of the earth ˜ 6.37×10^6 km

The refractive modulus profile can be obtained by using the transformation equation given by

$$M(h) = N(h) + 157\frac{h}{km} \tag{3}$$

However, the atmosphere is not under such ideal condition as assumed above. The vertical gradient of the refractive index is rarely constant and the electromagnetic field in the atmosphere can be calculated from Maxwell's fundamental equations as the solution of the wave equation:

$$\nabla^2\Psi + k^2M^2\Psi = 0 \tag{4}$$

Where

Ψ : Scalar component of Hertzian vector
k : $2\pi/\lambda$
λ : Wavelength
M : Modified refractive index, M(h)defined by

$$M(h) = M(h_1) + \int_{h1}^{h} \frac{dM}{dh} dh$$

The height range of propagation path, dM/dh can be expanded into Fourier's series between the heights h_1 and h_2 as:

$$\begin{aligned} M(h) &= M(h_1) + \int_{h1}^{h} Mdh = (Mh_1) + \alpha(h - h_1) \\ &+ \int_{h1}^{h} \Sigma\left[a_n \cos\frac{2\pi nh}{\Delta H} + b_n \sin\frac{2\pi nh}{\Delta H}\right] dh \end{aligned} \tag{5}$$

Where

$$\alpha = \frac{1}{\Delta H}\int Mdh$$

and $H = h_2\text{-}h_1$

Assuming that 3rd and higher terms in the expansion of M(h) of Eq. 5 are neglected, then Eq. 4 can be written as

$$\nabla^2\Psi + K^2[M(h_1) + \alpha(h - h_1)]^2\Psi = 0$$

If any M-profile may be approximately expressed by multi-linear M-profile, M(h) is written as

$$M(h) = M_s + \sum(\frac{dM}{dh})_i(\Delta h)_i$$

Where $M_s = M_0$
$(\Delta h)_i$ is the ith layer from the earth.

M-profile is determined according to the distribution of (dM/dh) and (Δh). By using α_i , ΔM can be written as

$$\Delta M = \sum\alpha_i.(\Delta h)_i \tag{6}$$

Hence, for height interval ΔH, k is defined by using the average gradient α of M-profile as:

$$\frac{1}{k} = \frac{a}{\Delta H}.\sum_i \alpha_i .(\Delta h)_i 10^{-6} \tag{7}$$

But, against height interval $(\Delta h)_i$ k is defined by

$$\frac{1}{k_i} = a.\alpha.10^{-6} \tag{8}$$

If $\Delta H = n\Delta h$

$$\frac{1}{k} = \frac{1}{n}\sum k_i \text{ and } \alpha = \frac{1}{n}\sum\alpha_i$$

MATERIALS AND METHODS

The air temperature, the water vapour pressure and the barometric pressure were measured at various heights of 50, 100, 150 and 200 m. The fixed-measuring method by a high tower equipped with 'Integrated Sensor Suite' (ISS) at these pre-determined heights, was used for this study because of its high accuracy for finding M-profile. A set of the measuring equipment was also positioned at the ground surface to measure the surface values of these parameters. The data obtained were used to calculate the mean gradient of M-profile for each range of the propagation path, defined by ΔH using Eq. 3 and 6.

The site of the measurement is located at the old site of the Nigerian Television Authority, NTA Akure located a few kilometers from the city. The sensors are mounted at the various pre-determined heights on a communication mast (220 m tall), while the receiver (console), together with the data-logger, is located in a measurement room adjacent to the communication mast.

The device used for the measurement is the Davis 6162 wireless Vantage Pro Plus, manufactured by Davis Instruments, Hayward, California, United States of America. It is equipped with the Integrated Sensor Suite (ISS), a solar panel (with an alternative battery power source) and wireless console, which provides the user interface, data display and analogue-to-digital conversion. The device uses the combination of fan-aspiration to minimize the effects of solar radiation induced temperature error. The ISS houses the external sensor array for measurements of pressure, temperature, relative humidity, UV index, solar radiation, rainfall rate among others. The console is connected to a computer through the data-logger from which the stored data are downloaded.

RESULTS AND DISCUSSION

In order to obtain the vertical distribution of atmospheric refractive index, the air temperature, the relative humidity and the barometric pressure were measured at various heights of 50, 100, 150 and 200 m. From the data collected at each measuring level, profiles of air temperature, humidity and M were prepared for each hour of the day.

