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Model of Radiation Transport at Cosmic Ray Shocks

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Abstract: Using a framework of the radiation approximation followed by a two-term perturbation expansion for cosmic ray transport in the spherical polar coordinates (r, θ, φ) researchers identify the effect of cosmic ray radiation on shock dominated transport. When the buoyancy parameter F_r is negligible, researchers find that the cosmic ray density at shock boundary (E_n) decreases with increasing temperature. It is also observed that the variation of radiation parameter N in cosmic ray transport has no significant effect in the temperature distribution. Thus, even when radiation is significant, it does not really modify the temperature within the cosmic ray region. However, for increases in the density at shock boundaries say (En), the temperature distribution decreases.

Key words: Cosmic rays, background plasma, radiative shocks, pressure tensor, fluid formalism

INTRODUCTION

The radiation mechanism of cosmic ray shocks including the transport of cosmic ray is a question of common interest in astrophysics. It has been known that cosmic rays contribute a viscosity to the collisionless plasma in which they propagate (Williams and Jokipii, 1993; Trotta *et al*., 2011). This is the result of the acceleration of particles in shear geometries in the fluid flow. It remains to explore the effect of radiation at cosmic ray dynamics. Cosmic ray energy also reservoir suffers some loss processes by way of scattering and particle decay (Pfrommer *et al*., 2008).

In the present study, researchers employ a two fluid formalism for cosmic ray shocks (Drury and Volk, 1981; Wagner *et al*., 2006) to include the full cosmic ray pressure tensor for small buoyancy parameter. The special case of a cold background fluid with negligible pressure is also considered in this model. Researchers start by considering the mathematical formulation of the problem for a curvilinear shock (spherical) with no average magnetic field.

The case of the influence of magnetic field on the effect of radiation at cosmic ray shocks is deferred to later study. The mathematical formulation is presented in non-dimensional form followed by a two term perturbation expansion.

MATHEMATICAL FORMULATION

The following is within the two-fluid (the cold background plasma and the cosmic rays) framework. The background plasma contributes mass, momentum and flow energy to the system. The cosmic rays dominate the internal energy and pressure in the spherical coordinate system.

The inner sphere cosmic ray radius r_0 is maintained at temperature T_0 and rotates with angular velocity ω. The outer cold background plasma is at temperature T_1.

Researchers assume that the temperatures T_0, 1 are large enough for radiative transfer to be significant, thus if (u', v', w') and (q', qφ, 0) are the velocity and radiative flux components, respectively in the spherical polar coordinates (r', φ', z) then considering a time dependent flow with the equations of motion referenced to the shock frame, the mass, momentum and energy equations gives:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho r^2 \mu\right)+\frac{1}{r\sin\kappa}\left(\rho v \sin\varphi\right)=0 \quad (1a)$$

$$\rho\left(u\frac{\partial u}{\partial r}+\frac{v}{r}\frac{\partial u}{\partial \varphi}-\frac{v^2+F_r w^2}{r}\right)$$
$$=\frac{\partial \rho}{\partial r}+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 Trr\right)+\frac{1}{r\sin\varphi}\frac{\partial}{\partial \varphi}\left(T r\sin\varphi\right)+ \quad (1b)$$
$$\frac{T_{r\varphi}\sin\varphi+T_{zz}}{r}-F_r\left(\rho-1\right)\cos\varphi$$

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$$\rho=\left(u\frac{\partial u}{\partial r}+\frac{v}{r}\frac{\partial u}{\partial \varphi}-\frac{F^2w^2\cot}{r}\right)=\frac{1}{r}\frac{\partial \rho}{\partial r}+\frac{1}{r^2}\frac{\partial}{\partial r}(r^2T_r\varphi)+ \frac{1}{r\sin\varphi}\frac{\partial}{\partial \varphi}(T_{\varphi\varphi}\sin\varphi)+ \frac{T_{r\varphi}}{r}-\frac{\cot\varphi}{r}T_{zz}+ F_r(\rho-1)\sin\varphi \quad (1c)$$

$$\rho\left(u\frac{\partial w}{\partial r}+\frac{v}{r}\frac{\partial u}{\partial \varphi}-\frac{uw}{r}+\frac{vw}{r}\cot\varphi\right) =\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2Trz\right)+\frac{1}{r}\frac{\partial T_{\varphi z}}{\partial \varphi}+\frac{T_{rz}}{r}\frac{2\cot\varphi}{r}T_{\varphi z} \quad (1d)$$

