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Non-Newtonian Flow of Blood Through an Atherosclerotic Artery

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Abstract: An attempt has been made to investigate the Non-Newtonian behavior on blood flow through a stenosed artery using Power-law fluid model. Numerical illustration presented at the end of the paper provides the results for the resistance to flow, apparent viscosity and the wall shear stress through their graphical representations. It has been shown that the resistance to flow, apparent viscosity and wall shear stress increases with the size of the stenosis but these increases is comparatively small due to Non-Newtonian behavior of the blood indicating the usefulness of its rheological character in the functioning of the diseased arterial circulation.

Key words: Non-Newtonian fluid, stenosis, power-law fluid, resistance to flow, apparent viscosity, India

INTRODUCTION

Atherosclerosis is the leading cause of death in many countries. There is considerable evidence that vascular fluid dynamics plays an important role in the development and progression of arterial stenosis which is one of the most widespread diseases in human beings. The fluid mechanical study of blood flow in artery bears some important aspects due to the engineering interest as well as the feasible medical applications.

The hemodynamic behavior of the blood flow is influenced by the presence of the arterial stenosis. If the stenosis is present in an artery, normal blood flow is disturbed. A Newtonian fluid, by definition is one in which the coefficient of viscosity is constant at all rates of shear. Homogeneous liquids may behave closely like Newtonian fluids. However, there are fluids that do not obey the linear relationship between shear stress and shear strain rate. Fluids that exhibit a non-linear relationship between the shear stress and the rate of shear strain are called Non-Newtonian fluids. Blood behaviour is referred to as Non-Newtonian properties. These properties are of two types as follows:

- At low shear rates, the apparent viscosity increases markedly-Sometimes even a certain yield stress is required for flow
- In small tubes, the apparent viscosity at higher rates of shear is smaller than it is in larger tubes

These two types of anomalies are often referred to as low shear and high shear effects, respectively. It is thus concluded that the behaviour of blood is almost Newtonian at high shear rate while at low shear rate the blood exhibits yield stress and non-Newtonian behaviour. In the series of the studies (Texon, 1957; Hershey and Cho, 1966; Forrester and Young, 1970; Caro *et al*., 1971) the effects on the cardiovascular system can be understood by studying the blood flow in its vicinity. In these studies the behavior of the blood has been considered as a Newtonian fluid. However, it may be noted that the blood does not behave as a Newtonian fluid under certain conditions. It is generally accepted that the blood being a suspension of cells, behaves as a Non-Newtonian fluid at low shear rate (Fry, 1972; Young and Tsai, 1973; Lee, 1974). It has been pointed out that the flow behaviour of blood in a tube of small diameter (<0.2 mm) and at <20 sec^{-1} shear rate can be represented by a power-low fluid (Kirkeeide *et al*., 1977; Charm and Kurland, 1965; Lih, 1975). It has also been suggested that at low shear rate (0.1 sec^{-1}) the blood exhibits yield stress and behaves like a Herschel-Bulkely fluid (May *et al*., 1963; Casson, 1959; Reiner and SottBaldair, 1959).

For blood flows in large arterial vessels (i.e., vessel diameter ≥1 mm) (LaBarbera, 1990; Haldar, 1985; Pontrelli, 2001) which can be considered as a large deformation flow, the predominant feature of the rheological behavior of blood is its shear rate dependent viscosity and its fact on the hemodynamics of large arterial vessel flows has not been understood well (Young, 1968; Tandon *et al*., 1991; Jung *et al*., 2004). In this study the effect of Non-Newtonian behaviour of blood flow has been investigated by considering blood as a power-law fluid model.

Formulation of the problem: In the present analysis, it is assumed that the stenosis develops in the arterial wall in an axially non-symmetric but radially symmetric manner

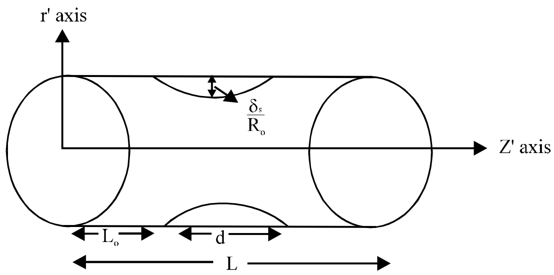

Fig. 1: Stenotic artery

and depends upon the axial distance z and the height of its growth. In such a case the radius of artery, R (z) can be written as follows (Fig. 1):

$$\left.\begin{aligned} \frac{R(z)}{R_0} &= 1 - A[L_0^{(m-1)}(z-d) - (z-d)^m], \\ d \le z \le d + L_0 \quad &= 1, \qquad \text{otherwise,} \end{aligned}\right\} \quad (1)$$

Where:

- R (z), R_0 = The radius of the artery with and without stenosis, respectively
- L_0 = The stenosis length and d indicates its location
- $m \ge 2A$ = The parameter determining the stenosis shape and is referred to as stenosis shape parameter

