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H-Infinity Controller Based on LMI Region for Flexible Robot Manipulator

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Abstract: This study presents investigations into the development of H-infinity controller with LMI region schemes for trajectory tracking and vibration control of a Flexible Robot Manipulator System. A constrained planar single-link flexible manipulator is considered and the dynamic model of the system is derived using the assume mode method. H-infinity synthesis with pole clustering schemes is used to control the hub angle with very minimal vibration at the end-point of the manipulator. The performance of the flexible robot manipulator system is examined in terms of time response specifications of hub angle, minimal vibration at the end-point of the manipulator and minimal control input torque. To examine the effectiveness of the proposed controller, it is compared with hybrid input shaping with collocated PD controller schemes. The implementation results show that H-infinity controller with confined closed-loop poles based on LMI region produce a fast tracking capability with very minimal end-point vibration and smooth control input torque.

Key words: Flexible Robot Manipulator System, vibration control, trajectory tracking, H-infinity, LMI region

INTRODUCTION

In the last three decades, various attempts in controlling flexible robot manipulator system based on feed-forward and feedback control system have been proposed. For example, the development of computed torque based on a dynamic model of the system (Moulin and Bayo, 1991), utilisation of single and multiple-switch bang-bang control functions (Onsay and Akay, 1991), construction of input functions from ramped sinusoids or versine functions (Meckl and Seering, 1990). Moreover, feed-forward control schemes with command shaping techniques have also been investigated in reducing system vibration. These include filtering techniques based on low-pass, band-stop and notch filters (Tokhi and Azad, 1996) and input shaping (Singer and Seering, 1990; Mohamed and Tokhi, 2002; Mohamed *et al*., 2006). Poor results were obtained in these studies because feed-forward strategy is sensitive to the system parameters and could not compensate for the effect of disturbance. On the other hand, feedback control which is well known to be less sensitive to disturbances and parameter variations has also been adopted for controlling the flexible robot manipulator system. These include linear state feedback control (Hasting and Book, 1990), adaptive control (Yang *et al*., 1992), robust control technique (Moser, 1993), variable structure control (Moallem *et al*., 1998) and intelligent control based on neural networks (Gutierrez *et al*., 1998), fuzzy logic control schemes (Moudgal *et al*., 1994; Ahmad *et al*., 2008) direct strain feedback with PID approach (Mohd Tumari *et al*., 2001) and PID controller based on metamodeling approach (Ali *et al*., 2008). Recently, hybrid control schemes for flexible robot manipulator by combining both feed-forward and feedback control have also been introduced (Tokhi and Mohamed, 2003; Zain *et al*., 2006; Ahmad and Mohamed, 2008).

In this study, H-infinity controller with pole clustering based on LMI techniques is used to control the hub angle of flexible robot manipulator with very minimal vibration. In order to design the controller, the linear model of flexible robot manipulator system as shown in Fig. 1 is obtained. The reason for choosing H-infinity synthesis is because of its good performance in handling with various types of control objectives such as disturbance cancellation, robust stabilization of uncertain systems input tracking capability or shaping of the open-loop response. As we all know, a good time response specifications and closed-loop damping of flexible robot manipulator system can be achieved by forcing the closed-loop poles to the left-half plane. Moreover, many literatures have proved that H-infinity synthesis can be formulated as a convex optimization problem involving

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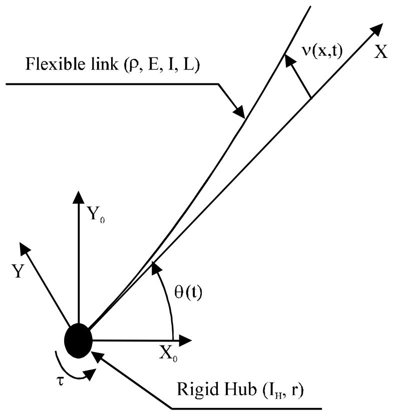

Fig. 1: Description of the flexible manipulator system

Linear Matrix Inequalities (LMI) (Gahinet and Apkarian, 1993; Iwasaki and Skelton, 1994; Packard, 1994). In this case, the normal Riccati equation with inequality condition was used. This behavior will give wide range of flexibility in combining several constraints on the closed loop system. This flexible nature of LMI schemes can be used to handle H-infinity controller with pole placement constraints. In this research, the pole placement constraints will refer directly to regional pole placement (Chilali *et al*., 1999). It is slightly difference with point-wise pole placement where poles are assigned to specific locations in the complex plane based on specific desired time response specifications. In this case, the closed-loop poles of flexible robot manipulator system are confined in a suitable region of the complex plane. This region consists of wide variety of useful clustering area such as half-planes, disks, sectors, vertical/horizontal strips and any intersection thereof (Chilali *et al*., 1999). Using LMI approach, the regional pole placement known as LMI region combined with design objective in H-infinity controller should guarantee a fast input tracking capability, precise hub angle position and very minimal vibration.

