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Transmission of Electron Charge Carrier in Type I MQW System

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Abstract: Transmission of electron charge carrier in type I MQW system has been investigated by solving the effective mass equation including off-diagonal effective mass tensor elements. The boundary condition for an electron wave function (under the effective mass approximation) at a hotero junction interface is suggested and included in the calculation. The calculation reveals that the transmission coefficient is dependent on potential well depth and it is not symmetric with the angel of incidence.

Key words: Transmission, electron, coefficient, incidence, symmetric, Nigeria

INTRODUCTION

The tunnelling phenomenon through a potential barrier has been discussed over the last half century and also is of present day interest in the study of charge transport in superlative and hetero-structures. One of the present researchers investigated transmission in Kroning-Penney model for SL and MQW systems (Adelabu and Ajala, 2005). Lee calculated the transmission coefficient and tunnelling time of election both under normal incidence through a one-dimensioned potential barrier considering the different effective masses at hereto-junction interface (Lee, 1993) and under non-normal incidence through a one dimensional potential barrier assuming constant effective mass (Lee and Lee, 1998). Recently, Paranjape (1995) studied the two-dimensional tunnelling time of an electron through an isotropic hetero-structure potential barrier with different effective masses and calculated the transmission coefficient. The researcher showed that the conservation of transverse (parallel to hereto-structure interface) momentum of the hetero-junction with different effective masses implies the loss or gain of kinetic energy for that direction and the energy difference is conferred on the longitudinal (perpendicular to the hetero structure interface) kinetic energy, helping or hindering the tunnelling. Therefore, under the effective-mass approximation, the longitudinal motion of an electron is coupled to transverse momentum due to different effective masses at the hetero-junction interface.

However, besides the energy coupling characteristic due to the hetero structure such coupling of the longitudinal motion to the transverse motion also exists in anisotropic materials via the off-diagonal effective mass tensor elements are well dependent which suggests that the coupling effects are not identical in all wells. Therefore, it is necessary to investigate its effect on tunnelling phenomenon quite rigorously. In this study we calculate and discuss the transmission coefficient of an electron charge carrier through multiple quantum well system. The use of the term multiple quantum well implies that the wells act independently of one another but this is only so if the barriers separating the wells are thick enough to prevent appreciable quantum mechanical tunnelling. If tunnelling occurs, this has the result of broadening the energy levels of the individual wells into minibands and thus te researchers speak of a superlattice.

MATERIALS AND METHODS

Theoretical considerations and calculations: The Hamiltonian for the general anisotropic material is (Yi and Quinn, 1983a, b):

$$H = \frac{1}{2m_e} P^T \bullet \alpha(r) \bullet P + V(r) \tag{1}$$

Where:

m_e = The mass of free electron
P = The momentum vector
$(1/m_e)\,\alpha$ = The inverse effective-mass tensor

It is noteworthy that in considering the spatially varying effective-mass tensor, the researchers simply place it without rigorous justification between the two derivative (momentum) operators as an extension of the scalar effective mass case. Figure 1 shows the potential profile we assumed in the normal direction (Z-direction)

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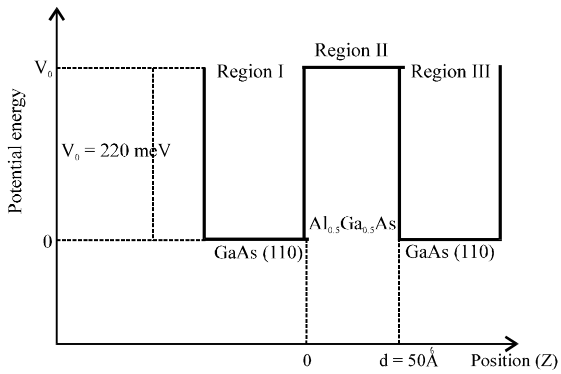