Axially symmetric stenosis occurs when m = 2 and a parameter A is given by:

$$A = \frac{\delta}{R_0 L_0^m} \frac{m^{m/(m-1)}}{(m-1)}$$

Where, δ denotes the maximum height of stenosis at:

$$z = d + L_0 m^{-1/(m-1)}$$

Conservation equation and boundary condition: The equation of motion for laminar and incompressible, steady, fully-developed, one-dimensional flow of blood whose viscosity varies along the radial direction in an artery reduces to Young (1968):

$$\left.\begin{aligned} 0 &= -\frac{\partial P}{\partial r} + \frac{1}{r}\frac{\partial (r\tau)}{\partial z} \\ 0 &= -\frac{\partial P}{\partial r} \end{aligned}\right\} \quad (2)$$

Where:

- z, r = Co-ordinates with z measured along the axis
- r = Measured normal to the axis of the artery

Following boundary conditions are introduced to solve the above equations:

$$\left.\begin{aligned} &\partial u/\partial r = 0 \quad \text{at } r = 0, u = 0 \quad \text{at } r = R(z) \\ &\tau \text{ is finite} \quad \text{at } r = 0, P = P_0 \quad \text{at } z = 0 \\ &P = P_L \quad \text{at } z = L \end{aligned}\right\} \quad (3)$$

ANALYSIS OF THE PROBLEM

Power-law fluid: Non-Newtonian fluid is that of power-law fluid which have constitutive equation:

$$\left.\begin{aligned} &\left(-\frac{du}{dr}\right) = \left(\frac{\tau}{\mu}\right)^{1/n} = f(\tau), \\ &\text{where:} \\ &\tau = \left(-\frac{dp}{dz}\right)\frac{Rc}{2} \end{aligned}\right\} \quad (4)$$

Where:

- u = The axial velocity
- μ = The viscosity of fluid
- -dp/dz = The pressure gradient
- n = The flow behaviour index of the fluid

Solving for u from Eq. 2, 4 and using the boundary conditions Eq. 3, we have:

$$\frac{du}{dr} = \left(\frac{P}{2\mu}\right)^{1/n} [(r - R_C)^{1/n}] \quad (5)$$

The volumetric flow rate Q can be defined as:

$$Q = \int_0^R 2\pi u r \, dr = \pi \int_0^R r\left(-\frac{du}{dr}\right) dr \quad (6)$$

By the help of Eq. 5 and 6, we have:

$$Q = \left(\frac{P}{2\mu}\right)^{1/n} \left(\frac{n\pi}{(3n+1)}\right) (R)^{[(1/n)+1]} \quad (7)$$

From Eq. 7 pressure gradient is written as follows:

$$\frac{dp}{dz} = -2\mu\left(\frac{(3n+1)}{n\pi}Q\right)^n \frac{1}{(R)^{3n+1}} \quad (8)$$

Integrating Eq. 8 using the condition $P = P_0$ at $z = 0$ and $P = P_L$ at $z = L$. We have:

$$P_L - P_0 = \left(\frac{(3n+1)}{np}Q\right)^n \frac{2\mu}{\left(R_0\right)^{3n+1}} \int_0^L \frac{dz}{\left(R/R_0\right)^{1+3n}} \quad (9)$$

The resistance to flow (resistive impedance) is denoted by λ and defined as follows (Shukla *et al*., 1980):

$$\lambda = \frac{P_L - P_0}{Q} \quad (10)$$

The resistance to flow from Eq. 10 using Eq. 9 can write as:

$$\lambda_0 = \left(\frac{(3n+1)Q}{np}\right)^n \frac{2\mu}{QR_0^{3n+1}}$$

$$\left(\int_0^d dz + \int_0^{d+L_0} \frac{dz}{\left(R/R_0\right)^{3n+1}} + \int_{d+L_0}^{L} dz\right) \quad (11)$$

When there is no stenosis in artery then $R = R_0$, the resistance to flow:

$$\lambda_N = \left(\frac{(3n+1)}{n\pi}Q\right)^n \frac{2\mu}{QR_0^{3n+1}} L \quad (12)$$

From Eq. 11 and 12 the ratio of (λ_0 / λ_N) is given as:

$$\lambda = \frac{\lambda_0}{\lambda_N} = 1 - \frac{L_0}{L} + \frac{1}{L} \int_d^{d+L_0} \frac{dz}{\left(R/R_0\right)^{3n+1}} \quad (13)$$

Now the ratio of shearing stress at the wall can be written as:

$$\frac{\tau_R}{\tau_N} = \left(\frac{R_0}{R}\right)^{-3n} \quad (14)$$

$$\tau = \frac{\tau_R}{\tau_N} = \frac{1}{\left(1 - \frac{\delta}{R_0}\right)^{3n}} \quad (15)$$

Figure 2 reveals the variation of resistance to flow (λ) with stenosis size (δ/R_0) for different values of flow behavior index (n). It is observed that the resistance to flow (λ) increases as stenosis size (δ/R_0) increases. It is also noticed here that resistance to flow (λ) increases as flow behavior index (n) increases. It is seen from the (Fig. 2 and 3) that the ratio is always >1 and decreases as n decreases from unity. This result is similar with the result of Shukla *et al*. (1980). In Fig. 3, resistance to flow (λ) decreases as stenosis shape parameter (m) increases and maximum resistance to flow (λ) occurs at (m = 2), i.e., in case of symmetric stenosis. This result is therefore, consisting to the result of Hershey *et al*. (1964) and Haldar (1985). It is also seen that for $\delta/R_0 = 0.1$ and $L_0/L = 1.0$. In Fig. 4 the variation of wall shear stress (τ) with stenosis length (L_0/L) for different values of flow behavior index (n) has been shown. This figure shows that wall shear stress (τ) increases as stenosis length increases. Also it has been seen from this graph that the wall shear stress (τ) increases as value of flow behavior index (n) increases (Whitmore, 1968; Srivastava and Mishra, 2010).

