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Modelling N-Team Interacting Decision Makers with Bounded Rationality Constraints

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Abstract: This presentation extends the research of Levis and Botcher on military decision analysis which lacked the ability to address hierarchical command structures. The research, though a theoretically analytic approach, employed games theory, combinatorics, induction and analytical geometry for derivations. The results show that the performance of the Nth Decision Maker (DMN) was greatly hindered due to the effects of the colossal amount of bounded rationality constraints (N^N) of command inputs. Hierarchical command organization diminishes in efficiency and thereby increases the work load of the DMN. Hence, Decision Centers (DC) are recommended for adaptation in order to remedy the anomaly arising from the effects of the imposed constraints. The threshold of the DM N was also derived.

Key words: Decision support, C^3I, interacting decision makers, hierarchical command structure, bounded rationality constraints, computational intelligence and information management

INTRODUCTION

Decision making, analysis and their applications to obtain optimal results, treated by Cooper and Klein (1980), Cooper *et al*. (1986), Cooper (1979), Hitchins (1989), Holt (1988) and Johnson (1981) are among the best researches that have considered extensively the various structures that support the process.

In another consideration of contribution, Dockery (1984), Earl and Johnson (1981), Edward and Lees (973) and Holt *et al*. (1992) provided some fundamental concepts and models in support of C^3I frame work. However, the best known work relating to a comprehensive consideration of complete decision-making process in the presence of a team of interacting Decision-Maker (DM) is that of Levis and Boettcher (1982). In that research, they considered a situation involving two teams of decision makers. The objective here is to extend their research to N-teams of decision makers at non-hierarchical and then at hierarchical levels of configuration. Some areas where these models have been applied are in the research of Iyang and Oladejo (1995), Oladejo (1995, 2006), Oladejo *et al*. (2006) and Oladejo and Ovuworie (2007).

The aim of this research is to develop a generalized model of C^3I for N-team interacting decision makers with bounded rationality constraints.

MATERIALS AND METHODS

The development of the generalized model of interacting decision makers with bounded rationality constraints and the resultant threshold was achieved through the application of games theory, combinatorics, induction and analytic geometry, respectively.

RESULTS AND DISCUSSION

Summary of the research of Levis and Boettcher: The cases of “Single Interacting Decision Maker with other members of his organization” and “A Team of two Decision Makers” have been treated by Levis and Boettcher (1982). They showed that the effectiveness of decision-makers diminishes with load due to bounded rationality constraints. They also showed that the bounded rationality constraint is proportional to the tempo of operation and that internal decision strategy is a complex combination of pure strategies for each decision-maker.

First, in some detail, a summary of their research is presented. The ultimate objective here is to generalize these results. In particular, efforts were made to model N (a positive integer) teams of interacting decision makers with bounded rationality.

Adopted from Levis and Boettcher (1982), Fig. 1 is the configuration of two interacting decision makers with bounded rationality constraints developed by Levis and Boettcher. By extension of Levis and Boettcher Model, consideration is given to N/2 pair of DMS, who have equal status (Oladejo, 1995). They are organized in parallel (equivalent to Divisional officers in the Navy). The results of analysis obtained were not different from those given by Levis and Boettcher.

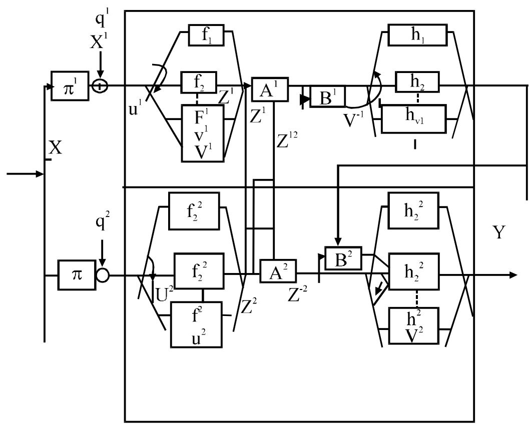

Fig. 1: A two decision maker N-team on none-hierarchical decision maker

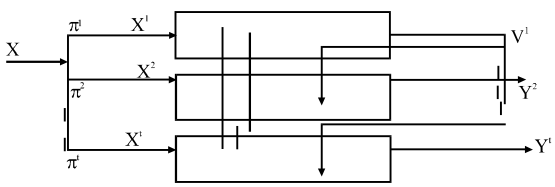

Fig. 2: Non-hierarchical command with semi-autonomous DM Analogous to CO and divisional officers' duties

Figure 2 shows the configuration of N interacting decision makers with bounded rationality constraints depicting non-hierarchical (parallel) structure. This is the researcher's configuration. This is an extension and modification of Fig. 1 by this research.

