Research Journal of Applied Sciences 7 (9-12): 444-450, 2012
ISSN: 1815-932X

Non-Geodesic Filament Winding of Axisymmetric Surfaces

[1]Milad Noorabadi, [1]Jafar Eskandari Jam, [2]S.H. Taghavian and
[2]Mohammad Reza Asadi Asad Abad
[1]Composite Materials and Technology Center, Tehran, Iran
[2]Department of Mechanical Engineering, Buinzahra Branch, Islamic Azad University,
Buinzahra, Iran

Abstract: Filament Winding is one of the widely used methods for fabrication of axisymmetric composite structures. Geodesic curve on the surface is the ideal path for placement of fibers since these curves are stable path and there is no possibility of slippage of fibers during winding. Pressure vessel with integral end domes having unequal opening at two ends is a typical example. All such examples require deviation from geodesic path during winding. Non-geodesic winding is inherently unstable and prone to slippage. Tendency of slippage can be resisted only if adequate friction is available. Thus, non-geodesic winding requires investigations of stability of fiber path. This study deals with theoretical investigations related to stability of non-geodesic winding on axi-symmetric surface. The relation between the slippage coefficient and the winding angle is obtained to meet stable winding requirements. Limits of deviation from geodesic path and stability of a predefined path is established through application of differential geometry. Simple design nomograms for cylindrical sections are established.

Key words: Filament winding, geodesic, non-geodesic, fabrication, slippage

INTRODUCTION

Filament winding is a popular production technique for composite structures. Filament winding technology has been so far increasingly applied to develop lightweight high pressure vessels in commercial and aerospace industries. Compared to their steel-based counterparts, filament-wound composite pressure vessels provide significant advantages such as high specific strength/stiffness and modulus, exceptional fatigue life, excellent corrosion and chemical resistance. In the filament winding process, a fiber bundle is placed on a rotating and removable mandrel. For manufacturing of composite components by filament winding, wet or prepreg fibers or tapes are placed on a predetermined path on a mandrel. The assembly is subsequently cured. Continuous filaments are an economical and excellent form of fiber reinforcement and can be oriented to match the direction of stress loaded in a structure. Fuel tanks, oxidizer tanks, motor cases and pipes are some examples of filament wound axisymmetric structures under internal pressure. The trajectory of the fiber path and the corresponding fiber angles cannot be chosen arbitrarily because of the stability requirement. The fiber path instability induced by fiber slippage on a mandrel surface is too complicated to be predicted because it is affected by many parameters such as temperature, mandrel shape, fiberresin combination, surface treatment and so on. Little research has been focused on the design method using the continuum theory in combination with non-geodesic winding law. Koussios *et al.* (2005) derived the uninterrupted hoop and polar fiber path equation on cylindrical pressure vessels using non-geodesic trajectories. Zu *et al.* (2010) performed parametric studies about the Shape optimization of filament wound articulated pressure vessels based on non-geodesic trajectories. Little research has been focused on the design method using the continuum theory in combination with non-geodesic winding law. Most of the earlier models have employed geodesic winding principle (Vasiliev *et al.*, 2003) and the netting theory assumption to design the rotationally convex structures (Krikanov, 2000). Accurate placement of fibers with certain tension, on predetermined path largely determines the quality and strength of the final component. The geodesic curve on a surface is the ideal curve for placement of fibers on the mandrel surface since this will ensure that there is no slippage of fibers during winding. However, for some applications such as a pressure vessel with integral end domes having unequal opening at two

Corresponding Author: Mohammad Reza Asadi Asad Abad, Department of Mechanical Engineering, Buinzahra Branch, Islamic Azad University, Buinzahra, Iran

ends. It becomes essential to deviate from the geodesic path. Placement of fiber under tension on a non-geodesic path generates a force component in the direction tangential to the mandrel surface. This force gives rise to tendency of slippage. The friction between the mandrel surface and the fiber is the only resisting force that can prevent this slippage. For slippage free winding, filament winding along a non-geodesic path requires effective balancing of slippage tendency (force component in the tangential direction) by the frictional force. The magnitude of the disturbing force depends on the geometry, of the mandrel surface and the fiber path on the surface. Similarly, the magnitude of the resisting friction force depends on the friction characteristics between fiber and surface together with the geometric parameters of the mandrel surface and the fiber path. Thus investigations on the stability of fiber on nongeodesic path as well as limits on deviation from geodesic path on any surface can be established by application of differential geometry.