Figure 1-3 show examples of the vertical distributions of the refractivity (profile), air temperature and relative humidity respectively, at every hour of the day. The figures show examples of such measured profiles at different hours of the day for the month of February 2007 (Table 1).

Taking a general view of the M-profiles prepared, it may be found that their characteristics depend on the occurrence of humidity inversion which occurs frequently between heights of 100 and 150 m during the day and night hours. Furthermore, the refractivity gradient observed for this layer is smaller than the values obtained in the other layers such as 50-100 and 150-200 m (Fig. 3). In most of the cases considered at the (100-150 m) layer, a sudden change of the vertical gradient of relative humidity occurs. It can, therefore, be stated that the occurrence of M inversion is almost always accompanied by a sudden decrease of water-vapour pressure against height (Fig. 1-3). It follows therefore, that the existence of temperature inversion layer alone may not be the only condition for the formation of M-inversion.

By calculating average gradients of the atmospheric refractive index between two points at different heights for a period of three months (Dec. 2006-Feb. 2007) and calculating its standard deviation, the statistics of the vertical gradient of the refractive index through the heights were determined (Table 2).

It is important to note that in the consideration of wave propagation in the troposphere, the earth's equivalent radius coefficient k is used to represent the vertical distribution of refractive index (ITU-R, 1987; Rec ITU-R, 2004). Furthermore, it is usual to define the k-factor by assuming ideal condition of the atmosphere, where the atmosphere is uniformly stratified and the vertical gradient of the refractive index is constant. However, theatmosphere is not under such ideal condition as