Figure 3 shows mixed strategies of two interacting decision makers with bounded rationality constraints. This was adopted from Levis and Boettcher (1982).

Figure 4 shows the throughput G and threshold J of two interacting decision makers with bounded rationality constraints.

Let z^{ij} be the situational assessment of 2 interacting Decision Makers (DM) transmitted between i and j where $i, j = 1, ..., t$. Researchers proceed by induction. For $t = 2, 3, ..., N$.

DM #1 (DM1):

$$G_t^1 = T(x^1, z^{12} : z^{21}, v') \quad (1)$$

$$G_b^1 = H(x^1, z^{12}) - G_t^1 \quad (2)$$

$$G_n^1 = H(u^1) + Hz^1(v^1) \quad (3)$$

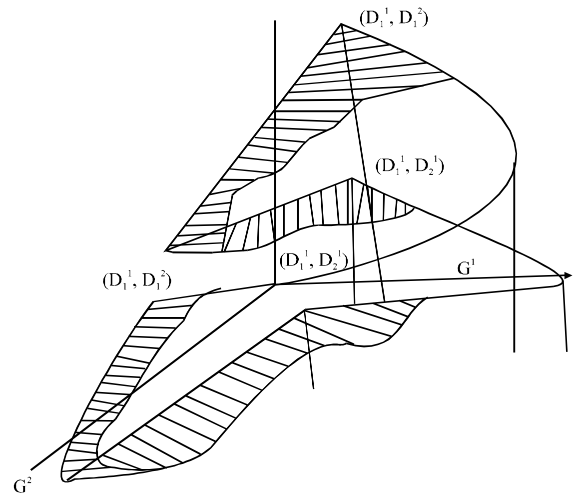

Fig. 3: The locus of binary variation of pure strategies for a team of two decision makers with two pure strategies each

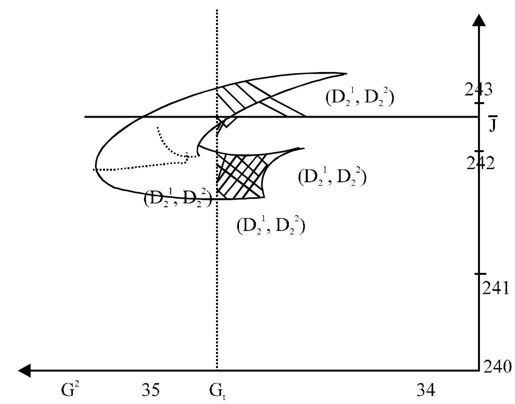

Fig. 4: Region of admissible (J, G^2) pairs

$$G_c^1 = \sum_{i=1}^{u^1}[p_i g_c^i(p(x^1)+\alpha_1 H(p_i)] + H(z^1, z^{21}) + g_c^{A^1}(p(z^1), p(z^{12})) + \sum_{i=1}^{v^1}[p_j g_c^i(p(z^1 | \bar{v}^1) + \alpha_j H(p_j))] + H(v') + H(z^1) + H(\bar{z}^1) + T_{z^1}(x^1 : z^{12}) \quad (4)$$

DM # 2 (DM): Copy DM#N and replace digit N with 2. Now for t = N, by way of extension.