MATERIALS AND METHODS

The single-link flexible manipulator system considered in this study is shown in Fig. 1 where XoOYo and XOY represent the stationary and moving coordinates frames respectively, τ represents the applied torque at the hub. E, I, ρ, A, I_h, ν (x, t) and θ (t) represent the young modulus, area moment of inertia, mass density per unit volume, cross-sectional area, hub inertia, displacement and hub angle of the manipulator respectively. In this study, an aluminium type flexible manipulator of dimensions $900\times19.008\times3.2004$ mm^3, $E = 71\times10^9$ N/m^2, $I = 5.1924\times10^{11}$ m^4, $\rho = 2710$ kg/m^3 and $I_H = 5.8598\times10^{-4}$ kg m^2 is considered.

This study provides a brief description on the modelling of the flexible robot manipulator system as a basis of a simulation environment for development and assessment of the H-infinity control techniques. The assume mode method with two modal displacement is considered in characterising the dynamic behaviour of the manipulator incorporating structural damping (Ahmad *et al*., 2008). The dynamic model has been validated with experimental exercises where a close agreement between both theoretical and experimental results has been achieved (Martins *et al*., 2003).

Considering revolute joints and motion of the manipulator on a two-dimensional plane, the kinetic energy of the system can thus be formulated as:

$$T = \frac{1}{2}\left(I_H + I_b\right)\dot{\theta}^2 + \frac{1}{2}\rho\int_0^L\left(\dot{v}^2 + 2\dot{v}x\dot{\theta}\right)dx \qquad (1)$$

where, I_b is the beam rotation inertia about the origin θ_0 as if it were rigid. The potential energy of the beam can be formulated as:

$$U = \frac{1}{2}\int_0^L EI\left(\frac{\partial^2 v}{\partial x^2}\right)^2 dx \qquad (2)$$

This expression states the internal energy due to the elastic deformation of the link as it bends. The potential energy due to gravity is not accounted for since only motion in the plane perpendicular to the gravitational field is considered.

To obtain a closed-form dynamic model of the manipulator, the energy expressions in Eq. 1 and 2 are used to formulate the Lagrangian L = T−U. Assembling the mass and stiffness matrices and utilising the Euler-Lagrange equation of motion, the dynamic equation of motion of the flexible manipulator system can be obtained as:

$$M\ddot{Q}(t) + DQ(t) + KQ(t) = F(t) \qquad (3)$$

Where:

M, D and K = The global mass, damping and stiffness matrices of the manipulator respectively. The damping matrix is obtained by assuming the manipulator exhibit the characteristic of rayleigh damping
F (t) = A vector of external forces
Q (t) = A modal displacement vector given as:

$$Q(t) = \begin{bmatrix}\theta & q_1 & q_2 & \dots & q_n\end{bmatrix}^T = \begin{bmatrix}\theta & q^T\end{bmatrix}^T \qquad (4)$$

$$F(t) = \begin{bmatrix}\tau & 0 & 0 & \dots & 0\end{bmatrix}^T \qquad (5)$$

Here, q_n is the modal amplitude of the ith clamped-free mode considered in the assumed modes method procedure and n represents the total number of assumed modes. Then, the state-space representation of the flexible robot manipulator system can be expressed as:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{6}$$

with the vector;

$$x = \begin{bmatrix} \theta & q_1 & q_2 & \dot{\theta} & \dot{q}_1 & \dot{q}_2 \end{bmatrix}^T$$

and the matrices A, B and C are given by:

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -6175.8 & -1.3527\times10^6 & 0 & 7.8681 & -341.145 \\ 0 & 2.1701\times10^4 & 2.5731\times10^6 & 0 & -2.5541 & 745.786 \\ 0 & -1629.67 & -1.4755\times10^6 & 0 & -0.3340 & -46.5154 \end{pmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 1173.24 & -1128.74 & 23.8429 \end{bmatrix}^T$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