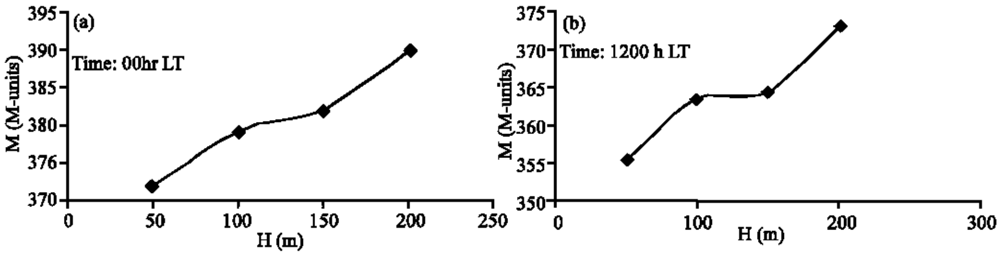

Fig 1: Typical M-profiles obtained for Akure

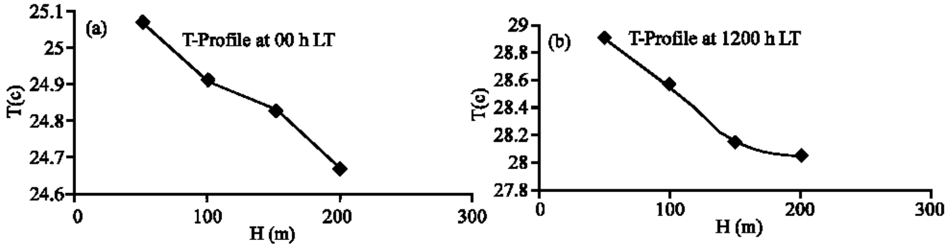

Fig. 2: Typical temperature-profile obtained for Akure

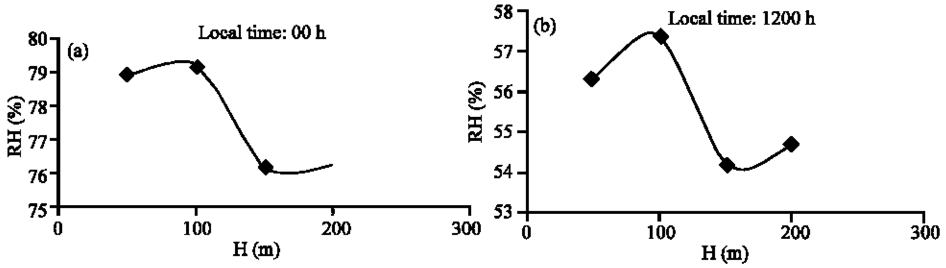

Fig. 3: Typical humidity-profile obtained for Akure

Table 1: Averaged refractivity data measured at 200 m height for February 2007

L T (hr)	T(C)	RH (%)	P (mb)	T (K)	Es	e(mb)	N	M
0:00	24.68571	76.25000	994.8036	297.8057	31.08176	23.69984	358.8940	390.2940
1:00	24.39286	78.32143	994.3179	297.5129	30.54193	23.92088	360.1502	391.5502
2:00	23.98929	80.50000	993.7286	297.1093	29.81146	23.99823	360.9495	392.3495
3:00	23.72857	82.46429	993.2750	296.8486	29.34773	24.20140	362.0970	393.4970
4:00	23.55357	83.21429	993.1429	296.6736	29.04001	24.16544	362.1841	393.5841
5:00	23.34286	82.82143	993.3357	296.4629	28.67323	23.74758	360.7915	392.1915
6:00	23.28929	82.35714	993.8321	296.4093	28.58063	23.53819	360.1160	391.5160
7:00	23.18214	82.67857	994.4321	296.3021	28.39622	23.47759	360.1820	391.5820
8:00	23.24643	82.03571	995.1393	296.3664	28.50674	23.38571	359.8772	391.2772
9:00	23.75714	78.82143	995.8714	296.8771	29.39824	23.17211	358.3750	389.7750
10:00	24.82857	73.46429	996.1786	297.9486	31.34810	23.02966	356.2163	387.6163
11:00	26.48214	63.78571	995.8893	299.6021	34.57929	22.05665	349.6010	381.0010
12:00	28.04643	54.67857	995.0250	301.1664	37.89993	20.72314	341.6049	373.0049
13:00	29.24286	47.07143	993.8929	302.3629	40.62421	19.12240	333.0958	364.4958
14:00	30.37857	41.96429	992.6214	303.4986	43.36663	18.19850	327.4921	358.8921
15:00	31.25357	38.53571	991.5464	304.3736	45.58817	17.56773	323.5257	354.9257
16:00	31.42500	37.71429	990.9464	304.5450	46.03483	17.36171	322.3224	353.7224
17:00	31.21429	38.35714	991.9714	304.3343	45.48634	17.44726	323.1999	354.5999
18:00	29.90741	42.92593	992.4037	303.0274	42.20993	18.11900	327.7374	359.1374
19:00	29.19630	45.96296	993.1593	302.3163	40.51508	18.62193	330.9283	362.3283
20:00	28.40741	50.29630	994.0778	301.5274	38.70457	19.46696	335.6967	367.0967
21:00	27.59259	56.81481	994.9519	300.7126	36.90904	20.96981	343.2478	374.6478
22:00	26.22222	66.29630	995.6556	299.3422	34.05290	22.57581	352.0847	383.4847
23:00	25.08889	74.96296	995.8741	298.2089	31.83857	23.86714	359.2544	390.6544

Table 2: Typical average refractivity values at different heights for February 2007