Fig. 1: The model used in the numerical calculation

to the layer. The electron is incident from region I to the potential barrier. The effective mass of the electron and potential are dependent only on the Z-direction. The wave function of the effective-mass equation with the Hamiltonian of Eq. 1 is given as:

$$\Psi(r) = \phi(Z)\exp(-i\gamma z)\exp\{i(K_x x + K_y y)\} \quad (2)$$

Where:

$$\gamma = \frac{K_x\alpha_{xz} + K_y\alpha_{yz}}{\alpha_{zz}} \quad (3)$$

ϕ (z) satisfies the one-dimensional Schrodinger like equation in the problem.

$$\frac{-\hbar^2}{2m_e}\alpha_{zz,\ell}\frac{\partial^2\phi(Z)}{\partial Z^2} + V(Z)\phi(Z) = E_z\phi(Z) \quad (4)$$

Where, the subscript l in $\alpha_{zz,\,1}$ denotes each region in Fig. 1 and:

$$E_z = E - \frac{\hbar^2}{2m_e}\sum_{i,j\in\{x,y\}}\beta_{ij}k_ik_j \quad (5)$$

$$\beta_{ij} = \alpha_{ij} - \frac{\alpha_{iz}\alpha_{zj}}{\alpha_{ZZ}} \quad (6)$$

The above equations are instructive in that K_z (say, ϕ (z) = exp (±ik'Z) in region I) is separated into two parts: on (-γ) linked to k_x and k_y and another (±k') that is independent of the momentum in the x or y direction. If the election has momentum hk_x and hk_y, respectiviely in the x and y directions, the off-diagonal effective-mass tensor elements α_{xz} and α_{yz}.

Their values are dependent on the well to which the electron belongs (In most cases, the magnitudes are the same for all Quantum wells but signs are not the same). Therefore, even though some electrons may have the same momentum in the x and y directions, the liked Z-directional momentum is dependent on the wells where the electrons are. In short, the motion in the x and y directions is not independent of that in the Z-direction but they are mutually coupled by the off diagonal-mass tensor elements. Note that k_x and k_y are conserved through all region in Fig. 1 as in Paranjape (1995). This is not as obvious as in the isotropic cases since, according to Newton's law:

$$\left[{}^{dv}\!/_{dt} = \left(\frac{1}{m_e}\right)\alpha F\right]$$

The force dependent on only the Z-direction can change the transverse velocities. However, it can be shown that k_x and k_y are conserved even though v_x and v_y are not.

The next problem is to find the appropriate boundary conditions for the Hamitionian in Eq. 1. In one-dimensional heterostructure where the potential and effective-mass tensor are dependent only on the Z-coordinate, the effective-mass equation is:

$$\frac{-\hbar^2}{2m_e}\sum_{i,j\in(x,y,z)}\left[\frac{\partial}{\partial i}\alpha_{ij}(z)\frac{\partial}{\partial i}\right]\psi(x,y,z) = \left[E - V(z)\right]\psi(x,y,z) \quad (7)$$

where, V (z) is the contribution of both the condition-band edge and the xexternal field. In the problem where α_{ij} and V are dependent only on the z coordinate:

$$\left({}^{\partial}\!/_{\partial i}\right)\left\{\alpha_{ij}(z)\,{}^{\partial\psi}\!/_{\partial j}\right\}(i,j\in\{x,y\})$$

does not have any δ-function singularity because the differential operations are taken in the same layer (with the condition that ϕ is continuous).

$$\left({}^{\partial}\!/_{di}\right)\left\{\alpha_{iz}(z)\,{}^{\partial\psi}\!/_{\partial z}\right\}(i\in\{x,y\})$$

does not have any δ-function singularity with the continuity condition of ψ because the;

$${}^{\partial}\!/_{\partial i}\ (i,j\in\{x,y\})$$

operation is taken in the same layer. Therefore, the requirement condition reduces to the condition that:

$$\left(\partial/\partial z\right)\sum_{j\in\{x,y,z\}}\left\{\alpha_{zj}(z)\partial\psi/\partial j\right\}$$

