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Fuzzy Multi-Objective Production Inventory Model of Deteriorating Items With Shortage and Various Types of Demands

[1]R. Kumar and [2]A. Solai Raju
[1]Department of Mathematics, Mookambigai College of Engineering, Kalamavur, India
[2]Department of Mathematics, Jamal Mohamed College, Tiruchirapalli, India

Abstract: In this study, a multi-item and multi-objective production inventory model of deteriorating items with constant and stock dependent demands under shortage is developed in crisp and fuzzy environment with finite time horizon, respectively. The total holding cost and the total shortage cost are limited and the rate of production is finite and uniform. Our objectives are maximizing the profit and minimizing the wastage cost. In fuzzy model the above said objectives are fuzzy in nature. The allowable total shortage cost and total holding cost are also, assumed to be vague and imprecise. The impreciseness in the above objectives and constraints goals have been expressed by fuzzy linear membership functions. Here, crisp and fuzzy models are developed and solved by the Weighted Fuzzy Non-linear Programming (WFNLP), Fuzzy Additive Goal Programming (FAGP) and integrated goal attainment and fuzzy non-linear programming technique (GAFNLP). The models are illustrated with numerical examples and the results of different techniques are compared.

Key words: Production inventory, multi-objective, shortage, demands, deterioration, membership function

INTRODUCTION

Bellan and Zadeh (1970) first introduced fuzzy set theory in fuzzy decision making process. Tanaka *et al.* (1974) applied the concepts of fuzzy sets to decision making problems by considering the objectives as fuzzy goals over the α-cuts of a fuzzy constraints set and Zimmermann (1976) showed the classical algorithms can be used to solve multi- objective fuzzy linear programming problems.

In an inventory model, generally 3 types of demands are considered. These are constant demand time-dependent demand and stock-dependent demand. Padmanabhan and Prem vrat (1990) derive multi-objective inventory model of deterioration items with stock-dependent demand by a non-linear goal programming method. Ben-Daya and Raouf (1993) discussed a multi-item inventory model with stochastic demand under two constraints. Roy and Maiti (1995) developed the classical EOQ model in a fuzzy environment with fuzzy goal, fuzzy inventory costs and storage area by fuzzy non-linear programming method using different types of membership function for inventory parameters. Roy and Maiti (1997) also developed the fuzzy EOQ model with demand-dependent unit price and imprecise storage area by both fuzzy geometric and non-linear programming methods. Roy and Maiti (1998) discussed the multi-objective inventory models of deteriorating items with two constraints of storage area and total average cost in fuzzy environment. Roy *et al.* (2003) developed multi- objective fuzzy inventory model for deteriorating items with shortages over a finite time horizon. They developed both crisp and fuzzy models with weighted fuzzy non-linear programming (WFNLP), integrated goal attainment and fuzzy non-linear programming technique (GAFNLP). Several researchers Silver and Meal (1973) and Donaldson (1991) developed and solved the inventory models with time-dependent demand.

In most of the earlier inventory model, life time of an item is assumed to be infinite, while it is in storage. But, in reality, many physical goods deteriorate due to dryness, spoilage, vaporization etc. and are damaged due to holding longer than their normal storage periods. The deterioration also depends on preserving facilities and environmental conditions of storage. So, due to deterioration effect, a certain fraction of the items is either damaged or decayed and is not in perfect condition to satisfy the future demand of customers as good items. Deterioration for such items is continuous and constant or time-dependent or stock-dependent. A number of research papers have already been published on the above type of items by Dave and Patel (1981), Hariga and Benkher (1994), De and Goswami (2001), Bhunia and Maiti (1997), Chang and Dye (1999), Kang and Kim (1983) and others.

Corresponding Author: R. Kumar, Department of Mathematics, Mookambigai College of Engineering, Kalamavur, India

In this study, we have developed a multi- item and multi-objective inventory problem along with two constraints namely allowable total shortage cost and allowable total holding cost. Demands are constant and stock-dependent. Here, production is finite and uniform. In the development of the model, the loss of production quantity due to faulty machine/aged machine, manufacturing defect etc. from the actual production quantity have also been taken into account. Two models are developed here. The objectives are to maximize the profit and to minimize the wastage cost. In the first model profit goal , wastage costs and constraints are crisp. In the second model, profit goal, wastage cost, allowable total shortage cost and total holding cost are fuzzy in nature. Fuzzy parameters are represented by linear membership functions. Both models are solved by Weighted Fuzzy Non-linear Programming (WFNLP), fuzzy additive goal programming (FAGP) and integrated goal attainment and fuzzy non-linear programming technique (GAFNLP). The models are illustrated numerically and results are obtained from WFNLP, FAGP and GAFNLP techniques. Optimum results are presented in a tabular form. The results are compared, as for as the different weights are concerned.