	H1 = 50 m		H2 = 100 m		H3 = 150 m		H4 = 200 m	
Hours	N1	M1	N2	M2	N3	M3	N4	M4
0:00	364.0592	371.9092	363.7281	379.4281	359.1168	382.6668	358.8940	390.2940
1:00	365.3077	373.1577	364.4714	380.1714	359.9114	383.4614	360.1502	391.5502
2:00	366.2554	374.1054	365.3895	381.0895	360.9171	384.4671	360.9495	392.3495
3:00	366.7675	374.6175	366.3507	382.0507	361.8020	385.3520	362.0970	393.4970
4:00	366.8638	374.7138	366.2058	381.9058	361.6469	385.1969	362.1841	393.5841
5:00	365.9535	373.8035	365.0920	380.7920	360.2754	383.8254	360.7915	392.1915
6:00	365.0423	372.8923	364.1461	379.8461	359.7001	383.2501	360.1160	391.5160
7:00	364.9149	372.7649	364.1324	379.8324	359.7307	383.2807	360.1820	391.5820
8:00	364.4768	372.3268	364.0801	379.7801	359.4994	383.0494	359.8772	391.2772
9:00	364.5428	372.3928	363.3988	379.0988	358.4931	382.0431	358.3750	389.7750
10:00	362.3355	370.1855	362.0192	377.7192	355.6250	379.1750	356.2163	387.6163
11:00	355.3101	363.1601	355.8450	371.5450	349.5199	373.0699	349.6010	381.0010
12:00	347.4219	355.2719	347.5409	363.2409	340.8294	364.3794	341.6049	373.0049
13:00	338.5588	346.4088	339.2556	354.9556	332.2708	355.8208	333.0958	364.4958
14:00	333.2375	341.0875	333.9113	349.6113	326.8625	350.4125	327.4921	358.8921
15:00	329.2188	337.0688	329.9857	345.6857	322.3296	345.8796	323.5257	354.9257
16:00	328.4014	336.2514	328.8369	344.5369	321.3455	344.8955	322.3224	353.7224
17:00	329.4974	337.3474	329.8426	345.5426	322.4476	345.9976	323.1999	354.5999
18:00	334.5014	342.3514	334.6256	350.3256	327.4503	351.0003	327.7374	359.1374
19:00	337.7176	345.5676	337.3636	353.0636	330.9967	354.5467	330.9283	362.3283
20:00	341.7904	349.6404	341.8464	357.5464	335.8131	359.3631	335.6967	367.0967
21:00	348.7930	356.6430	348.9074	364.6074	343.8752	367.4252	343.2478	374.6478
22:00	358.5748	366.4248	358.1891	373.8891	352.5305	376.0805	352.0847	383.4847
23:00	363.8034	371.6534	363.5453	379.2453	359.2063	382.7563	359.2544	390.6544

Table 3: Typical values of Refractivity Gradient and k-factor calculated for February 2007

	200-150 m		150-100 m		100-50 m	
Hours	$\alpha 1$	k1	$\alpha 2$	k2	$\alpha 3$	k3
0:00	0.152544	1.029120	0.064773	2.423628	0.150379	1.043936
1:00	0.161777	0.970382	0.065799	2.385823	0.140274	1.119134
2:00	0.157647	0.995806	0.067553	2.323901	0.139681	1.123891
3:00	0.162900	0.963692	0.066026	2.377640	0.148665	1.055967
4:00	0.167744	0.935866	0.065822	2.384989	0.143841	1.091386
5:00	0.167324	0.938216	0.060668	2.587623	0.139768	1.123186
6:00	0.165317	0.949605	0.068081	2.305867	0.139075	1.128787
7:00	0.166026	0.945551	0.068965	2.276304	0.141352	1.110606
8:00	0.164556	0.953995	0.065387	2.400888	0.149066	1.053130
9:00	0.154637	1.015187	0.058886	2.665931	0.134121	1.170480
10:00	0.168827	0.929861	0.029116	5.391759	0.150674	1.041894
11:00	0.158621	0.989692	0.030498	5.147402	0.167698	0.936125
12:00	0.172509	0.910018	0.022770	6.894374	0.159389	0.984978
13:00	0.173500	0.904820	0.017305	9.071769	0.170936	0.918389
14:00	0.169592	0.925670	0.016025	9.796577	0.170475	0.920872
15:00	0.162279	0.967385	0.020225	7.761866	0.159924	0.981630
16:00	0.176538	0.889246	0.007171	21.89090	0.165710	0.947352
17:00	0.172045	0.912472	0.009101	17.24880	0.163903	0.957798
18:00	0.162741	0.964633	0.013495	11.63252	0.159483	0.984342
19:00	0.155633	1.008692	0.029662	5.292488	0.149920	1.047133
20:00	0.154671	1.014966	0.036335	4.320515	0.158120	0.992828
21:00	0.144453	1.086760	0.056356	2.785616	0.159287	0.985551
22:00	0.148084	1.060116	0.043827	3.581932	0.149286	1.051576
23:00	0.157962	0.993818	0.070220	2.235643	0.151840	1.033891