should not have any δ-function singularity i.e.,

$$\sum_{j\in\{x,y,z\}}\left\{\alpha_{zj}(z)\partial\psi/\partial j\right\}$$

is continuous along the Z-direction. With these boundary conditions the researchers obtain the transmission amplitude T_a as:

$$T_a = G\exp(i\varphi) \quad (8)$$

Where:

$$G = \begin{cases} \dfrac{2k'\gamma'}{\left[p^2\sinh^2(r'd)+4(k'\gamma')^2\cosh^2(\gamma'd)\right]^{1/2}}, (\gamma')^2 \succ 0 \\ \dfrac{2k'\gamma'}{\left[Q^2\sin^2(r''d)+4(k'\gamma'')^2\cos^2(\gamma''d)\right]^{1/2}}, (\gamma')^2 \prec 0 \end{cases} \quad (9)$$

$$\varphi = \begin{cases} \tan^{-1}\left(\dfrac{p}{2k'\gamma'}\tanh(\gamma'd)\right) - k'd + (\gamma_1-\gamma_2)d, (\gamma')^2 \succ 0 \\ \tan^{-1}\left(\dfrac{Q}{2k'r''}\tan(\gamma''d)\right) - k'd + (\gamma_1-\gamma_2)d, (\gamma')^2 \prec 0 \end{cases} \quad (10)$$

$$P = \frac{\alpha_{zz,I}}{\alpha_{zz,II}}(k')^2 - \frac{\alpha_{zz,II}}{\alpha_{zz,I}}(\gamma')^2 \quad (11)$$

$$Q = \frac{\alpha_{zz,I}}{\alpha_{zz,II}}(k')^2 + \frac{\alpha_{zz,II}}{\alpha_{zz,I}}(\gamma'')^2 \quad (12)$$

Here, d is the width of the barriers (Fig. 1) and:

$$(\gamma')^2 = \frac{2m_e}{\hbar^2}\cdot\frac{1}{\alpha_{zz,II}}V_0 - \frac{\alpha_{zz,I}}{\alpha_{zz,II}}(k')^2 - \frac{1}{\alpha_{zz,II}}\sum_{i,j\in\{x,y\}}(\beta_{ij,I}-\beta_{ij,II})k_ik_j \quad (13)$$

$$(\gamma'')^2 = -(\gamma')^2 \quad (14)$$

$$(k')^2 = \frac{2m_eE_z}{\hbar^2}\cdot\frac{1}{\alpha_{zz,I}} \quad (15)$$

The $(\gamma')^2$ is positive (negative) when the kinetic energy of the incident electron is lower (higher) than the potential barrier.

The model used as an example in the numerical calculation is shown in Fig. 1. There is a strained

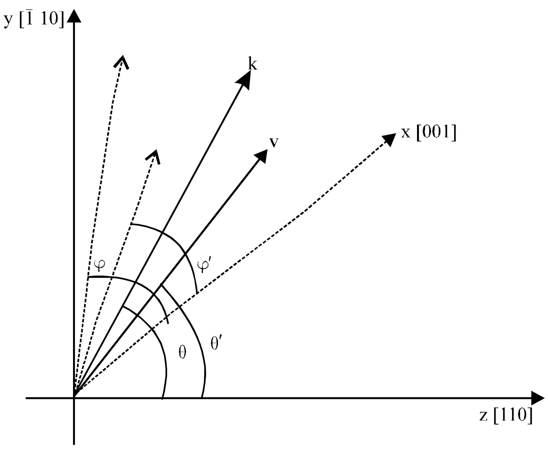

Fig. 2: The coordinates used in the analysis. Where the electron hits the barrier, the researchers take as the origin of the coordinate system

Table 1: Inverse effective-mass tensors used in the numerical calculation

Valleys	Regions I and III	Region II
1	$\begin{bmatrix} 6.58 & 0 & 0 \\ 0 & 3.93 & 2.65 \\ 0 & 2.65 & 3.93 \end{bmatrix}$	$\begin{bmatrix} 8.06 & 0 & 0 \\ 0 & 5.70 & 3.43 \\ 0 & 3.43 & 5.70 \end{bmatrix}$
2	$\begin{bmatrix} 6.58 & 0 & 0 \\ 0 & 3.93 & -2.65 \\ 0 & -2.65 & 3.93 \end{bmatrix}$	$\begin{bmatrix} 8.06 & 0 & 0 \\ 0 & 5.70 & -3.43 \\ 0 & -3.43 & 5.70 \end{bmatrix}$

(AlGaAs) potential barrier grown on GaAs (110). The width of the barrier is 50 Å and the conduction-band discontinuity is taken as 220 meV.