$$\rho\left(u\frac{\partial \theta}{\partial r}+\frac{v}{r}\frac{\partial \theta}{\partial \varphi}\right)=\frac{1}{p^r}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\theta\frac{\partial \theta}{\partial r}\right)+\frac{1}{r\sin\varphi}\frac{\partial}{\partial \varphi}\left(\frac{\sin\varphi}{r}\theta\frac{\partial \theta}{\partial \varphi}\right)\right]- \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2q_r\right)-\frac{1}{r\sin\varphi}\frac{\partial}{\partial \varphi}(\sin\varphi q\varphi)=0 \quad (1e)$$

But:

$$\left(q_r,q\varphi,0\right)=4N\theta^3\left(\frac{\partial \theta}{\partial r}.\frac{1}{r}\frac{\partial \theta}{\partial \varphi}.0\right) \quad (2)$$

The cosmic ray energy density is defined as:

$$\rho=4\pi\int\left(\sqrt{p^2c^2+m_0^2c^4.m_0c^2}\right)fp^2dp$$

Where:

- m_0 = The particle rest mass
- m = The relativistic mass measured in the frame of the fluid

The momentum P is measured relative to the fluid frame and c is the speed of light, the integrals are over the isotropic part of the cosmic ray distribution function. Researchers define:

$$T_{rr}=\theta\left(2\frac{\partial u}{\partial \theta}-\frac{2}{3}\nabla.\vec{q}\right) \quad (3a)$$

$$T_{\varphi\varphi}=\theta\left(2\left(\frac{1}{r}\frac{\partial v}{\partial \theta}+\frac{u}{r}\right)-\frac{2}{3}\nabla.\vec{q}\right) \quad (3b)$$

$$T_{zz}=\theta\left(2\left(\frac{u}{r}\frac{v\cot\varphi}{r}\right)-\frac{2}{3}\nabla.\vec{q}\right) \quad (3c)$$

$$T_{r\varphi}=\theta\left(r\frac{1}{r}\frac{\partial}{\partial \varphi}\left(\frac{v}{r}\right)-\frac{1}{r}\frac{\partial u}{\partial \varphi}\right) \quad (3d)$$

$$T_{rz}=\theta r\frac{\sin}{r}\frac{\partial}{\partial \varphi}\left(\frac{w}{\sin\varphi}\right) \quad (3e)$$

$$T_{r\varphi}=\theta r\frac{\partial}{\partial r}\left(\frac{w}{r}\right) \quad (3f)$$

TWO TERM PERTURBATION EXPANSION SOLUTION

First, researchers take the boundary conditions for:

$$r=1 \quad (4a)$$

$$u=0 \quad (4b)$$

$$v=\frac{2-\Gamma}{r}\frac{5r\pi}{16}E_n\left[\frac{1}{r}\frac{\partial}{\partial \varphi}r\frac{\partial}{\partial r}\left(\frac{v}{r}\right)\right]+\frac{15}{32}\left(\frac{\pi}{2\delta}\right)^{1/2}\frac{E_n}{M_\infty}\left(\frac{1}{\frac{1}{\theta^2}}\frac{1}{r}\frac{\partial \theta}{\partial \varphi}\right) \quad (4c)$$

$$w=\Omega\sin\varphi-\frac{2-\lambda 75\pi}{128}E_n\frac{1}{\theta}\frac{\partial \theta}{\partial r}-\frac{5}{48}\left(\frac{\pi\delta}{2}\right)^{1/2}M_\infty E_n\frac{1}{\frac{1}{\theta r^2}} \quad (4d)$$

$$\frac{2}{r}u+\frac{1}{r}\frac{\partial v}{\partial \varphi}+\frac{\cot\varphi}{r}v-2\frac{\partial u}{\partial r} \quad (4e)$$