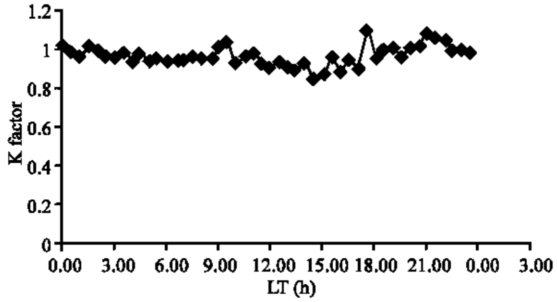

Fig. 4 : Typical diurnal variation of k-factor against $\Delta H = 50$ m for February 2007

assumed above; the vertical gradient of refractive index varies with space and time (Grabner and Kvicera, 2003a). In this study, therefore, the k-factor has been evaluated using the average gradient of the M-profile calculated using Eq. 8. The M-profile was determined according to the distribution of (dM/dH) and H. The values of k_i were determined against height interval $(H)_i$ and typical results are presented in Table 3. Using the average value of the refractivity gradient, an average value of the k-factor was calculated for Akure to be 1.86. This result is higher than that obtained by Kolawole and Owonubi (1982) using radiosonde data. They found that the equatorial climatic region has a k-factor value of 1.52. It is important to note however, that the results obtained in this study is based on *in-situ* measurements of parameters rather than extrapolated data often used in previous studies (Fig. 4).

Also, in order to examine the diurnal variation of the gradient of the M-profiles, the thickness of the layer was designated as H and the M-profile was approximated as a linear one in such thickness. Typical diurnal variation of the gradient of the M-profile is shown in Fig. 5 for the different atmospheric layers. They show the diurnal variation characteristics of the average gradient, , of M-profile in the dry season. Such variation characteristics are found to be more remarkable in the (100-150 m) layer than (50-100 m) and or (150-200 m). Whereas the variation is similar in the (50-100 m) layer and (150-200 m) layer, it follows the variation pattern of the vapour pressure in the (100-150 m) layer with the minimum value in the afternoon hours. This further shows that the occurrence of M-inversion is associated with sudden fall in the value of vapour pressure. The occurrence of M-inversion is an important condition for observing some abnormal propagation phenomenon (such as ducting) especially when such occurrence is accompanied with temperature inversion (Falodun and Ajewole, 2006). It was also observed that the smaller the thickness of the atmospheric layer becomes, the larger was the standard deviation of the mean gradient of refractive index for the thickness. For example, the standard deviation is 0.0089 for $\Delta H = 50$ m while the value is 0.0078 for $\Delta H = 100$ m. This implies that for large atmospheric layers, the dispersion of the values from the mean gradient is very small. Figure 5 shows the diurnal variations of the refractivity gradient against the layer thickness of H = 50 m (50-100, 100-150 and 150-200 m) and for those against H = 100 m (50-159, 100-200 m) are shown in Fig. 6, while Fig. 7 shows that for those against H = 150 m (50-200 m). They show the diurnal

Table 4: Average k-factor calculated for February 2007

ΔH	$\overline{\alpha}_i$	k-factor
(299-150 m)	0.162115	0.968364
(150-100 m)	0.04479	3.504907
(100-50 m)	0.153104	1.025354

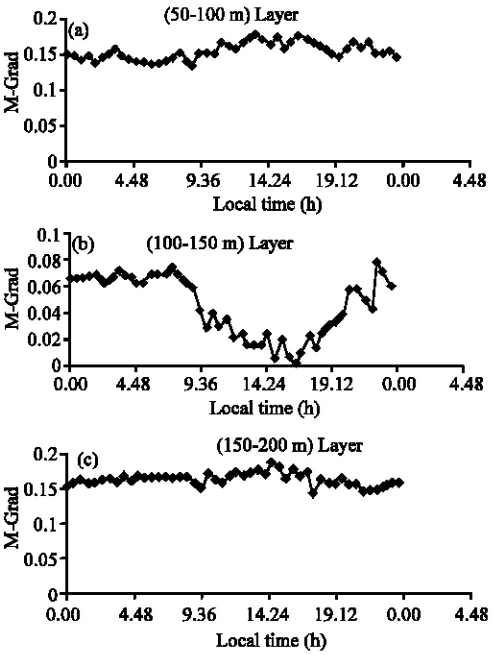

Fig. 5: Typical diurnal variation of M-gradient against ΔH = 50 m

variation characteristics of the average gradient of M-profile in the dry season months (December-February) for this locality.

The mean value of α for the layer (50-100 m) is rather smaller than that for the layer (100-150 m), but the standard deviations of α for both are nearly equal. However, the standard deviation of for the layer with the thickness of 100m is smaller than that for layer 50 m thick. In any case, it may be seen that, against the layers with the same ΔH, the standard deviations of α are nearly equal and according as H becomes greater, the standard deviation of becomes rather smaller. The variation characteristics of α is useful for the estimation of the seasonal characteristics of the fading range (Table 4).

