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Global Relative Controlability for Nonlinear Neutral Systems with Delays in the Control

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Abstract: Sufficient conditions for global relative controllability of certain types of nonlinear neutral time-varying systems with time-variable delay in the control are given. By a careful linearisation of our system, the results are obtained by use of Schauder's fixed-point theorem and a generalization of Davison.

Key words: Global relative controllability, time-variable delay, Arzela-Ascoli theorem, Schauder's fixed-point theorem

INTRODUCTION

The problem of controllability of nonlinear systems has been handled by many authors as in (Davison and Kunze, 1964; Eke, 1990). Controllability of systems with delays in the control has been considered in the process (Chyung, 1970). The purpose of this study, is to extend the work of Davison and Kunze (1964) by including the time-variable delay in the control. The results obtained in this study provide sufficient conditions for global relative controllability of certain types of nonlinear neutral time-varying systems with time-variable delay in the control given by

$$\frac{d}{dt}\left[D(t,x(t))\right] = L(t,x(t))x(t) + C(t,x(t))u(v(t)) \quad (1)$$

Where x(t) is an n×1 state vector, u(t) is a p×1 input vector, L(t,x(t)) is an n×n matrix and C(t,x(t)) is an n×m matrix satisfying the following conditions

$$|\, l_{i,k}(t,x)\,| \le M \quad \text{for } t \in [t_0,t_1] \quad \text{and } x \in R^n \quad (2)$$

$$|c_{i,k}(t,x)| \le N \quad \text{for } t \in [t_0,t_1] \quad \text{and } x \in R^n \quad (3)$$

Where M and N are positive real constants. The operator D is given by

$$D(t,\phi) = x(t) - g(t,x(t)) \quad (4)$$

and

$$g: [0,t1] \times Bn \rightarrow Rn$$

R^n is a real n-dimensional vector space . We denote B_n $[t_0,t_1]$ the Banach space of continuous R^n valued functions on $[t_0,t_1]$ with the norm

$$|z| = \underset{t_0 \le t \le t_1}{\text{Max}} |z(t)| \text{ where } z(t) = \sum_{i=1}^{n} |z_i(t)| \quad (5)$$

We assume that the function $v:[t_0,t_1]\rightarrow R^n$ is absolutely continuous and strictly increasing on $[t_0,t_1]$ and satisfy the following condition

$$v(t) \le t \quad \text{for } t \in [t_0,t_1]$$

We introduce a time-lead function r(s) such that

$$r(v(t)) = t \quad \text{for } t \in [t_0,t_1] \quad (6)$$

It is well known in Chyung (1970) that the complete state of the system, (1) at time t is defined as in the case of delay control.

Definition 1 (complete state): The set y(t) = {x(t),w(t)} where w(t,s)=u(s) for $s \in [v(t),t)$ is said to be the complete state of the system (1) at t.

Definition 2 (global relative control): The system (1) is said to be globally relatively controllable on $[t_0,t_1]$ if for every complete state $y(t_0)$ and every vector $x_1 \in R^n$, there exists a control u(t) defined on $[t_0,t_1]$ such that the corresponding trajectory of the system satisfies $x(t_1)=x_1$.

PRELIMINARIES

In this study, we gather the tools necessary in the development of the main result of this study. Consider the system (1), which is nonlinear. By replacing the arguments x(t) of L and C by a specified function $z \in B_n[t_0,t_1]$, the system (1) becomes

$$\frac{d}{dt}\left[D(t,x(t))\right] = L(t,z(t))x(t) + C(t,z(t))u(v(t)) \quad (7)$$

The system (7) is a linear approximation of (1). Owing to the difficulty in controlling the system (1) directly, we usually solve the system by controlling system (7). We now obtain the variation of constant formula for (7). The solution with initial complete state $y(t_0)$ is of the form

$$x(t) = x(t,t_0,\phi) + F(t,t_0,z) + \int_0^{t_1} F(t,s,z)C(s,z)u(v(s))ds \quad (8)$$

Where $F(t,t_0;z)$ is the transition matrix of the system $x(t) = L(t,z(t))x(t)$. with $F(t_0,t_0;z) = I$. System (8) can be written in the following form as in Davison and Ball (1972), Davison and Kunze (1964):

$$\begin{aligned} x(t) = x(t,t_0,\phi) + F(t,t_0,z)\Big[x(t_0) + \int_{v(t_0)}^{t_0} F(t_0,r(s);z) \\ C(r(s);z)w(t_0,s)r^{(1)}(s)ds + \int_{t_0}^{v(t)} F(t_0,r(s),z) \\ C(r(s);z)w(t_0,s)r^{(1)}(s)u(s)ds\Big] \end{aligned} \quad (9)$$

$$\begin{aligned} x(t) = x(t,t_0,\phi) + F(t,t_0,z)\Big[x(t_0) + \int_{t_0}^{v(t)} F(t_0,r(s);z) \\ C(r(s);z)w(t_0,s)r^{(1)}(s)ds + \int_{v(t_0)}^{t_0} F(t_0,r(s),z) \\ C(r(s);z)w(t_0,s)r^{(1)}(s)u(s)ds\Big] \end{aligned} \quad (10)$$