For a simple case when:

$$r=R$$
$$w=\frac{2-\Gamma}{\Gamma}\frac{5\pi}{16}E_nr\frac{\partial}{\partial r}\left(\frac{w}{r}\right) \quad (5)$$

where, R is the spherical radius of the outer plasma. Researchers introduce the following non-dimensional quantities:

$$r=\frac{r'}{r_0},(u,v,w)=\frac{1}{F_r}(F_ru',w')$$
$$(\theta,\theta_{0.1})=\frac{TT_{0.1}}{T_\infty}$$
$$(P.P_\infty)=(P'.P'_\infty)\frac{r_0^2}{\rho_\infty v^2} \quad (6)$$
$$\Omega=\frac{1}{F_r}\frac{\rho_\infty gr_0}{vw},\ Pr=\frac{\mu C_P}{k}$$
$$M_\infty=\frac{v_\infty}{a_\infty r_0},\quad E_n=\frac{1}{r_0}$$
$$N=4\sigma T_\infty^3 3\alpha\mu_\infty C_P$$

Where:

- Ω = The gyro-frequency of the cosmic ray particles in the average background radiation field

α and σ = The absorption coefficient and Stefan-Boltzmann constant
Γ and λ = The reflection and accommodation coefficient on the cosmic ray dominant shock

The problem is highly non-linear and thus not amenable to easy analytical treatment. Asymptotic approximation is then invoked. For a simplified cosmic ray dominant shock in which there is a small buoyancy factor F_r, for ω researchers write:

$$\omega = \omega^{(0)}(r)\sin\varphi + F_r\omega^{(1)}(r,\varphi) + \dots$$

for P and θ, researchers put:

$$P = P^{(0)}(r) + F_n P^{(1)}(r\varphi) + \dots \quad (7)$$

while for u and v researchers set:

$$U = F_n u^{(1)}(r,\varphi) + \dots$$

Substituting Eq. 7 into the governing equations, researchers have the sequence of approximations 0 (1):

$$\frac{dP^{(0)}}{dr} = 0,\ P^{(0)} = \frac{1}{M_\infty^2}\left(P^{(0)}\theta^{(0)} - 1\right)\frac{1}{r^2}\frac{d}{dr}\left(r^2\tau_{rz}^{(0)}\right) + \frac{\tau_{rz}^{(0)}}{r}0,$$
$$\tau_{rz}^{(0)} = \theta^{(0)} r\frac{d}{dr}\left(\frac{w^{(0)}}{r}\right)\frac{1}{r^2}\frac{d}{dr}\left(r^2\theta^{(0)}\frac{d\theta^{(0)}}{dr}\right) - \frac{1}{r^2}\frac{d}{dr}\left(r^2 q^{(0)} r\right) = 0$$
$$\tau_{rz}^{(0)} = \theta_r^{(0)}\frac{d}{dr}\left(\frac{\omega^{(0)}}{r}\right),\ q_r^{(0)} = -4N\theta^{(0)^3}\frac{d\theta^{(0)}}{dr} \quad (8)$$

Where:

$$\omega^{(0)} = \Omega - \frac{2-\Gamma}{\Gamma}\frac{5\pi}{16}E_n r\frac{d}{dr}\left(\frac{\omega^{(0)}}{r}\right) \text{ on } r = 1 \quad (9)$$

$$\omega^{(0)} = \Omega - \frac{2-\Gamma}{\Gamma}\frac{5\pi}{16}E_n r\frac{d}{dr}\left(\frac{\omega^{(0)}}{r}\right) \text{ on } r = R \quad (10)$$

$$\frac{\theta^{(0)}}{\theta_{0,1}} = +\frac{2-\lambda}{\lambda}\frac{75\pi}{128}E_n\frac{1}{\theta(0)}\frac{d\theta^{(0)}}{dr} \text{ on } r = 1, R \quad (11)$$

On 0 (F_r) with $u^{(1)}$, $p^{(1)}$, $\theta^{(1)} = u^{(1)}(r)$, $u_r^{(1)}(r)$ and:

$$\theta^{(1)} = \theta_1^{(1)}(r)\cos\varphi, \omega^{(1)}\ \omega_1^{(1)}(r)\sin\varphi\cos\varphi\, v^{(1)} = (r)\sin\varphi$$