Theoretical investigation on the stability of non-geodesic filament winding on an axisymmetric surface has been presented in this study. As already mentioned, it is essentially an application of differential geometry. Although, many of the expressions are applicable to any arbitrary shape of the surface, these are finally specialized to axi-symmetric surface as this is the shape of the surface of practical importance.

INTRODUCTION FORCE EQUILIBRIUM IN FILAMENT WINDING

In filament winding, the fibers are wound on the mandrel surface under preset tension. Since, the surface and consequently the fiber is curved, a force of magnitude F in the direction normal to the fiber will be developed to balance the tension T at two adjacent points on the fiber. For non-geodesic winding, the direction of normal to the fiber will be different from the direction of normal to the mandrel surface. Researchers can therefore visualize two components F_n and F_t of F in the directions normal and tangential to the surface, respectively. Let the angle between the normal to the surface and the normal to the fiber be ψ. The components F_n and F_t may be expressed as:

$$F_n = F\cos\psi$$
$$F_t = F\sin\psi \quad (1)$$

The tendency of slippage in non-geodesic winding is due to the destabilizing force Ft in the direction tangential to the surface. For stable winding, the destabilizing force Ft has to be resisted by friction force μF_n where, μ is the friction coefficient. Thus, the fiber will remain in position (stable winding) even on a non-geodesic path as long as:

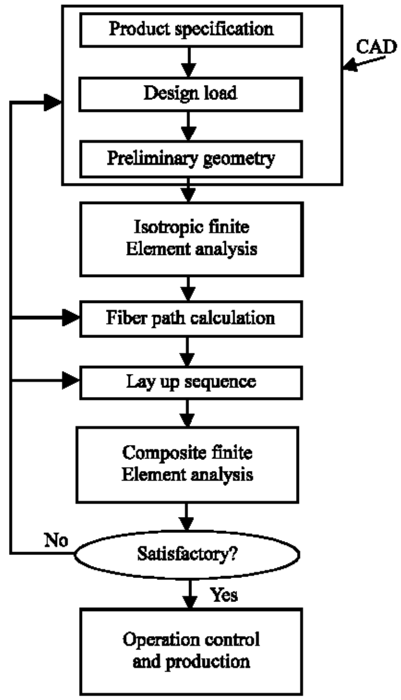

Fig. 1: The design methodology for filament winding

$$F_t \leq |\mu| F_n \quad (2)$$

For geodesic winding, the direction of normal to the fiber coincides with the direction of normal to the mandrel surface, thus making the tangential component of force F as zero. Hence, there will not be any fiber slippage. The design of a filament wound structure consists of the design of the mandrel shape and the calculation of the fiber path. When designing filament-wound parts use of an integrated strategy is recommended to make use of all composite benefits in spite of the restrictions imposed by the process. The basic methodology used for the design of filament wound parts is shown in Fig. 1.

SURFACE PARAMETERS AND FIBER SLIPPAGE

In filament winding, fibers are laid on a mandrel surface. Researchers can visualize the mandrel as a surface S and the fiber path as a curve C on the surface. Slippage condition Eq. 2 earlier is based on force balance. In order to extend the relation to useful equations, it is necessary to express Eq. 2 in terms of geometric parameters of the mandrel surface and the fiber path. This study establishes the slippage condition in terms of geometric parameters of the surface and curve on the surface.

Geodesic and normal curvatures: Consider a surface S (u, v) defined by the parameters u and v. The base vectors $\vec{a}^1$ and $\vec{a}^2$ at any point on the surface are tangential to the parametric curves v = const. and u = const, respectively. The plane T containing the base vectors $\vec{a}^1$ and $\vec{a}^2$ at any point is the plane tangential to the surface S at the point under consideration (the base vectors $\vec{a}^1$ and $\vec{a}^2$ at the point lies completely on the tangent plane). The unit normal to the surface n is normal to the tangent plane. Let C (s) be a curve on the surface. It is defined by the surface coordinates u (s) and v (s) of any point on the curve in terms of a parameter S which in this case is chosen to be the distance measured along the curve C. Let t_c be the unit tangent vector to the curve at any point. The unit tangent vector to the curve lies on the plane T tangent to the surface at the point. The curvature vector k is normal to the curve C with magnitude equal to the curvature k of the curve. The curvature vector k can be resolved into two components is shown in Fig. 2:

$$\vec{k} = \vec{k}_n + \vec{k}_g \tag{3}$$

Where:

$\vec{k}_n$ = The direction normal to the surface ($\vec{n}_s \equiv \vec{a}^3$) called Normal Curvature Vector the curve

$\vec{k}_g$ = Lies in the tangent plane T. This is called geodesic curvature vector of the curve

The direction of the curvature vector $\vec{k}_g$ is normal to the plane defined by tangent vector of the curve $\vec{t}_s$ and normal vector to the surface ($\vec{n}_s$). Let b be a unit vector in the direction of the curvature vector $\vec{k}_g$. Then:

$$\vec{b} = \vec{n}_s \times \vec{t}_s \tag{4}$$

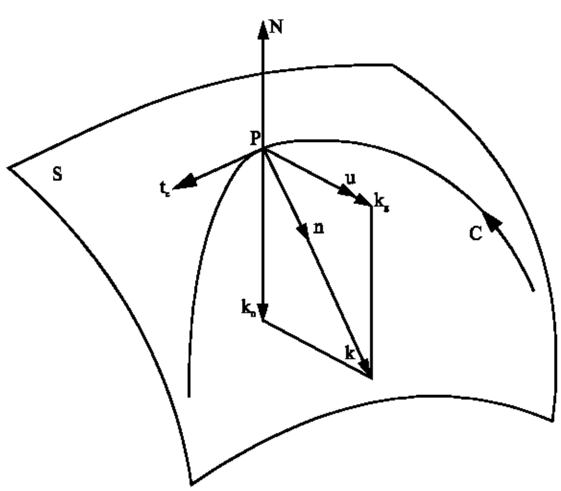

Fig. 2: Definition of geodesic curvature

The three unit vectors $\vec{n}_s$, $\vec{t}_c$ and $\vec{b}$ form right handed triad. The magnitudes of normal and geodesic curvature vectors of a curve are called the normal curvature and geodesic curvature, respectively. These are denoted by $\vec{k}_n$ and $\vec{k}_g$. These are therefore given by:

$$\begin{aligned} k_n &= \vec{k}.\vec{n}_s \\ k_g &= \vec{k}.\vec{b} \end{aligned} \tag{5}$$

The Curvature vector $\vec{k}$ follows as:

$$\vec{k} = k_n.\vec{n}_s + k_g\vec{b} \tag{6}$$

Consider the projection of the surface curve C on the tangent plane T and denote this by C*. For an arbitrary curve C, the projection C* will be curve with a non-zero curvature. It may be shown that the curvature vector of C* (projection of curve C on tangent plane T) is identical to the projection of the curvature vector of the original curve on the tangent plane T. Therefore, the geodesic curvature vector k_g may be interpreted in either of the following ways:

- Projection of curvature vector of C on the tangent plane T
- Curvature vector of projection C of on the tangent plane T

For a geodesic curve C. The projection C* is a straight line. The curvature of this path is always zero. Therefore, the curvature constraint will never be violated.

Stability of filament winding: Equation 2 expresses the stability of filament winding in terms of the force. From the definition of normal and geodesic curvatures in Eq. 3 and definition of F_n and F_t in Eq. 1, it is obvious that:

$$\frac{F_n}{F_t} = \frac{k_n}{k_g} \tag{7}$$

Therefore, the Eq. 2 representing stability of filament winding may be expressed in terms of geometric parameters by:

$$k_g \leq |\mu| k_n \tag{8}$$

Equation 8 determines the stability of winding along a (non geodesic) path in which the terms k_g and k_n depend on geometry of the mandrel surface together with orientation of fiber path on the mandrel and μ is the coefficient of friction. At this point, it may be appropriate to take note of the properties of a geodesic curve. For a geodesic curve:

- Normal to the curve and normal to the surface are identical
- Projection of geodesic curve C on the tangent plane T of the surface S is a straight line
- Force F does not have a tangential component: $F_t = 0$
- Geodesic curvature of a geodesic curve is zero: $k_g = 0$

Consequently, there is no tendency of slippage even for negligible friction. Therefore the curvature constraint will never be violated.