In this study, an integral state feedback control is used as a platform to design the proposed controller. The block diagram of integral state feed back control is shown in Fig. 2. The main objective of the proposed controller is to find the gain parameter matrix, F and G such that it fulfils the design requirement. From the block diagram of Fig. 2, the control input of the system is derived as follow:

$$u(t) = Fx(t) + Gv(t) \tag{7}$$

where, $v(t) = \int_0^t e(\tau)d\tau$ and e (t) = r−y (t). Using new state variable and Eq. 7 the representation of state space equation can be rewrite as:

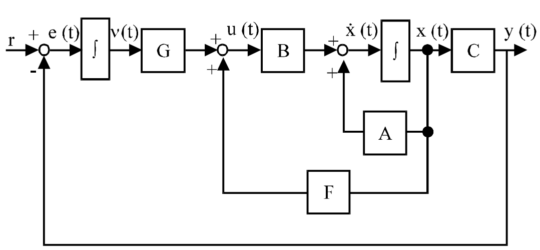

Fig. 2: Block diagram of integral state feedback control

$$\begin{bmatrix} \dot{x}(t) \\ \dot{v}(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r \tag{8}$$
$$e(t) = r - Cx(t)$$

Next, at the steady state condition as t→∞ the state space equation can be written in the following form:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(\infty) \\ v(\infty) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(\infty) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r \tag{9}$$
$$0 = r - Cx(\infty)$$

By subtracting Eq. 8 to 9, the state space form is converted to:

$$\begin{aligned} \dot{\tilde{x}}_e(t) &= \tilde{A}\tilde{x}_e(t) + \tilde{B}_2\tilde{u}(t) \\ \tilde{e}(t) &= \tilde{C}_1\tilde{x}_e(t) \end{aligned} \tag{10}$$

Where:

$$\tilde{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}, \tilde{B}_2 = \begin{bmatrix} B \\ 0 \end{bmatrix}, \tilde{x}_e = \begin{bmatrix} \tilde{x} \\ \tilde{v} \end{bmatrix} = \begin{bmatrix} x - x(\infty) \\ v - v(\infty) \end{bmatrix}$$
$$\tilde{C}_1 = \begin{bmatrix} -C & 0 \end{bmatrix}, \tilde{e}(t) = e - e(\infty)$$

Then, the new control input function is described as follow:

$$\tilde{u}(t) = F\tilde{x}(t) + G\tilde{v}(t) = K\tilde{x}_e(t) \tag{11}$$

Finally, a closed loop state space equation with controller gain, K can be obtained below:

$$\begin{aligned} \dot{\tilde{x}}_e(t) &= \tilde{A}_d\tilde{x}_e(t) + \tilde{B}_1 w \\ \tilde{y}(t) &= \tilde{C}_1\tilde{x}_e(t) + \tilde{D}_{11}w + \tilde{D}_{12}u \end{aligned} \tag{12}$$

Where:

$$\tilde{A}_d = (\tilde{A} + \tilde{B}_2K), \tilde{B}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$
$$\tilde{D}_{11} = 1, \tilde{D}_{12} = 0$$

and w is exogenous input disturbance or reference input to the system. Let G_{yw} (S) denote the closed loop transfer function from w to y under state feedback control u = Kx. Then, for a prescribed closed loop H-infinity performance y>0, the constrained H_∞ problem consists of finding a state feedback gain K that fulfil the following objectives:

- The closed loop poles are required to lie in some LMI stability region D contained in the left-half plane
- Guarantees the H_∞ performance $\|G_{yw}\|_\infty < \gamma$

Quote from the definition by Chilali and Gahinet (1996), a subset D of the complex plane is called an LMI region if there exist a symmetric matrix $\alpha \in R^{m\times m}$ and a matrix $\beta \in R^{m\times m}$ such that:

$$D=\{z\in C: f_D(z)<0\} \tag{13}$$

Where:

$$f_D(z):=\alpha + z\beta + \bar{z}\beta^T$$

Then, pole location in a given LMI region can be characterized in terms of the m×m block matrix:

$$M_D(\tilde{A}_{cl}, X_D):=\alpha\otimes X_D + \beta\otimes(\tilde{A}_d X_D) + \beta^T\otimes(\tilde{A}_d X_D)^T \tag{14}$$