There are n items and they are separated in 2 categories I_1 and I_2 depending upon their demands. Demands of the items in I_1 are inventory level dependent and those of I_2 are constant.

$$d_i = D_i + \gamma_i \, q_i(t) \quad \text{for } i \in I_1$$
$$\quad = D_i \quad \text{for } i \in I_2 \qquad (1)$$

where, $q_i(t)$ be the inventory level at time t of the i-th item, γ_i $(0 < \gamma_i < 1)$ and D_i $(D_i > 0)$ are constants.

MODEL FORMULATION

Here, a multi- item, multi-objective inventory model for deteriorating items are formulated, considering two types of demands described in Eq. 1. The loss of production quantity due to faulty machine/aged machine, manufacturing defect etc. from the actual production quantity has also been taken into account.

The production starts at the beginning of the each cycle at time t = 0 and continues up to time $t = t_{1i}$, due to constraint of allowable total holding cost. The actual production rate becomes less than the original production rate due to faulty machines and some production quantity deteriorates at the time of production. The inventory, accumulated during the production period t_{1i} after meeting up demand during the period and the deterioration, the inventory reaches to the zero level at time $t = t_{2i}$. We allow the shortages upto time $t = t_{3i}$, where the allowable total shortage cost is the constraint. Again production starts at $t = t_{3i}$ and back- lock is filled during time T_i, till the back log becomes zero. This is complete one cycle T_i.

$$\frac{dq_{i1}(t)}{dt} = K_i(1-\phi_i) - \theta_i \, q_i - d_i \qquad 0 \le t \le t_{1i} \qquad (2)$$

$$\frac{dq_{i2}(t)}{dt} = -\theta_i \, q_i - d_i \qquad t_{1i} \le t \le t_{2i} \qquad (3)$$

$$\frac{dq_{i3}(t)}{dt} = -d_i \qquad t_{2i} \le t \le t_{3i} \qquad (4)$$

$$\frac{dq_{i4}(t)}{dt} = K_i(1-\phi_i) - \theta_i \, q_i - d_i \qquad t_{3i} \le t \le T_i \qquad (5)$$

With the conditions are

$$q_{i1}(0) = 0,\ q_{i2}(t_{2i}) = 0,\ q_{i3}(t_{2i}) = 0,\ q_{i4}(T_i) = 0$$

and

$$q_{i1}(t_{1i}) = q_{i2}(t_{1i}),\ q_{i3}(t_{3i}) = q_{i4}(t_{3i})$$

Using condition at q_{i1} (t) = 0 at t = 0 in (2), we get the production quantity during the time $0 \le t \le t_{1i}$ is:

$$q_{i1}(t) = \int_0^t (K_i(1-\phi_i) - D_i)\exp((\theta_i + \gamma_i)(x-t))dx \qquad 0 \le t \le t_{1i},\ i \in I_1$$
$$= \int_0^t (K_i(1-\phi_i) - D_i)\exp((\theta_i(x-t))dx \qquad 0 \le t \le t_{1i},\ i \in I_2 \qquad (6)$$

Since, Q_i is the maximum production level for the i-th item at $t = t_{1i}$, $q_{i1}(t_{1i}) = Q_1$, then

$$Q_i = \frac{(K_i(1-\phi_i) - D_i)}{(\gamma_i + \theta_i)}(1 - \exp(-(\gamma_i + \theta_i)t_{1i})) \quad i \in I_1$$
$$\frac{(K_i(1-\phi_i) - D_i)}{\theta_i}(1 - \exp(-\theta_i t_{1i})) \quad i \in I_2 \qquad (7)$$

Where,

$$t_{1i} = \frac{1}{(\gamma_i+\theta_i)} \ln\left(\frac{(K_i(1-\phi_i)-D_i)}{K_i(1-\phi_i)-D_i-(\gamma_i+\theta_i)Q_i}\right) \quad i \in I_1$$
$$= \frac{1}{\theta_i} \ln\left(\frac{(K_i(1-\phi_i)-D_i)}{K_i(1-\phi_i)-D_i-\theta_i Q_i}\right) \quad i \in I_2 \qquad (8)$$

Solving the differential Eq. 3 using condition

$$q_{i2}(t) = 0 \text{ at } t = t_{2i}$$

$$q_{i2}(t) = \frac{D_i}{(\gamma_i+\theta_i)}(\exp((\gamma_i+\theta_i)(t_{2i}-t))-1) \quad t_{1i} \le t \le t_{2i} \quad i \in I_1$$
$$= \frac{D_i}{\theta_i}(\exp(\theta_i(t_{2i}-t))-1) \quad t_{1i} \le t \le t_{2i} \quad i \in I_2 \qquad (9)$$

with the condition $q_{i1}(t_{1i}) = q_{i2}(t_{1i})$

$$t_{2i} = t_{1i} + \frac{1}{(\gamma_i+\theta_i)} \ln\left(\frac{D_i+(\gamma_i+\theta_i)Q_i}{D_i}\right) \quad i \in I_1$$
$$= t_{1i} + \frac{1}{\theta_i} \ln\left(\frac{D_i+\theta_i Q_i}{D_i}\right) \quad i \in I_2 \qquad (10)$$