FILAMENT WINDING EQUATIONS FOR AXISYMETRIC SURFACE

In this study, the filament winding equations are derived for an axi-symmetric surface. For axi-symmetric surfaces, the meridional and circumferential directions are chosen as the parametric curves. In this case, the meridional and circumferential directions are chosen as u and v parameter curves, respectively.

In filament winding, the angle of winding α is an important parameter. In the context of surface and curve parameters, the angle of winding is essentially the angle between the tangent to the curve $\vec{t}_c$ and one of the parametric curves u = const. or v = const. Let the angle of winding α be defined as the angle between the fiber and the meridional curve v = const. The well known expressions of geodesic and normal curvatures in terms of the angle α will be given by the governing equation of stability.

Geodesic curvature: For a naturally represented curve C (s) (fiber path) on a surface S (u, v) (mandrel), Liouville's formula directly gives the geodesic curvature of the curve (fiber path) kg in terms of geodesic curvatures of u and v parameter curves and the angle α:

$$k_g = \frac{d\alpha}{ds} + (k_g)_u \cos\alpha + (k_g)_v \sin\alpha \tag{9}$$

where, $(k_g)_u$ and $(k_g)_v$ are geodesic curvatures of u and v parameter curves, respectively. If the It and v parameter curves are orthogonal (as in this case), the geodesic curvatures $(k_g)_u$ and $(k_g)_v$ are given by:

$$(k_g)_u = -\frac{E_v}{2E}\frac{1}{\sqrt{G}} \qquad (k_g)_v = \frac{G_u}{2G}\frac{1}{\sqrt{E}} \tag{10}$$

where, E and G are the coefficients of first fundamental fonn of the surface S (u, v). For an axi-symmetric surface, E and G are independent of v (circumferential parameter). Thus:

$$(k_g)_u = 0 \tag{11}$$

The geodesic curvature kg of a curve C (s) on an axi-symmetric surface is given by:

$$k_g = \frac{d\alpha}{ds} + \frac{1}{2}\frac{G_u}{G}\frac{1}{\sqrt{E}}\sin\alpha \tag{12}$$

Normal curvature: Euler's theorem expresses normal curvature k_n of a curve at a point on a surface in terms of Principal Curvatures k_1 and k_2 and the angle α. The principal curvatures k_1 and k_2 are curvatures along u and v parameter curves. The Euler's theorem states that:

$$k_n = k_1\cos\alpha + k_2\sin\alpha \tag{13}$$

The principal curvatures k1 and k2 can be expressed in terms of coefficients of the first and second fundamental form. The normal curvature kn is then expressed as: where L and N are the coefficients of the second fundamental form of the surface:

$$k_n = \left(\frac{L}{E}\right)\cos^2\alpha + \left(\frac{N}{G}\right)\sin^2\alpha \tag{14}$$

Filament winding equations: The equation of stability (no slippage condition) of filament winding along a non-geodesic path is obtained by substituting Eq. 12 and 14 in Eq. 8 which gives:

$$\frac{d\alpha}{ds} = -\frac{1}{2}\frac{G_u}{G}\frac{1}{\sqrt{E}}\sin\alpha + \mu\left\{\left(\frac{L}{E}\right)\cos^2\alpha + \left(\frac{N}{G}\right)\sin^2\alpha\right\} \tag{15}$$

The independent variable in the above equation is the path length along the curves whereas the parameters E, G, L, N and the derivative G_u are function of surface coordinates u and v. In order to solve the equation, the above equation has to be supplemented by additional equation. In order to achieve this it may be noted that for natural representation of a curve C (s) on a surface S (u, v), the scalar product of unit vectors along u parameter curve at a point on the curve and a tangent vector along the curve at that point gives:

$$\cos\alpha = \sqrt{E}\frac{du}{ds} \quad (16)$$

where, s is the arc length. Similarly, the scalar product of unit vectors along v parameter curve at a point on the curve and a tangent vector along the curve at that point gives:

$$\sin\alpha = \sqrt{G}\frac{dv}{ds} \quad (17)$$

The above two equations, rewritten as:

$$\frac{du}{ds} = \frac{1}{\sqrt{E}}\cos\alpha \quad (18)$$

$$\frac{dv}{ds} = \frac{1}{\sqrt{G}}\sin\alpha \quad (19)$$

have to be supplemented with the Eq. 15. The variables a and u may be obtained as a function of path length s by simultaneous solution of Eq. 15 and 18. Subsequently, v may be directly evaluated from the third relation Eq. 19.