Quote from the theorem by Chilali and Gahinet (1996), the matrix $\tilde{A}_{cl}$ is D-stable if and only if there exists a symmetric matrix X such that:

$$M_D(\tilde{A}_{cl}, X_D)<0, X_D>0 \tag{15}$$

In this study, the region S (λ, r, θ) of complex numbers x+jy such that:

$$x<-\lambda<0,\ |x+jy|<r,\ \tan\theta\ x<-|y| \tag{16}$$

as shown in Fig. 3 is considered. The advantages of placing the closed loop poles to this region are the hub angle position response ensures a minimum decay rate λ, a minimum damping ratio $\zeta = \cos\theta$ and a maximum undamped natural frequency $\omega_d = r \sin\theta$ (Chilali and Gahinet, 1996). Equation 17-19 show the clustering region used in this study which are λ-stability region, a disk and the conic sector, respectively.

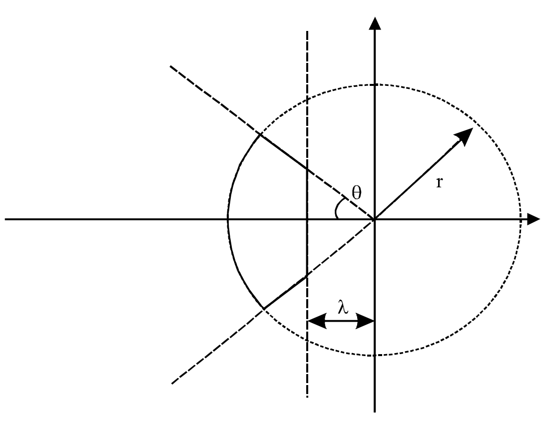

Fig. 3: Region S (λ, r, θ)

$$M_{D1}(\tilde{A}_{cl}, X_{D1}):=\tilde{A}_d X_{D1} + X_{D1}\tilde{A}_{cl}^T + 2\lambda X_{D1}<0 \tag{17}$$

$$M_{D2}(\tilde{A}_{cl}, X_{Dl}):=\begin{pmatrix} -rX_{D2} & \tilde{A}_d X_{D2} \\ X_{D2}\tilde{A}_{cl}^T & -rX_{D2} \end{pmatrix}<0 \tag{18}$$

$$M_{D3}(\tilde{A}_{cl}, X_{D3}):= \begin{pmatrix} \sin\theta(\tilde{A}_d X_{D3} + \tilde{A}_{cl} X_{D3}{}^T) & \cos\theta(\tilde{A}_{cl} X_{D3} - \tilde{A}_{cl} X_{D3}{}^T) \\ \cos\theta(X_{D3}\tilde{A}_d{}^T - \tilde{A}_{cl} X_{D3}) & \sin\theta(\tilde{A}_{cl} X_{D3} + \tilde{A}_d X_{D3}{}^T) \end{pmatrix}<0 \tag{19}$$

where, this region is the intersection of three elementary LMI regions ($M_{D1\cap D2\cap D3}$ ($\tilde{A}_{cl}, X_D$)). Meanwhile, the H_∞ constraint is equivalent to the existence of a solution $X_\infty>0$ to the LMI:

$$\begin{pmatrix} \tilde{A}_{cl} X_\infty + X_\infty \tilde{A}_{cl}{}^T & X_\infty \tilde{C}_1{}^T & \tilde{B}_1 \\ \tilde{C}_1 X_\infty & -\gamma I & \tilde{D}_{11} \\ \tilde{B}_1{}^T & \tilde{D}_{11}{}^T & -\gamma I \end{pmatrix}<0 \tag{20}$$

Equation 20 is also known as the Bounded Real Lemma (Boyd *et al.*, 1994). As mentioned before, the main objective of this study is to minimize the H_∞ norm of G_{yw} (S) over all state feedback gains K that enforce the pole constraints. However, this problem is not jointly convex in the variables, X_{D1}, X_{D2}, X_{D3} and K. The convexity can be enforced by seeking a common solution:

$$X = X_{D1} = X_{D2} = X_{D3} = X_\infty > 0 \tag{21}$$

to Eq. 17-20 and rewriting these equations using the auxiliary variable Y = KX. These changes of variables lead to the suboptimal LMI approach to H-infinity synthesis with pole assignment in LMI regions. As a result, the new representations of 17-20 are shown in the following equation.