Holding cost in each cycle for i-th item is $C_{1i}\, H(q_{ii})$, Where,

$$H(q_i) = \int_0^{t_{1i}} q_{i1}(t)\,dt + \int_{t_{1i}}^{t_{2i}} q_{i2}(t)\,dt$$

$$H(q_i) = \frac{1}{(\gamma_i+\theta_i)} (K_i(1-\phi_i)\, t_{1i} - D_i\, t_{2i}) \quad i \in I_1 \qquad (11)$$

$$= \frac{1}{\theta_i} (K_i(1-\phi_i)\, t_{1i} - D_i\, t_{2i}) \quad i \in I_2 \qquad (12)$$

Using condition $q_{i3}(t_{2i}) = 0$ in (4) ,we get the shortage quantity during period $t_{2i} \le t \le t_{3i}$ is:

$$q_{i3}(t) = -D_i \int_{t_{2i}}^{t} \exp(\gamma_i (x-t))\, dx \quad t_{2i} \le t \le t_{3i} \quad i \in I_1$$
$$= -D_i \int_{t_{2i}}^{t} dx \quad t_{2i} \le t \le t_{3i} \quad i \in I_2 \qquad (13)$$

Since, SQ_i is the maximum shortage level at $t = t_{3i}$, (i.e.) $q_{i3}(t_{3i}) = SQ_i$, then

$$SQ_i = \frac{D_i}{\gamma_i}(1-\exp(\gamma_i (t_{2i}-t_{3i}))) \quad i \in I_1$$
$$= \frac{D_i}{\gamma_i}(t_{3i}-t_{2i}) \quad i \in I_2 \qquad (14)$$

Where,

$$t_{3i} = t_{2i} - \frac{1}{\gamma_i} \ln\left(\frac{D_i-\gamma_i SQ_i}{D_i}\right) \quad i \in I_1$$
$$= \frac{SQ_i}{D_i} + t_{2i} \quad i \in I_2 \qquad (15)$$

The shortage quantity accumulated over the period $t_{2i} \le t \le t_{3i}$

$$SA(q_i) = \int_{t_{2i}}^{t_{3i}} q_{i3}(t)\,dt \quad i \in I_1 \cup I_2 \qquad (16)$$

$$\text{(i.e.,) } SA(q_i) = \frac{D_i}{\gamma_i}\left[(t_{3i}-t_{2i}) + \frac{1}{\gamma_i}(\exp(\gamma_i(t_{2i}-t_{3i}))-1)\right] \quad i \in I_1$$
$$= \frac{D_i}{2}(t_{3i}-t_{2i})^2 \quad i \in I_2 \qquad (17)$$

Solve the differential Eq. 5, using the condition $q_{i4}(t) = 0$ at $t = T_i$. We get the production quantity during the period $t_{3i} \le t \le T_i$.

$$q_{i4}(t) = \int_t^{T_i} (K_i(1-\phi_i)-D_i) \exp((\gamma_i+\theta_i)(x-t))\,dx, \quad i \in I_1$$
$$= \int_t^{T_i} (K_i(1-\phi_i)-D_i) \exp(\theta_i (x-t))\,dx, \quad i \in I_2 \qquad (18)$$

$$\text{(i.e.,) } q_{i4}(t) = \frac{(K_i(1-\phi_i)-D_i)}{(\gamma_i+\theta_i)}(\exp((\gamma_i+\theta_i)(T_i-t))-1) \quad i \in I_1$$
$$= \frac{(K_i(1-\phi_i)-D_i)}{\theta_i}(\exp(\theta_i(T_i-t))-1) \quad i \in I_2 \qquad (19)$$

The Production quantity accumulated over the period $t_{3i} \le t \le T_i$ is:

$$RA(q_i) = \int_{t_{3i}}^{T_i} q_{i4}(t)\,dt$$

$$RA(q_i) = \frac{(K_i(1-\phi_i)-D_i)}{(\gamma_i+\theta_i)}[\frac{1}{(\gamma_i+\theta_i)}(\exp((\gamma_i+\theta_i)(T_i-t_{3i}))-1)-(T_i-t_{3i})], \quad i \in I_1 \quad (20)$$

$$= \frac{(K_i(1-\phi_i)-D_i)}{\theta_i}[\frac{1}{\theta_i}(\exp(\theta_i(T_i-t_{3i}))-1)-(T_i-t_{3i})] \quad i \in I_2 \quad (21)$$