There are four equivalent valleys in the conduction band of Ga As (110) with a strained $Al_{0.5}Cra_{0.5}As$ potential barrier (Einevoll *et al*., 1990). The effective-mass tensor elements of these four valleys are not the same. There are two groups of valleys in GaAs (110). Each of them has two valleys. In both groups of valleys, α_{xz} is zero (Fig. 2). However, one group has positive α_{yz} while the other has negative α_{yz} (Yi and Quinn, 1983a, b). Therefore, the calculated results are dependent on the group to which the electron belongs. For simplicity, the researchers denote the group that has positive α_{yz} as valley 1 and the other as valley 2. The effective masses used in the example are shown in Table 1 where, the effective-mass tensor elements of $Al_{0.5}Ga_{0.5}As$ are calculated through linear interpolation as for the case of $Si_{0.5}Ge_{0.5}$ (Shur, 1990). Figure 2 shows the chosen coordinate system. It is note-worthy that k (the wave vector of incident electron) and v (the velocity vector of the incident electron) are not necessarily parallel. The unprimed and primed angle coordinates are used for k and v, respectively.

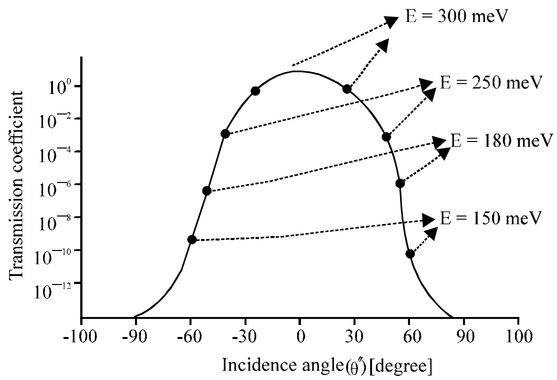

Fig. 3: The numerical values of the transmission coefficient for the angle of incidence varying from −90° to 90° with incident energies of 150, 180, 250 and 300 meV

RESULTS AND DISCUSSION

Figure 3 shows the numerical values of the transmission coefficients ($=T_a^*T_a$) for the angel of incidence for v varying form −90° to 90° with incident energies of 150 and 180 meV (which is lower than the potential barrier) and 250 and 300 meV (higher than the potential barrier). The incidence angles are θ′ and φ′ but the researchers fix φ′ to π/2 for simplicity and change only θ′.

The most remarkable result is that the transmission coefficient is dependent on the valley where the electron belongs. This suggests a method of determining the valley where the electrons are and also the valley occupancy (the relative population of electrons in that specific valley) of the electrons. It is also noteworthy that the transmission coefficient as maximum not for normal incidence but for about ±30° incidence (The sign ± corresponds to valley 1 and 2, respectively. This difference in direction also indicates the anisotropy of the material).

The researchers also see that in all valleys the transmission coefficient is not symmetric with the change of sign of incidence angle (θ′→-θ′) which confirms the anisotropy of the material.

CONCLUSION

In this study, the researchers have discussed the transmission of electron charge carrier in type 1 MQW system grown on an anisotropic material under a normal incidence. The researchers included the effect of different effective masses at heterojunction interfaces. The boundary condition for an electron wave function (under the effective-mass approximation) at a superlattice heterojunction is suggested and included in the calculation. The numerical calculations are done with a $Al_{0.5}Ga_{0.5}$ as potential barrier grown on GaAs (110). The calculation shows that the transmission coefficient is dependent on the valley and it is not symmetric with the angle of incidence.

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