Although, the independent variable s may. Not be as physically meaningful as other alternatives, namely, parameter u, the above equations in terms of variable s is most natural and most robust since the solution can be continued beyond the point of reversal without any difficulty (not so with u as independent variable).

Sometimes (e.g., cylindrical surfaces) it may be meaningful to rewrite the set of equations in terms of the independent variable u. Such alternative form is given below:

$$\frac{d\alpha}{ds} = -\frac{1}{2}\frac{G_u}{G}\frac{1}{\sqrt{E}}\tan\alpha + \mu\frac{\sqrt{E}}{\cos a}\left\{\left(\frac{L}{E}\right)\cos^2\alpha + \left(\frac{N}{G}\right)\sin^2\alpha\right\} \quad (20)$$

$$\frac{dv}{du} = \sqrt{\frac{E}{G}}\tan\alpha \quad (21)$$

This form however has the disadvantage of singularity around the point of reversal.

PARAMETERS FOR AXI-SYMMETRIC SURFACE

The coefficients of First and Second Fundamental Forms appear in the governing Eq. 15, 18 and 19. These functions will depend on the surface and will be different for different types of surfaces. In the context of filament winding on a non-geodesic path, a typical problem is determination of shape (profile) of isotensoid end dome of an internal pressure vessel. In this case, the shape of the surface is not known but is to be determined. Thus, the coefficients are not known apriori but are to be determined such that the surface represents an Isotensoid dome. In either case, it is necessary to define the coefficients of fundamental forms of an axi-symmetric surface in terms of function describing the profile.

Consider a global Cartesian Coordinate System (x, y and z). Let z axis be the axis of revolution of the axi-symmetric surface. The surface coordinate v has been assumed as the coordinate in the circumferential direction. The angle θ, measured in the circumferential direction is most appropriate for this. If researchers assume r (u) as the radius of the axi-symmetric surface for a particular value of the longitudinal parameter u then:

$$\begin{aligned} x &= r(u)\cos\theta \\ y &= r(u)\sin\theta \end{aligned} \quad (22)$$

The nature of function for z will depend on physical interpretation of u. During the course of study, two different forms were found appropriate. These, together with the expressions for coefficients are described.

Most general form for u: In this case, the surface coordinates are given by:

$$S(\theta) = \begin{Bmatrix} r(u)\cos\theta \\ r(u)\sin\theta \\ g(u) \end{Bmatrix} \quad (23)$$

for which the coefficients are given by:

$$\begin{aligned} E &= r'^2 + g'^2 \\ G &= r^2 \\ L &= \frac{r'g'' - g'r''}{\sqrt{r'^2 + g'^2}} \\ N &= \frac{rg'}{\sqrt{r'^2 + g'^2}} \end{aligned} \quad (24)$$

Above form was found most convenient for end domes of pressure vessel.

u is identical to axial distance z: In this case, the surface coordinates are given by:

$$x = r(u)\cos\theta$$
$$x = r(u)\sin\theta$$
$$z = u \quad (25)$$

for which the coefficients are given by:

$$E = 1 + r'^2$$
$$G = r^2$$
$$L = -\frac{r''}{\sqrt{1+r'^2}} \quad (26)$$
$$N = \frac{r}{\sqrt{1+r'^2}}$$

This form was found most convenient for a cylindrical surface.

NON GEODESIC WINDING: CASE STUDIES

A general purpose computer program, based on the above was developed for non-geodesic winding of axi-symmetric sections. The axi-symmetric section can be defined to the program either by specifying the equation of the profile (r (u)) or by specifying the functions describing the coefficients of fundamental forms or simply by specifying the surface coordinates. Either of the independent variables (s or u), may be chosen.

Iri this study, researchers shall consider two examples. The examples were selected keeping in mind filament winding of pressure vessel with end domes. Although, very simple shape has been chosen (closed form solution can indeed be obtained for both), it essentially demonstrates the working of the computer program.