Using the condition $q_{i3}(t_{3i}) = q_{i4}(t_{3i}) = SQ_i$, we get

$$T_i = t_{3i} + \frac{1}{(\gamma_i+\theta_i)} \ln (\frac{(SQ_i*(\gamma_i+\theta_i)+K_i(1-\phi_i)-D_i)}{(K_i(1-\phi_i)-D_i)}) \quad i \in I_1$$

$$= t_{3i} + \frac{1}{\theta_i} \ln (\frac{(SQ_i*\theta_i)+K_i(1-\phi_i)-D_i)}{(K_i(1-\phi_i)-D_i)}) \quad i \in I_2 \quad (22)$$

Total shortage cost in each cycle for i-th items is $C_{2i}\,S(q_i)$, Where,

$$S(q_i) = SA(q_i) + RA(q_i) \quad i \in I_1 \cup I_2 \quad (23)$$

The total number of deteriorating units of the i-th items is:

$$\theta_i(q_i) = \theta_i(H(q_i) + RA(q_i)) \quad i \in I_1 \cup I_2 \quad (24)$$

The cost due to deteriorating of i-th item.

$$S_i * \theta_i(q_i). \quad i \in I_1 \cup I_2 \quad (25)$$

Hence, revenue from sale

$$R(q_i) = (S_i - P_i)(K_i(1-\phi_i) * (t_1 + (T - t_3)). \quad i \in I_1 \cup I_2 \quad (26)$$

The total cost

$$TC(q_i) = \sum_{i=1}^{n} P_i * [K_i(1-\phi_i)*(t_{1i} - (T_i - t_{3i})) + C_{3i} + C_{1i} * H(q_i) + C_{2i} * S(q_i) + S_i * \theta_i(q_i)]/T_i \quad i \in I_1 \cup I_2 \quad (27)$$

Total profit =Revenue form sale –Set up cost-Holding cost -Shortage cost–Deteriorating cost.

$$PF(q_i) = \sum_{i=1}^{n} [R(q_i) - C_{3i} - C_{1i} * H(q_i) - C_{2i} * S(q_i) - S_i * \theta_i(q_i)] \quad i \in I_1 \cup I_2 \quad (28)$$

The total wastage cost is:

$$WS(q_i) = \sum_{i=1}^{n} P_i * [(K_i\ \phi_i)(t_{1i} + (T_i - t_{3i})) + \theta_i(q_i)] \quad i \in I_1 \cup I_2 \quad (29)$$

Hence, the problem is to maximize the total average profit per unit time and minimize the total average wastage cost per unit time is:

$$MaxPF(q_i) = \sum_{i=1}^{n} [R(q_i) - C_{3i} - C_{1i}H(q_i) - C_{2i}\,S(q_i) - S_i * \theta_i(q_i)]/T_i \quad i \in I_1 \cup I_2 \quad (30)$$

$$MinWS(q_i) = \sum_{i=1}^{n} P_i * [(K_i\ \phi_i)(t_{1i} + (T_i - t_{3i})) + \theta_i(q_i)]/T_i \quad i \in I_1 \cup I_2 \quad (31)$$

There are some restrictions on available resources in inventory problems that cannot be ignored to derive the optimal profit.

- The maximum permissible total holding cost per cycle

$$\sum_{i}^{n} C_{1i}\,H(q_i) \le THC \quad i \in I_1 \cup I_2 \quad (32)$$

- The maximum permissible total shortage cost per cycle

$$\sum_{i}^{n} C_{2i}\,S(q_i) \le TSC \quad i \in I_1 \cup I_2 \quad (33)$$

Hence, our problem is to maximize the total average profit per unit time, subject to the available total holding cost and total shortage cost restrictions (i.e.,):

$$MaxPF(q_i) = \sum_{i=1}^{n} [R(q_i) - C_{3i} - C_{1i}H(q_i) - C_{2i}\,S(q_i) - S_i * \theta_i(q_i)]/T_i \quad i \in I_1 \cup I_2$$

$$\text{Min}WS(q_i) = \sum_{i=1}^{n} P_i *[(K_i\ \phi_i)(t_{1i} + (T_i - t_{3i})) + \theta_i(q_i)]/T_i \qquad i \in I_1 \cup I_2$$

subject to inequality constraints

$$\sum_{i}^{n} C_{1i}\ H(q_i) \le THC \qquad i \in I_1 \cup I_2$$
$$\sum_{i}^{n} C_{2i}\ S(q_i) \le TSC \qquad i \in I_1 \cup I_2 \qquad (34)$$

CRISP MODEL

In crisp environment muti-item, multi-objective production inventory problems with 2 constraints in (34) will become,
$\text{Max}PF(q_i)$
$\text{Min}WC(q_i)$
subject to inequality constraints

$$\sum_{i}^{n} C_{1i}\ H(q_i) \le THC \qquad i \in I_1 \cup I_2$$
$$\sum_{i}^{n} C_{2i}\ S(q_i) \le TSC \qquad i \in I_1 \cup I_2 \qquad (35)$$

where, THC and TSC are maximum allowable total holing cost and total shortage cost, respectively.