Then:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 u_1^{(1)}\right) + \frac{2}{r}v_1^{(1)} = 0 \quad (12)$$

$$-\frac{dp_1^{(1)}}{dr} + \frac{1}{r^2}\frac{d}{dr}\left(r^2\tau_{rr}^{(1)}\right) + \frac{2}{r}\left(\tau_{r\varphi}^{(1)} + \tau^{(1)}\right) = 0 \quad (13)$$

$$\frac{1}{r^2}p_1^{(1)} + \frac{1}{r^2}\frac{d}{dr}\left(r^2\tau_{r\varphi}^{(1)}\right) + \frac{\tau_{r\varphi}^{(1)} - \tau^{(1)}}{r} - 1 = 0 \quad (14)$$

Thus, within the two fluid (the cold background plasma sphere and the cosmic rays) framework, the velocity and temperature be conserved across the shock, this leads to the equations:

$$\left(\frac{d\omega^{(0)}}{dr}u_1^{(1)} + \frac{\omega^{(0)}}{r}u_1^{(1)} + \frac{2\omega^{(0)}}{r}v_1^{(1)}\right) = \frac{1}{r^2}\frac{d}{dr}\left(r^2\tau_{rz}^{(1)} + \frac{\tau_{rz}^{(1)} - 4\tau_{\varphi z}^{(1)}}{r}\right) \quad (15)$$

$$\left(\frac{d\theta^{(0)}}{dr}\right)u_1^{(1)} = \frac{1}{r^2}\frac{d}{dr}\left(r^2\left(\theta^{(0)}\frac{d\theta_1^{(1)}}{dr} + \frac{d\theta^{(0)}}{dr}\theta_1^{(1)}\right)\right) - \frac{2}{r}\theta^{(0)}\theta_1^{(1)} - \frac{1}{r^2}\frac{d}{dr}\left(r^2 q_r^{(1)}\right) + \frac{2}{r}q_\varphi^{(1)} \quad (16)$$

And the cosmic ray pressure with one index fixed:

$$p_1^{(1)} = \frac{1}{M_\infty^2}\left(\rho^{(0)}\theta_1^{(1)} + \theta^{(0)}\rho_1^{(1)}\right) \quad (17)$$

$\rho^{(0)}$ and $\rho^{(1)}$ are the zero and non-zero components of cosmic ray energy density. Researchers define:

$$q_r^{(1)} = 4N\left(\theta^{(0)3}\frac{d\theta_1^{(1)}}{dr} + \frac{d\theta^{(0)3}}{dr}\theta_1^{(1)}\right) \quad (18)$$

$$q_0^{(1)} = \frac{1}{r}\theta^{(0)3}\theta_1^{(1)} \quad (19)$$

And:

$$q_1^{(1)} = \frac{du_1^{(1)}}{dr} + \frac{2}{r}\left(u^{(1)} + v_1^{(1)}\right) \quad (20)$$

$$\tau_{rr}^{(1)} = \theta^{(0)}\left(2\frac{du_1^{(1)}}{dr} - \frac{2}{3}q_1^{(1)}\right) \quad (21)$$

$$\tau_{\varphi\varphi}^{(1)} = \theta^{(0)}\left(\frac{2}{r}\left(u_1^{(1)} + v_1^{(1)}\right) - \frac{2}{3}q_1^{(1)}\right) = \tau_{zz}^{(1)} = \tau \quad (22)$$

$$\tau_{z\varphi}^{(1)} = \frac{1}{r}\theta^{(0)}\omega_1^{(1)} \quad (23)$$

$$\tau_{zr}^{(1)} = \left(\theta^{(0)} r \frac{d}{dr}\left(\frac{\omega^{(1)}}{r}\right) + \theta_1^{(1)} r \frac{d}{dr}\left(\frac{\omega^{(0)}}{r}\right)\right) \quad (24)$$

Subject to the boundary conditions:

$$u_1^{(1)} = 0$$

$$v_1^{(1)} = \frac{2-\Gamma}{\Gamma}\frac{5\pi}{16}E_n\left(r\frac{d}{dr}\left(\frac{v_1^{(1)}}{r}\right)\right) - \frac{1}{r}u_1^{(1)} - \frac{15}{32}\left(\frac{\pi}{2s}\right)^{1/2} E_n \frac{1}{\theta^{(0)\frac{1}{2}}}\frac{1}{r}\theta_1^{(1)}$$

$$\omega_1^{(1)} = \frac{2-\Gamma}{\Gamma}\frac{5\pi}{16}E_n r\frac{d}{dr}\left(\frac{\omega_1^{(1)}}{r}\right) \quad (25)$$