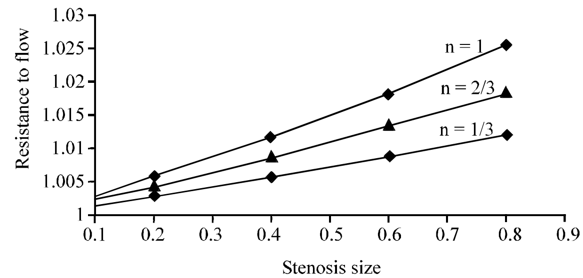

Fig. 2: Variation of resistance to flow with stenosis size for different value of n

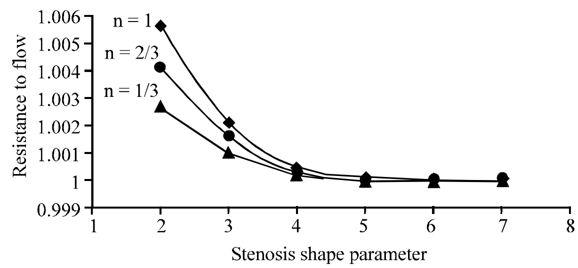

Fig. 3: Variation of resistance to flow with stenosis shape parameter for n

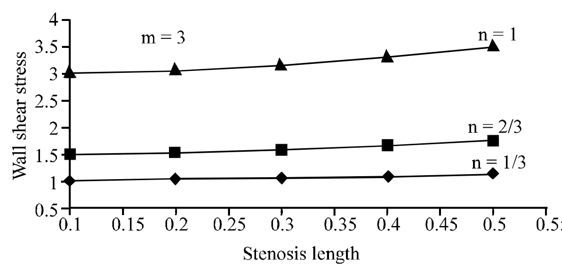

Fig. 4: Variation of wall shear stress with stenosis length for different value of n

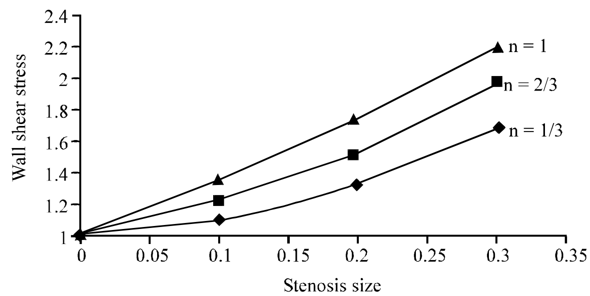

Fig. 5: Variation of wall shear stress with stenosis size for different value n

As the, stenosis grows, the wall shearing stress (τ) increases in the stenotic region. It is also noted that the shear ratio given by Lih (1975) is >1 and decreases as n decreases (n<1). These results are similar with the results of (Shukla *et al*., 1980). Figure 5 shows the variation of wall shear stress (τ) with stenosis size for different values of flow behavior index (n). This figure depicts that wall shear stress (τ) increases as stenosis size increases. Also it has been seen from this graph that the wall shear stress (τ) increases as value of flow behavior index (n) increases. These results are consistent to the observation of Shukla *et al*. (1980) and Mishra *et al*. (2010). It is also seen that the shear ratio is always >1 and decreases as n decreases. For δ/R_0 = 0.1 the increases in wall shear due to stenosis is about 37% when compared to the wall shear corresponding to the normal artery in the Newtonian case (n = 1) but for n = 2/3 this increase is only 23% approximately. However for δ/R_0 = 0.2, the corresponding increase in Newtonian (n = 1) and non-Newtonian (n = 2/3) cases are 95 and 56%, respectively.

CONCLUSION

Blood flow through an artery mainly depends on the pressure gradient and resistance to flow. Resistance to flow increases as the stenosis grows and remains constant outside the stenotic region. In this study researchers has studied the behavior of Non-Newtonian flow in an stenosed artery by considering the blood as power-law fluid model. It has been concluded that the resistance to flow and wall shear stress increases as the size of stenosis increases for a given Non-Newtonian model of the blood. These increases are however, small due to Non-Newtonian behaviour of the blood. It has also been concluded that the apparent viscosity increases as yield stress increases and decreases as stenosis shape parameter increases. The results were greatly influenced by the change of stenosis shape parameter. It appears that the Non-Newtonian behaviour of the blood is helpful in the functioning of diseased arterial circulation.

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