Here, to solve the above Model-1, given by the Eq. 35, we use the Weighted fuzzy non-linear programming (WFNLP) method based on Zimmermawn (1978) and Lee and Li (1993). Fuzzy additive goal programming (FAGP) technique based on Tiwari *et al.* (1987) and Integrated goal attainment and fuzzy non-linear programming (GAFNLP) method based on Das *et al.* (2003).

Weighted Fuzzy Non-linear Programming Technique to solve crisp model (WFNLP): In this technique, the following steps are used to solve the fuzzy model.

Step 1: Solve the multi- objective programming problem (35) as a single objective problem using only one objective at a time ignoring the other.

Step 2: From the results of Step 1, determine the corresponding values of all objectives at each solution derived.

Step 3: From Step 2, for each objective, one can find the values L_i and U_i (i = PF, WS) corresponding to the set of solutions where L_i and U_i are the lower and upper level of achievements.

Step 4: The multi-objective fuzzy model in Eq, (35) the membership function (MF) μ_i, corresponding to the objective i, (i = PF, WS), which may be linear or non-linear, is defined here, for simplicity, linearly as a:

$$\mu_{PF}(q_i) = \begin{cases} 0 & \text{for } PF(q_i) \le L_{PF} \\ 1 + \dfrac{PF(q_i) - U_{PF}}{U_{PF} - L_{PF}} & \text{for } L_{PF} \le PF(q_i) \le U_{PF} \\ 1 & \text{for } PF(q_i) \ge U_{PF} \end{cases}$$

$$\mu_{WS}(q_i) = \begin{cases} 1 & \text{for } WS(q_i) \le L_{WS} \\ 1 - \dfrac{WS(q_i) - L_{WS}}{U_{WS} - L_{WS}} & \text{for } L_{WS} \le WS(q_i) \le U_{WS} \\ 0 & \text{for } WS(q_i) \ge U_{WS} \end{cases}$$

Step 5: Now decision maker may use positive weight to reflect their decision regarding the relative importance of each fuzzy goals.

Step 6: Using the above MFs and positive weights, the model in Eq. 35 is reduced to an equivalent crisp non-linear programming problem as follows:
Max α
subject to

$$w_1 * \left(1 + \frac{PF(q_i) - U_{PF}}{U_{PF} - L_{PF}}\right) \ge \alpha$$
$$w_2 * \left(1 - \frac{WS(q_i) - L_{WS}}{U_{WS} - L_{WS}}\right) \ge \alpha$$

$$\sum_{i}^{n} C_{1i}\ H(q_i) \le THC \qquad i \in I_1 \cup I_2$$
$$\sum_{i}^{n} C_{2i}\ S(q_i) \le TSC \qquad i \in I_1 \cup I_2 \qquad (36)$$

$$\alpha \in (0,1) \quad w_1 + w_2 = 1.$$

Fuzzy Additive Goal Programming algorithm (FAGP) (Bhunia and Maiti, 1997) to solve crisp model: To solve the crisp model Eq. 35 besides WFNLP technique, FAGP (Tlwari *et al.*, 1987) technique can be used. First convert the crisp model Eq. 35 into WFNLP model Eq. 36. Now the model is formulated as:

$$\text{Max } P(\mu_1, \mu_2) = (w_1\, \mu_1 + w_2\, \mu_2)$$

subject to

$$(1+\frac{PF(q_i)-U_{PF}}{U_{PF}-L_{PF}})=\mu_1 \quad (37)$$

$$(1-\frac{WS(q_i)-L_{WS}}{U_{WS}-L_{WS}})=\mu_2$$

$$\sum_i^n C_{1i}\ H(q_i)\le THC \qquad i\in I_1\cup I_2$$

$$\sum_i^n C_{2i}\ S(q_i)\le TSC \qquad i\in I_1\cup I_2$$

μ_1, μ_2 ϵ (0,1), where, P (μ_1, μ_2) be a simple additive fuzzy achievement.

Integrated Goal Attainment and Fuzzy Non-linear Programming(GAFNLP) to solve crisp model: The GAFNLP (Das *et al.*, 2003) technique can be used in crisp multi-objective non-linear programming problem. First convert the model in Eq. 35 in to crisp problem as in Eq. 36, then the problem is formulated based on Das *et al.* (2003) as follows:
Min β
subject to

$$w_1*(1+\frac{PF(q_i)-U_{PF}}{U_{PF}-L_{PF}})\ \ge\alpha$$

$$w_2*(1-\frac{WS(q_i)-L_{WS}}{U_{WS}-L_{WS}})\ \ge\alpha$$

$$\sum_i^n C_{1i}\ H(q_i)\le THC \qquad i\in I_1\cup I_2$$

$$\sum_i^n C_{2i}\ S(q_i)\le TSC \qquad i\in I_1\cup I_2 \quad (38)$$

α ϵ (0,1), α + β ≥ α_0, β ≥ 0.
Here, α_0 is the goal of the objective function in Eq. 36.