It is pertinent to note that monthly average values were used for this study. Although, hourly data were collected everyday throughout the period of the measurement, hourly averages for each month were computed to reduce the volume of the data involved. Therefore, daily occurrences (that may occur for a short period of time) such as temperature inversion may be missed out.

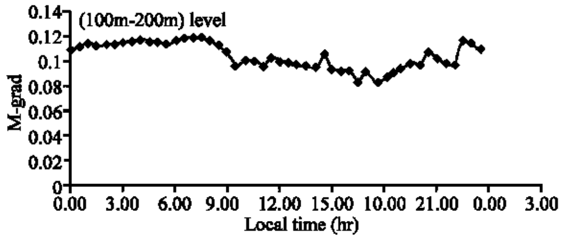

Fig. 6: Typical diurnal variation of M-gradient against H = 100 m

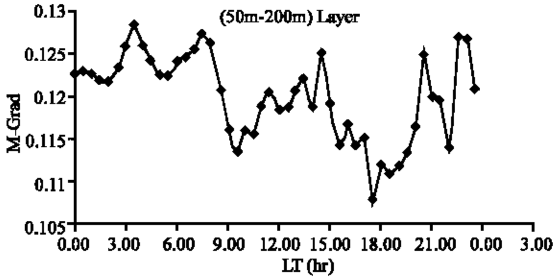

Fig. 7: Typical diurnal variation of M-gradient against H = 150 m

CONCLUSION

The results obtained from the statistical study of the atmospheric refractive index based on experiments can be summarized as follows

- By introducing ΔH, the height range of the propagation path and defining the mean gradient of M profile in this range, the conception of k-factor was better appreciated.
- The vertical distribution of the refractive index was statistically considered and it was found that the average gradient of the atmospheric layer which is H (m) in thickness approximately follows the normal distribution.
- By measuring the average gradient of the atmospheric refractive index between two points at different heights for a period of three months, the mean value of the k-factor was determined for Akure to be 1.85
- The smaller the thickness of the atmospheric layer becomes, the larger was the standard deviation of the mean gradient of refractive index for the thickness.

The results obtained from this study have shown that the exponential atmosphere differs considerably from the basic reference atmosphere. Hence, the ITU-R recommendation may not be appropriate to describe the decrease of refractivity with height in this tropical region, especially in the 0-200 m atmospheric layer.

Since it has been shown that the radio wave fading depends on the time variation of the shape of atmospheric refractive index profile, the statistics of the fading range is therefore expected to be quantitatively related to the standard deviation. Therefore, from this view point, the refractivity data are important to estimate the fading range of season and locality.

REFERENCES

Barclay, L., 2003. Propagation of Radio-waves (2nd Edn.), The Institution of Electrical Engineers. London. U.K., pp: 103-127.

Bean, B.R. and E.J. Dutton, 1968. Radio Meteorology. Dover Publication Co. New York, pp: 259-273.

Bonkoungou, Z. and K. Low, 1993. Radiowave propagation measurement in Burkina Faso. SMR on Radiowave propagation in Tropical regions, Trieste, Italy.

Falodun, S.E. and M.O. Ajewole, 2006. Radio refractive index in the lowest 100-m layer of the troposphere in Akure, south-western Nigeria. J. Atmospheric and Solar-Terrestrial Physics (68) . Elsevier, U.S.

Grabner, M. and V. Kvicera, 2003a. Refractive index measurement at TV tower Prague. Radioengineering, 12: 5-7.

Grabner, M. and V. Kvicera, 2003b. Clear-air propagation modeling using parabolic equation method. Radioengineering, 12: 50-54.

Hall, M.P.M., 1979. Effect of the troposphere on radio communication, IEEE Electromagnetic waves series, Peter Peregrinus Ltd, United Kingdom, pp: 105-116.

ITU, R., 1987. The Radio Refractive Index ; Its Formula and Refractivity Data. Rec. 370 ITU-R, pp: 453-456.

Kolawole, L.B. and J.J. Owonubi, 1982. The surface radio refractivity over Africa. Nig. J. Sci., 16: 1-2, 441-454.

Rec. Itu, R.P. 453-9, 2004. The radio refractive index: its formula and refractivity data, pp: 455-460.