Let us assume that $r > r(t_0)$ and for simplicity of notations, we demote

$$\begin{aligned} G(t_0 t,z) = \int_{t_0}^{v(t)} [F(t_0,r(s);z)C(r(s):z)r^{(1)}(s)]\cdot \\ [F(t_0,r(s);z)C(r(s;z)r^{(1)}(s)]^T ds \end{aligned} \quad (11)$$

Where T denotes transposition of matrix and

$$\begin{aligned} q(y(t_0,x(t);z) = F(t_9,t,z)x(t) - x(t_0) + \\ \int_{v(t_0)}^{t_0} F(t_0,r(s);z)C(r(s);z)w(t_0,s)r^{(1)})s)u(s)ds \end{aligned} \quad (12)$$

RESULTS AND DISCUSSION

Theorem 1: Given system (7). Assume $r_1 > r(t_0)$ and

$$\inf_{z\in Bn[to,t1]} \det G(t_0,t_1;z) > 0 \quad (13)$$

Then the system (1) is globally relatively controllable on $[t_0, t_1]$.

Proof: We define the control u(t) in $[t_0,t_1]$ as follows

$$u(t) = \begin{cases} F(t_0,r(t);z)C(r(t);z)r^{(1)}(t)G^{-1}(t_0,t_1;z)q \\ (y(t_0,x_x;z) \ldots\ldots\ldots\ldots\ldots\ldots t\in[t_0,v(t_1)] \\ \\ F^*(t_0,t;z)G^{-1}(t_0,t_1;z)q(y(t_0),x_1;z) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots t\in[v(t),t_1] \end{cases} \quad (14)$$

where $y(t_0)$ and x_1 are chosen arbitrarily. Inserting (14) into (10) we have

$$\begin{aligned} x(t) = x(t,t_0,\phi) + F(t,t_0;z)\Big\{x(t_0) + \int_{v(t_0)}^{t_0} F(t_0,r(s);z) \\ C(r(s);z)w(t_0,s)r^{(1)}(s)ds + \int_{t_0}^{v(t)} [F(t_0,r(s);z) \\ C(r(s);z)r^{(1)}(s)]G^{-1}(t_0,t_1;z)q(y(t_0),x_1;z)ds\Big\} \end{aligned} \quad (15)$$

By using Eq. 11 and 12 it is easily verified that the control u(t) defined by (14) transfer the system (7) for every $z\,^a\, B_n[t_0,t_1]$ from complete state $y(t_0)$ to the desired state x_1 at t_1. We now consider the right hand side of (15) as an operator P(z)(t) which maps the Banach space into itself. Hence we can write (15) as

$$x(t) = P(z)(t) \quad (16)$$

Consider a closed and convex subset of $B_n[t_0,t_1]$: $A=\{z/z\,^a\, B_n[t_0,t_1]:|z| \le k\}$ with k as defined in Davison and Kunze (1964). We denote the range of the operator P when A is in its domain by $H\subset B_n[t_0,t_1]$.
Where

$$H = \{x(t)/x(t) = P(z)(t) : z \in A \quad (17)$$

P is continuous (Davison and Kunze, 1964) and from Arzela-Ascoli theorem (Kantovich and Akilov, 1964) the image set H defined by (17) is compact and is a subset of A. Hence by the Schauder's Fixed Point Theorem, the operator P has at least one fixed point. Therefore, there always exists at least one function $z^* \in B_n[t_0,t_1]$ such that

$$x^*\{t\} = z^*(t) = P(z^*)(t) \quad (18)$$

It is easily verifiable by differentiating with respect to t that x(t) given by (18) is a solution of the system (1) for the control u(t) given by (14). The control u(t) steers the system from the complete state $y(t_0)$ to x_1 on $[t_0,t_1]$ and since $y(t_0)$ and x_1 are chosen arbitrarily, the system (1.1) is globally relatively controllable on $[t_0,t_1]$.

CONCLUSION

Using Schauder's Fixed Point Theorem, sufficient conditions for global relative controllability on $[t_0,t_1]$ of certain types of nonlinear neutral time-varying systems with time-variable delay in the control have been derived. In the case $t_1 = r(t_0)$, the global relative controllability on $[t_0,t_1]$ does not depend on the delay.

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