Numerical results of Model-1 of the above three techniques are presented in the study, with different weights to objectives.

FUZZY MODEL

When the profit, wastage cost and constraints such as allowable total holding and total shortage cost become imprecise, the crisp model Eq. 35 is transformed to

$$M\tilde{a}x PF(q_i)$$

$$M\tilde{i}n WS(q_i)$$

subject to inequality constraints

$$\sum_i^n C_{1i}\ H(q_i)\le TH\tilde{C} \qquad i\in I_1\cup I_2$$

$$\sum_i^n C_{2i}\ S(q_i)\le TS\tilde{C} \qquad i\in I_1\cup I_2 \quad (39)$$

Here, to solve the above fuzzy, given by the Eq. 39, we use the Weighted fuzzy non- linear programming (WFNLP) method, Fuzzy Additive Goal Programming (FAGP) technique and Integrated goal attainment and fuzzy non-linear programming (GAFNLP) method.

Weighted fuzzy non-linear programming technique to solve fuzzy model (WFNLP): In fuzzy set theory, the fuzzy objectives and fuzzy constraints are defined by their membership functions.

Here, we assume μ_{PF} (q_i), μ_{WS} (q_i) and μ_{TSC} q_i and are the linear membership functions for the objectives and 2 constraints, respectively.

$$\mu_{PF}(q_i)=\begin{cases}0 & \text{for } PF(q_i)<U_{PF}-L_{PF}\\ 1+\frac{PF(q_i)-U_{PF}}{P_{PF}} & \text{for } U_{PF}-L_{PF}\le PF(q_i)\le U_{PF}\\ 1 & \text{for } PF(q_i)>U_{PF}\end{cases}$$

$$\mu_{WS}(q_i)=\begin{cases}0 & \text{for } WS\ (q_i)>L_{WS}+P_{WS}\\ 1-\frac{WS(q_i)-L_{WS}}{P_{WS}} & \text{for } L_{WS}\le WS\ (q_i)\le L_{WS}+P_{WS}\\ 1 & \text{for } WS\ (q_i)<L_{WS}\end{cases}$$

$$\mu_{THC}(q_i) = \begin{cases} 0 & \text{for } \sum_{i=1}^{n} C_{1i}\, H(q_i) > THC + P_{THC} \\ 1 - \dfrac{\sum_{i=1}^{n} C_{1i} H(q_i) - THC}{P_{THC}} & \text{for } THC \le \sum_{i=1}^{n} C_{1i} H(q_i) \le THC + P_{THC} \\ 1 & \text{for } \sum_{i=1}^{n} C_{1i}\, H(q_i) < THC \end{cases}$$

$$\mu_{TSC}(q_i) = \begin{cases} 0 & \text{for } \sum_{i=1}^{n} C_{3i}\, S(q_i) > TSC + P_{TSC} \\ 1 - \dfrac{\sum_{i=1}^{n} C_{3i} S(q_i) - TSC}{P_{TSC}} & \text{for } TSC \le \sum_{i=1}^{n} C_{3i}\, S(q_i) \le TSC + P_{TSC} \\ 1 & \text{for } \sum_{i=1}^{n} C_{3i}\, S(q_i) < TSC \end{cases}$$

where, $P_{PF} = U_{PF}$-L_{PF} and $P_{WS} = U_{WS}$ - L_{WS}. Here p_{PF} is the minimum and P_{WS}, P_{THC} and P_{TSC} are the maximum acceptable violation of the aspiration level of U_{PF} and L_{WS}, THC, TSC, respectively.

Now, using the WFLP technique, the solution of fuzzy multi-objective inventory model can be obtained from

Max α

subject to

$$\begin{aligned} & w_1 * \left(1 + \frac{PF(q_i) - U_{PF}}{P_{PF}}\right) \ge \alpha \\ & w_2 * \left(1 - \frac{WS(q_i) - L_{WS}}{P_{WS}}\right) \ge \alpha \\ & w_3 * \left(1 - \frac{\sum_{i=1}^{n} C_{1i}\, H(q_i) - THC}{P_{THC}}\right) \ge \alpha \\ & w_4 * \left(1 - \frac{\sum_{i=1}^{n} C_{3i}\, S(q_i) - TSC}{P_{TSC}}\right) \ge \alpha \end{aligned} \quad (40)$$

$\alpha \in (0,1)$, where, $w_1 + w_2 + w_3 + w_4 = 1$, $w_1 + w_2 + w_3$ and w_4 are positive weight for the objectives and constraints, respectively.