$$\frac{\theta_1^{(1)}}{\theta_{0,1}} = \frac{2-\lambda}{\lambda}\frac{75\pi}{128}E_n\left(\frac{1}{\theta^{(0)}}\frac{d\theta_1^{(1)}}{dr} - \frac{1}{\theta^{(0)2}}\frac{d\theta^{(0)}}{dr}\theta_1^{(1)}\right) - \frac{5}{58}\left(\frac{\pi\delta}{2}\right)^{1/2} E_n \frac{1}{\theta(0)^{\frac{1}{2}}}\left(\frac{2}{r}\left(u_1^{(0)} + v_1^{(1)}\right) - 2\frac{du_1^{(1)}}{dr}\right)$$

And:

$$\frac{\rho^{(1)}}{\rho_{0,1}} = \frac{2-\Gamma}{\Gamma}\frac{15}{16}E_n\left(\frac{1}{\theta^{(0)}}\frac{d\theta_1^{(1)}}{dr} - \frac{1}{\theta^{(0)2}}\frac{d\theta^{(0)}}{dr}\theta_1^{(1)}\right) - \frac{5}{12}\left(\frac{\pi\delta}{2}\right)^{1/2} E_n \frac{1}{\theta(0)^{\frac{1}{2}}}\left(\frac{2}{r}\left(u_1^{(0)} + v_1^{(1)}\right) - 2\frac{du_1^{(1)}}{dr}\right) \text{ on } r = 1, R \quad (26)$$

The Eq. 8-24 allow us to obtain solutions subject to boundary conditions (Eq. 25). By virtue of the two fluids (the cold background plasma and cosmic rays) framework, the first set of Eq. 8 give:

$$p^{(0)} = 0$$

and therefore:

$$p^{(0)} = \frac{1}{\theta_{(0)}}$$

Next the equation involving temperature in Eq. 8 could be integrated twice to give:

$$\frac{1}{2}\theta^{\left((0)^2\right)} + N\theta^{(0)4} = A^{(0)}\frac{1}{r}B^{(0)} \quad (27)$$

If researchers put:

$$\theta^{(0)}(1) = \theta_0^*$$

And:

$$\theta^{(0)}(R) = \theta_1^*$$

then Eq. 27 with the temperature boundary condition in Eq. 11 furnish the four equations:

$$\frac{1}{2}\theta_0^2 + N\theta_0^4 = A^{(0)} + B^{(0)} \quad (28a)$$

$$\frac{1}{2}\theta_0^2 + N\theta_0^4 = \frac{A^{(0)}}{R} + B^{(0)} \quad (28b)$$

And:

$$\frac{\theta_0^*}{\theta_0} = 1 - \frac{2-\lambda}{\lambda}\frac{75\pi}{128}E_n\frac{A^{(1)}}{\theta_0^*\left(\theta_0^* + 4N\theta_0^{*3}\right)} \quad (28c)$$

$$\frac{\theta_1^*}{\theta_1} = 1 - \frac{2-\lambda}{\lambda}\frac{75\pi}{128}E_n\frac{A^{(1)}}{R^2\theta_1^*\left(\theta_1^* + 4N\theta_1^{*3}\right)} \quad (28d)$$

for the four unknown an appeal to Newton-Raphson algorithm readily gives a solution to these non-linear equations. The equation for the velocity could also be reduced to:

$$\theta^{(0)} r \frac{d}{dr}\left(\frac{\omega^{(0)}}{r}\right) = \frac{Y^{(0)}}{r^3} \quad (29a)$$

$$\omega^{(0)} = r\left(Y^{(0)}\right)\int_1^r \frac{dz}{z^4\theta^{(0)}(z)} + k^{(0)} \quad (29b)$$

The integral involved in Eq. 29a closed form if 0 is adapted as independent variable. Thus:

$$\int^r \frac{dr}{r^4\theta^{(0)}(r)} = \frac{1}{A^{(0)}}\int^{\theta^0}\left(1 + 4N\theta^{(0)2}\right)\left(N\theta^{(0)4} + \frac{1}{2}\theta^{(0)2-B^{(0)}}\right)^2 d\theta^{(0)} - H\left(\theta^{(0)}\right) \quad (30)$$