$$\text{Herm}[\tilde{A}X + \tilde{B}_2 Y] + 2\lambda X < 0 \tag{22}$$

$$\begin{pmatrix} -rX & \tilde{A}X + \tilde{B}_2 Y \\ * & -rX \end{pmatrix}<0 \tag{23}$$

$$\begin{pmatrix} \sin\theta(\text{Herm}[\tilde{A}X + \tilde{B}_2 Y]) & \cos\theta(\text{Herm}[\tilde{A}X - \tilde{B}_2 Y]) \\ * & \sin\theta(\text{Herm}[\tilde{A}X + \tilde{B}_2 Y]) \end{pmatrix}<0 \tag{24}$$

$$\begin{pmatrix} \text{Herm}[\tilde{A}X + \tilde{B}_2 Y] & X\tilde{C}_1{}^T & \tilde{B}_1 \\ * & -\gamma I & \tilde{D}_{11} \\ * & * & -\gamma I \end{pmatrix}<0 \tag{25}$$

Where:

$$\text{Herm}[\tilde{A}X + \tilde{B}_2Y] = \tilde{A}X + \tilde{B}_2Y + X\tilde{A}^T + Y\tilde{B}_2^T$$

and * is an ellipsis for terms induced by symmetry (Chilali *et al*., 1999). In this study, the entire LMI problem is solved using well known LMI optimization software which is LMI Control Toolbox.

To evaluate the effectiveness of the proposed controller, the results of this study is compared with hybrid input shaping and collocated PD control schemes as reported by Ahmad and Mohamed (2008). In particular, only positive input shaper with collocated PD controller is considered for the comparative assessment. Initially, a collocated PD control is designed. This is then extended to incorporate positive input shaping schemes for control of vibration of the system. The tracking performance o f the collocated PD control applied to on the root locus analysis.

The closed loop parameters with the PD control will subsequently be used to design and evaluate the performance of hybrid control schemes in terms of input tracking capability and level of vibration reduction. For the vibration suppression schemes, the positive input shaper is designed based on the vibration frequencies and damping ratios of the flexible robot manipulator system. The natural frequencies were obtained by exciting the flexible manipulator with an unshaped unit step reference input under a collocated PD controller.

The input shapers were designed for pre-processing the unit step reference input and applied to the system in a closed-loop configuration as shown in Fig. 4. The detail of the controller design is omitted in the interest of space. The design of the hybrid input shaping with collocated PD and the proposed controller must fulfil the following specifications:

- Settling time of <1.5 sec with overshoot <1% and zero steady state error for the hub angle
- End-point acceleration is less than ±200 m/sec^2
- Control input torque does not exceed ±1 Nm

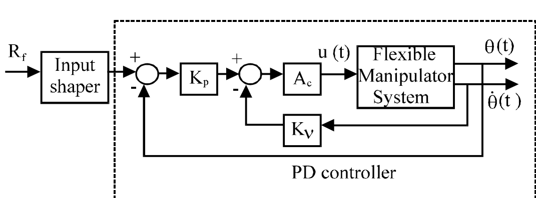

Fig. 4: Block diagram of hybrid control scheme configuration

RESULTS AND DISCUSSION

Applying the LMI conditions in Eq. 22-25, the parameter of conic sectors and disk that fulfil the design requirement is at λ = -6, r = 200 and θ = 28°. Then, the state feedback gain, K is obtained as followed:

$$K=[-12.16 \quad -62.58 \quad -4009.03 \quad -1.08 \quad -1.80 \quad -57.76 \quad 39.43]$$

with γ = 3.4145. This state feedback gain also guarantees the H_∞ performance $\|G_{yw}\|_\infty < \gamma$. The result shows that the location of poles has been confined in the selected LMI region with the value of -38.032, -65.073, -191.049, -7.243±j2.188 and -178.619±j59.46. On the other hand, using root locus technique for collocated PD, the parameter of K_p, K_v and A_c were deduced as 60, 19 and 0.02, respectively. Then, the positive ZVDD input shaper is designed based on two modes of vibration frequencies (15 and 55 Hz) from the analysis of closed-loop configuration with collocated PD control.