Fuzzy additive goal programming technique to solve fuzzy model (FAGP): Again following the FAGP based on Tiwari *et al*. (1987) the above problem can be formulated as:

$$\text{Max}P(\mu_1, \mu_2, \mu_3, \mu_4) = w_1\mu_1 + w_2\mu_2 + w_3\mu_3 + w_4\mu_4$$

subject to

$$\begin{aligned} & \left(1 + \frac{PF(q_i) - U_{PF}}{P_{PF}}\right) = \mu_1 \\ & \left(1 - \frac{WS(q_i) - L_{WS}}{P_{WS}}\right) = \mu_2 \\ & \left(1 - \frac{\sum_{i=1}^{n} C_{1i}\, H(q_i) - THC}{P_{THC}}\right) = \mu_3 \\ & \left(1 - \frac{\sum_{i=1}^{n} C_{3i}\, S(q_i) - TSC}{P_{TSC}}\right) = \mu_4 \end{aligned} \quad (41)$$

where, μ_1, μ_2, μ_3, $\mu_4 \in (0,1)$ and $P(\mu_1, \mu_2, \mu_3, \mu_4)$ be a simple additive fuzzy achievement function .

Integrated goal attainment and fuzzy non-linear programming technique to solve fuzzy mode (GAFNLP): To solve the fuzzy model besides WFNLP, FAGP technique, based on Das *et al*. (2003), the FAGP method can be used. First convert the fuzzy model Eq. 39 into crisp problem as in Eq. 40. Now, if α_0 be the goal of the objective function Eq. 40, then the problem is constructed as:

Min β

subject to

$$w_1 * (1 + \frac{PF(q_i) - U_{PF}}{P_{PF}}) \geq \alpha$$

$$w_2 * (1 - \frac{WS(q_i) - L_{WS}}{P_{WS}}) \geq \alpha$$

$$w_3 * (1 - \frac{\sum_{i=1}^{n} C_{1i} H(q_i) - THC}{P_{THC}}) \geq \alpha$$

$$w_4 * (1 - \frac{\sum_{i=1}^{n} C_{3i} S(q_i) - TSC}{P_{TSC}}) \geq \alpha, \alpha + \beta \geq \alpha_0, \beta \geq 0 \quad (42)$$

With above 3 techniques, the fuzzy model is illustrated numerically. The results are presented in the study, for different weights to objectives and constraints.

The Eq. (36-39 and 40-42) are highly non-linear equations. It is difficult to solve the cost functions analytically. To solve them standard soft ware package LINGO is used.

WEIGHTS IN NON-LINEAR PROGRAMMING

When several objectives appear in an inventory model, all the objectives may or may not be equally important to the decision maker and optimum compromise decision for the system varies depending upon the weights/importance attached to the objective by decision maker. Here, positive weights w_i ((i = 1, 2) for model -1 and i = (1, 2, 3, 4) for model-2) reflect the decision maker's preference regarding the relative importance of each fuzzy goal. Smallest of the i-weighted MFs gives the most importance to the respective objective than others. These weights can be normalized by taking

$$\sum_{i=1}^{n} w_i = 1$$

Numerical examples: To illustrate the above models, following input data is considered. For simplicity, it is assumed that there are two items (i.e., n = 2) of which demand of the first item depends on inventory level dependent and that for the second item is constant demand. For both the models, the required different parameter with their appropriate units is in Table 1.

In addition to these data, values of other required parameters specially for the fuzzy models are U_{PF} = \$180, L_{WS} = \$19, THC = \$228, TSC = \$200, P_{PF} = \$20, P_{WS} = \$3, P_{THC} = \$157 and P_{TSC} = \$220. Also for crisp model THC = \$38s and TSC = \$420. Moreover, to solve GAFNLP technique in crisp and fuzzy model, we assumed the aspiration level α_0 = 0.07. By using WFNLP,FAGP and GAFNLP techniques, with different weights to objectives in crisp model and to objectives and to objectives and constraints fuzzy model, we obtain optimum values. The optimum values are presented in Table 2 and 3. In this numerical representation, another criterion,(ie) ratio of profit to the total cost has been considered to classify the suitability of the methods. Each technique has some limitations with respect to its applicability and due to that a particular method does not yield the best results for all models. Table 4 gives the most suitable techniques to be adopted for the solution of a particular model.

Table 1: Different parameters with their appropriate units

Items	S_i(in\$)	P_i(in\$)	K_i	θ_i	ϕ_i	C_{1i}(in\$)	C_{3i}(in\$)	C_{2i}(in\$)	D_i	γ_i	T_i
1	15	10	30	0.04	0.03	4	20	5	26	0.03	9
2	12	8	38	0.04	0.02	2	25	3	30	-	10