Helical winding of cylindrical section: Consider a cylinder of radius R. It is most appropriate to use the form given in u is identical to axial distance z. The filament winding equation is then given as:

$$\frac{d\alpha}{dz} = \pm\frac{\mu}{R}\sin\alpha\tan\alpha \quad (27)$$

The above equation determines the angle of winding on a cylindrical section for nongeodesic winding. This gives maximum angle of winding that can be realized for the friction coefficient μ. Introducing normalization by:

$$\xi = \mu\frac{z}{R} \quad (28)$$

Leads to:

$$\frac{d\alpha}{d\xi} = \pm\sin\alpha\tan\alpha \quad (29)$$

The normalization leads to an Eq. 29 which is independent of and R and thus valid for all cylinders. The variation of angle of winding with normalized distance ξ is in the form of a single curve as shown in Fig. 3 and can be used as a nomogram for preliminary design studies.

Helical wincling on spherical section: Consider a sphere of radius R. Most appropriate choice for the parameter u is the spherical angle ϕ, measured from the base of the hemisphere. For this, it is most appropriate to utilize the form given in most general form for u. The filament winding equation is then given by:

$$\frac{d\alpha}{d\phi} = \pm\frac{\mu}{\cos\alpha} + \tan\phi\tan\alpha \quad (30)$$

In connection with filament winding, most important parameter is the pole opening radius (radius at the point of fiber reversal, $\alpha = 90°$). Since, the equation for spherical dome is independent of radius R, it is obvious that for same initial angle of winding and friction coefficient. The ratio of pole opening radius to the radius of the sphere will be same for all spheres, irrespective of diameter of the sphere.

The friction can be utilized to deviate on either side of the geodesic path. Therefore for any sphere, researchers can get two values of pole opening (by using positive and negative values of μ), one higher and the other lower than the pole opening for geodesic winding. For a given value of friction and initial angle of winding, these are the two bounds within which pole opening can

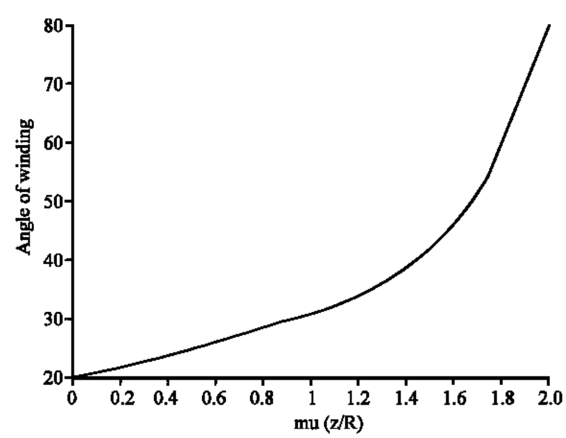

Fig. 3: Design nomogram for cylindrical section

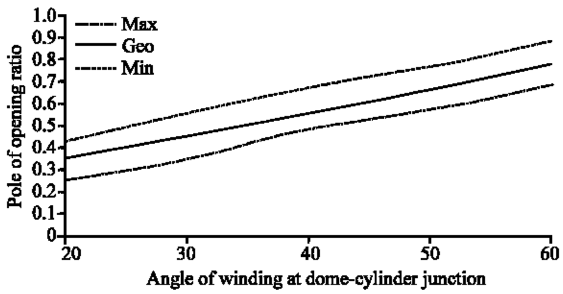

Fig. 4: Pole opening ratio for hemispherical section ($\mu = 0.1$)

be maintained. The variation of pole opening ratio with angle of winding at dome-cylinder junction for a typical value of friction coefficient $\mu = 0.10$ is shown in Fig. 4.

CONCLUSION

For development of a composite pressure vessel with integral domes having unequal pole opening by filament winding, it is essential to carry out winding on a non-geodesic path. Such is prone to slippage. It is therefore essential to have an apriori estimate of the maximum realizable deviation from the geodesic path. This research summarizes the efforts in this direction.

REFERENCES

Koussios, S., O.K. Bergsma and A. Beukers, 2005. Isotensoid pressure vessels based on nongeodesic roving trajectories. Proceedings of 15th International Conference on Composite Materials, June 27-July 1, 2005, Durban, South Africa.

Krikanov, A.A., 2000. Composite pressure vessels with higher stiffness. Compos. Struct., 48: 119-127.

Vasiliev, V.V., A.A. Krikanov and A.F. Razin, 2003. New generation of filament-wound composite pressure vessels for commercial applications. Compos. Struct., 62: 449-459.

Zu, L., S. Koussios, A. Beukers, 2010. Shape optimization of filament wound articulated pressure vessels based on non-geodesic trajectories. Compos. Struct., 92: 339-346.