Where:

$$\int^r \frac{dr}{r^4\theta^{(0)}(r)} = \frac{1}{A^{(0)}}\int^{\theta^0}\left(1 + 4N\theta^{(0)2}\right)\left(N\theta^{(0)4} + \frac{1}{2}\theta^{(0)2-B^{(0)}}\right)^2 d\theta^{(0)} - H\left(\theta^{(0)}\right) \quad (31)$$

And in terms of $\theta^{(0)}$:

$$r\frac{d}{dr}\left(\frac{\omega^{(0)}}{r}\right) = \frac{Y^{(0)}}{A^{(0)3}}\left(\frac{N\theta^{(0)4} + \frac{1}{2}\theta^{(0)2} - B^{(0)}}{\theta^{(0)3}}\right) \quad (32)$$

While:

$$\omega^{(0)} = \frac{A^{(0)}}{N\theta^{(0)4} + \frac{1}{2}\theta^{(0)2} - B^{(0)}}\left[Y^{(0)}\left(H\left(\theta^{(0)}\right) - H\left(\theta_0\right)\right) + k^{(0)}\right] \quad (33)$$

By virtue of Eq. 27, the velocity boundary conditions in Eq. 9 and 10 reduce to:

$$\theta_0^* k^{(0)} = \Omega - \frac{2-\Gamma}{\Gamma}\frac{5\pi}{16}E_n\left(\frac{N\theta_0^{*4} + \frac{1}{2}\theta_0^{*2} - B^{(0)3}}{A(0)^3\theta_0^{*3}}\right) x Y^{(0)} \quad (34)$$

And:

$$\theta_1^*\left[Y^{(0)}\left(H(\theta_1^*) - H(\theta_1^*)\right) + k^{(0)}\right] = \frac{2-\Gamma}{\Gamma}\frac{5\pi}{16}E_n\left(\frac{N\theta_0^* + \frac{1}{2}\theta_0^{*2} - B^{(0)3}}{A(0)^3\theta^{*3}}\right)Y^{(0)} \quad (35)$$

Equation 34 and 35 are two linear simultaneous equations for the unknown $Y^{(0)}$ and $k^{(0)}$. Finally, from the pressure boundary conditions in Eq. 25 the shock pressure (between the two fluid approximation) can be obtained as:

$$\frac{\rho_\infty}{\rho_{0,1}} = 1 - \frac{2-\Gamma}{\Gamma}\frac{5\pi}{16}E_n\frac{A^{(0)}}{(1,R^2)(\theta_{0,1}^* + 4N\theta_{0,1}^{*3})} \quad (36)$$

The solution of order 0 (1) problem is now completed.

HIGHER APPROXIMATE SOLUTIONS

Researchers know that:

$$\frac{d\theta^{(0)}}{dr} = \frac{-A}{r^2\left(\theta^{(0)} + 4N\theta^{(0)3}\right)} \quad (37)$$

from which follows:

$$\frac{d^2\theta^{(0)}}{dr^2} = \frac{2A^{(0)}}{r^3\left(\theta^{(0)} + 4N\theta^{(0)3}\right)} - \frac{A^{(0)}\left(1 + 12N\theta^{(0)2}\right)}{r^4\left(\theta^{(0)} + 4N\theta^{(0)3}\right)^3} \quad (38)$$

While from (Eq. 33) researchers obtain:

$$\frac{d\omega^{(0)}}{dr} = \frac{-A^{(0)}\left(4N\theta^{(0)3} + \theta^{(0)}\right)}{\left(4N\theta^4 + 2\theta^{(0)2} - B^{(0)}\right)^2}\left[Y^{(0)}\left[H(\theta_0) - H(\theta_0^*)\right] + k^{(0)}\right] \frac{d\theta^{(0)}}{dr} + \frac{A^{(0)}Y^{(0)}}{N\theta^{(0)4}} + \frac{1}{2}\theta^{(0)2} - B^{(0)}\frac{dH\left(\theta^{(0)}\right)}{d\theta^{(0)}}\frac{d\theta^{(0)}}{dr} \quad (39)$$

and in virtue of Eq. 27 the above derivatives are expressible in terms of r. Furthermore using Eq. 26 and neglecting the shock pressure, the zero order buoyancy factor 0 (Fr) may then be expressed in compact form as:

$$\frac{1}{\theta^{(0)}}\frac{1}{r^2}\frac{d}{dr}\left(r^2u_1^{(1)}\right) - \frac{1}{\theta^{(0)2}}\frac{d\theta^{(0)}}{dr}u_1^{(1)} + \frac{2}{r}\frac{1}{\theta^{(0)}}v_1^{(1)} = 0 \quad (40a)$$

$$\frac{1}{M_\infty}\left(\frac{1}{\theta^{(0)}}\frac{d\theta_1^{(1)}}{dr} - \frac{1}{\theta^{(0)2}}\frac{d\theta^{(0)}}{dr}\theta_1^{(1)} + \theta^{(0)}\frac{d\rho_1^{(1)}}{dr} + \frac{d\theta^{(0)}}{dr}\rho_1^{(1)}\right) + \frac{1}{r^2}\frac{d}{dr}\left(r^2\tau_{rr}^{(1)}\right) + \frac{r}{2}\left(\tau_{r\varphi}^{(1)} + \tau^{(1)}\right) - \left(\frac{1}{\theta^{(0)}} - 1\right) = 0 \quad (40b)$$

$$+\frac{1}{r^2}\frac{d}{dr}\left[r^2\left(\theta^{(0)}\frac{d\theta_1^{(1)}}{dr} + \frac{d\theta^{(0)}}{dr}\theta_1^{(1)}\right)\right] - \frac{2}{r}\theta^{(0)}\theta_1^{(1)} \quad (40c)$$

And:

$$-\frac{1}{r^2}\frac{d}{dr}\left(r^2q_r^{(1)}\right) + \frac{2}{r}q_\varphi^{(1)} = \frac{1}{\theta^{(0)}}\frac{d\theta^{(0)}}{dr}u_1^{(1)} \quad (40d)$$

It is now convenient to put $r_j = r_j + \Delta r$, j = 1, 2, ...; r_0 = 1 and r_j = R. For the dependent variables researchers set then all the basic approximations and their derivatives in Eq. 33-40 are expressible in terms of j. In the current approximation, researchers discretise by replacing derivatives by finite differences employing central difference as used by Bestman *et al.* (1993). In the boundary conditions it is expedient to replace derivative in by an expression involving the single unknown obtained easily in Eq. 40; since $u_1^{(1)}(0) = u_1(j)$:

RESULTS AND DISCUSSION

Researchers have illustrated the effect of cosmic-ray radiation on the structure of cosmic-ray modified shocks following the two fluid (the cold background plasma and cosmic-ray) framework. The basic approximation for small buoyancy parameter, though non-linear was integrated in a closed from. However, the arbitrary constants of integration in the temperature field as well as the temperature jumps at the spheres surface which satisfies the non-linear algebraic equations which are easily solved by the Newton-Raphson Method. Of interest is that the cosmic ray radiation thickens the shock if buoyancy factor 0 (F_r) is negligible. This is in agreement with the results obtained by Wagner *et al.* (2006). In the absence of a cold dense layer the post-shock produces no sharp boundary with the downstream flow.

In the quantitative discussion, researchers take Γ = 1.4 (cosmic rays), F_r = 0.1, -R = 2 and Ω = 1 which are varied when equations are solved numerically. The results of the temperature distribution are shown in Fig. 1 for velocity profiles r. In Fig. 1, researchers observe that variation in the cosmic ray radiation parameter (N) has no significant effect in the temperature distribution.

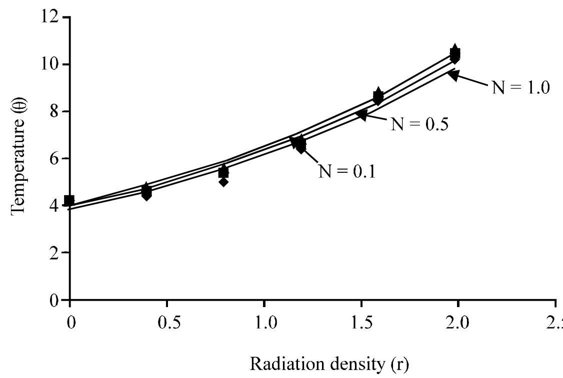

Fig. 1: Relationship between radiation density and temperature distribution

CONCLUSION

The result of study shows that once radiation is significant, the amount of radiation does not really modify the temperature within the cosmic ray-dominated shock.

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