Table 2: Optimum results due to different weight in crisp model

Method	Weight of objective w = (w_1, w_2)	PF(q_i)	WS(q_i)	$\sum_i^n C_{1i} * H(q_i)$	$\sum_i^n C_{2i} * S(q_i)$	Profit ratio
WNLP		174.1128	19.5887	303.7780	293.0544	0.30059
FAGP	.5, .5	174.1306	19.0000	304.1979	292.5014	0.30063
GAFNLP		162.8000	19.9591	279.2788	420.0000	0.27628
WNLP		175.7650	20.0587	385.0000	200.8967	0.30249
FAGP	.1, .9	169.7681	19.4096	237.3089	392.8227	0.29260
GAFNLP		174.0000	20.0287	385.0000	219.0475	0.29830
WNLP		168.2608	19.4009	221.8802	420.0000	0.28964
FAGP	.9, .1	175.7058	19.9492	36902602	216.6124	0.30227
GAFNLP		166.2848	19.6000	245.7033	420.0000	0.28456

Table 3: Optimum results due to different weight in fuzzy model

Method	Weight of objective $w = (w_1, w_2)$	$PF(q_i)$	$WS(q_i)$	$\sum_i^n C_{1i} * H(q_i)$	$\sum_i^n C_{2i} * S(q_i)$	Profit ratio
WNLP		173.8056	19.5894	296.7219	300.8516	0.30016
FAGP	.25,.25,.25,.25	174.3862	19.6209	309.5436	284.7989	0.30110
GAFNLP		169.2000	20.3312	339.3600	289.8466	0.28980
WNLP		175.4361	19.8193	347.1980	239.8229	0.30259
FAGP	.1, .3, .3, .3	173.0078	19.5093	280.4770	323.9931	0.29873
GAFNLP		175.4361	19.8183	347.1980	239.8230	0.30259
WNLP		171.2849	19.4372	255.8101	362.6962	0.29547
FAGP	.3, .1, .3, .3	175.1108	19.7365	332.1263	256.9259	0.30221
GAFNLP		173.6575	19.6000	295.4662	302.7265	0.29992
WNLP		171.2743	19.4599	254.9975	361.5122	0.29557
FAGP	.3, .3, .1, .3	175.6751	19.9286	367.0973	219.1715	0.30265
GAFNLP		168.5000	19.8242	270.9000	362.2775	0.28962
WNLP		175.2973	19.8081	341.8883	245.4373	0.30244
FAGP	.3, .3, .3, .1	171.5789	19.4487	258.4934	356.8576	0.29609
GAFNLP		171.4540	20.2221	346.9667	266.0000	0.29440

Table 4: Appropriate methods for optimum results for different weights with respect to profit, wastage cost, holding cost and shortage cost

	Equal weightage		More weightage to profit		More weightage to wastage cost		More weightage to total holding cost		More weightage to total shortage cost	
Environment	PF (ratio)	WS	PF (ratio)	WS	PF (ratio)	WS	PF (ratio)	WS	PF (ratio)	WS
Crisp	FAGP	FAGP	WFNLP	FAGP	FAGP	WFNLP	------		------	
Fuzzy	FAGP	FAGP	WFNLP	FAGP (or) GAFNLP	FAGP	WFNLP	FAGP	WFNLP	FAGP	WFNLP

CONCLUSION

So for, a very little research has been done for the solution of muti-objective fuzzy non-linear very few research papers are available for the solution of multi-objective fuzzy non-linear problems in production inventory models. As for as our knowledge is concerned, for first time the multi- objective production inventory problems with limitations of total holding cost and total shortage cost with varies types of demands have been constructed in crisp and fuzzy environment and solved by WFNLP, FAGP and GAFNLP techniques . The results are presented with different types of weights to objectives in first model and to objectives and constraints in second model in the fuzzy environment. Exact weight which implies the relative importance for these goals can be determined through practical experience. Though the models considered here is multi-objective production inventory models of deteriorating items with various types of demands with shortages and finite production rate, the present model can be extended to other types of inventory models with characteristics like discount, inflation rate, infinite production rate and fuzzy-stochastic environment. This technique is an appropriate tool to tackle the real life inventory problems in realistic environment.

NOTATIONS

The following notations have been used in developing the models.

K_i = Production quantity of the i-th item per unit time.

Q_i = The maximum production quantity of the i-th item at $t = t_{1i}$.

Sq_i = The maximum shortage quantity of the i- th item at $t = t_{3i}$.

P_i = Production cost of each product of i-th item.

S_i = Selling cost of each product of i-th item.

C_{1i} = Holding cost per unit quantity per unit time of i-th item.

C_{3i} = Set-up cost for i-th item per cycle.

C_{2i} = Shortage cost of each product of i-th item.

THC = Maximum permissible total holding cost per cycle.

TSC = Maximum permissible total shortage cost per cycle.

T_i = Time period for each cycle for the i-th item .

θ_i = Deterioration rate of the on-hand inventory per unit time of the i-th item.

ϕ_i = Deterioration fraction of production rate per unit time of the i-th item.

PF $(_{qi})$ = Total average profit of the i-th item.

WS $(_{qi})$ = Total wastage cost of the i-th item.

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