The response of hub angle, end-point acceleration and control input torque of the flexible robot manipulator are shown in Fig. 5-7 for both H-infinity and hybrid input shaping with PD controller. It shows that both controller can track the desired trajectory input with zero steady state error and achieve zero vibration from the response of end-point acceleration. Hence, in overall both controllers successfully fulfil th e design requirement. Table 1 shows the time response specifications of hub angle position. It is noted that the H-infinity controller

Table 1: Time response specifications of hub angle

Controller	H-infinity	PD with input shaper
Settling time (sec)	0.964	1.064
Rise time (sec)	0.529	0.513
Overshoot (%)	0.000	0.490

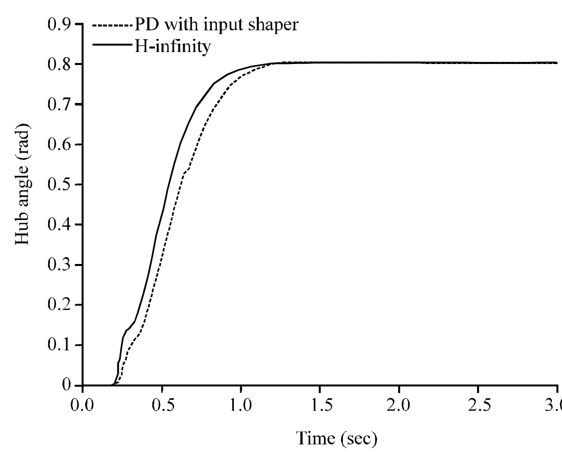

Fig. 5: Response of the hub angle of the flexible robot mainipulator

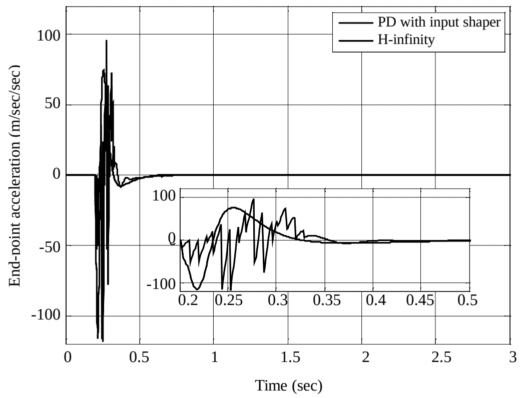

Fig. 6: Response of the end-point acceleration of the flexible robot manipulator

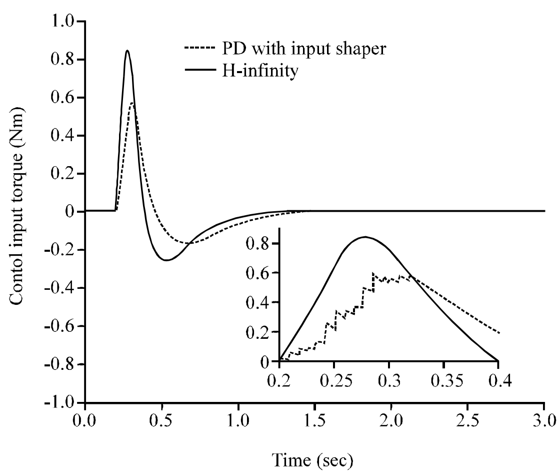

Fig. 7: Response of the control input torque of the flexible robot manipulator

produces a fast settling time with very minimal overshoot as compared to hybrid input shaping with PD controller. In addition, the H-infinity controller also shows a very minimal oscillation (low frequency) at the end-point during the movement of the manipulator as compared to hybrid input shaping with PD controller.

In terms of magnitude of oscillation, the end-point acceleration response of the H-infinity controller was found to oscillate between±116 m/sec^2 as compared to hybrid control schemes with±119 m/sec^2. On the other hand, in terms of control input torque response, the hybrid controller produced lower magnitude of input torque than H-infinity controller. However, during 0.2-0.4 sec, H-infinity controller produced a smooth torque response as compared to hybrid controller which gives advantage to the lifetime of the flexible robot manipulator actuator.

CONCLUSION

The development of H-infinity controller based on LMI region schemes has been developed for Flexible Robot Manipulator System. The results show that by confining the closed-loop poles of the Flexible Robot Manipulator System based on LMI region, one can successfully achieve the desired specifications and the designed state feedback gain also guarantees the H-infinity performance. The effectiveness of the proposed scheme has been compared with hybrid input shaping with collocated PD controller. Particularly, the proposed controller has produced a fast input tracking capability with very minimal end-point vibration and smooth control input torque than hybrid control schemes.

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