DM #N (DMN):

$$G_t^N = T(x^N, z^{N1}, v' : z^{1N}, y) \quad (5)$$

$$G_b^N = H(x^N, z^{N1}, v') - G_b^N \quad (6)$$

$$G_n^N = H(\mu^N) + H_{\bar{Z}^N}(\nu^N) \quad (7)$$

$$G_c^N = \sum_{i=1}^{U^N}[p_j g_c^i(p(x^N)) + \alpha_j H(p_j)] + H(z^N, z^{1N}) + g_c^{AN}(p(z^N), p(z^{N1})) + g_c^{B^2}(p(\bar{z}^N), p(\nu')) \sum_{j=1}^{\nu^N}[p_j g_c^j(p(z^N \mid \nu^{-N})) + \alpha_j H(p_j)] + H(Y) + H(z^N) + H(z^{-N}) + H(z^{-N}, \nu^{-N}) + T^{zN}(x^N : z^{N1}) + Tz^{-N}(x^N, z^{N1} : \nu^1) \quad (8)$$

By the partition law of information, the total processing activities for each DM is given by $G^t = G_t^t + G_b^t + G_n^t + G_c^t$ which depends on maximum rate of processing information F as they arrive and the tempo of activities τ (i.e., symbol of mean inter-arrival time).

It was established that internal decision strategy is a convex combination of pure strategies for each DM. Their behavioral strategies are generated by independent decision strategies. The total processing activities is in turn generated by the combined behavioral strategies. The DM processes his inputs at a rate that is at least equal to the rate with which they arrive (rate of processing cannot exceed the rate with which they arrive). The implication of this is that the configuration of, the universal norm of non-infallibility of human decision makers predicates rationality boundedness which in turn, was being modeled as a constraint on their processing capabilities when various modes of interactions between and among the teams of communicating decision makers were considered. Furthermore, the performance/efficiency of DMs decision makers cannot exceed a specified level due to these constraints, giving rise to the constraint:

$$G^t = G_t^t + G_b^t + G_n^t + G_c^t \le F^t.\tau \text{ or } G^t = G^t(\Delta_b) \le F^t.\tau \quad (9)$$

where, Δ_b is the behavioural strategy and also a function of decision randomized strategies:

$$p(\nu' \mid \bar{z}^1), p(\nu^N \mid \bar{z}^N)$$

The total processing activities of DM can be expressed as a convex function of Δ_b thus:

$$G^t(\Delta_b) \ge \Sigma_{k,l} G^t(\Delta_{kl}) p_k p_l \quad (10)$$

where, p_k, p_l are probabilities associated with the respective mixed strategies. Given that Y is actual response while Y^1 is the desired response, let the function $d(Y, Y^1)$ be a cost on the pair for any observable deviation. Also, let the performance index of the DM be J where:

$$J(\Delta_b) = E\{d(Y, Y')\} \ne p(Y \ne Y') \quad (11)$$

The probability of making the wrong decision in response to the input information. So, it is desired to minimize this probability. Since, $\Delta_{lk's}$ have associated probability and J is a distribution function for every Δ_{lk}, there is an associated value J_{kl} of performance index which implies the mapping $\Delta_{kl} \rightarrow J_{kl}$ is invertible or one to one and onto. The organizational performance can be represented thus:

$$J(\Delta) = \Sigma_{k,l} J_{kl} p_k p_l \quad (12)$$

Then, Eq. 1 and 2 are stochastically parametric. This stochastical parametricity brings these geometric entities $G^t(\Delta_{kl})p_k p_l$ and $J_{kl} p_k p_l$ into one envelope. Hence, $\sum_{k,l} G^t(\Delta_{kl}) p_k p_l$ and $\sum_{k,l} J_{kl} p_k p_l$ and would yield similar results. Since, the objective is to minimize J or find its lower bound, it would be pertinent to consider the locus of all admissible (J, G^1, G^N) triples which is obtained by constructing first, all binary variations between pure strategies; each of which produce a line in the three dimensional space (J, G^1, G^N) from (D_1^1, D_2^1) and (D^N_1, D^N_2) for DM1, DMN, respectively. Since, there are two sets of decision makers (DM1, DMN) each of the two pure strategies (one with v', the other free of v') there are altogether four pure strategies ($2^2 = 4$) with associated binary variations and (D_1^1, D_1^N), (D_1^1, D_2^N), (D_2^1, D_1^N) and (D_2^1, D_2^N). The triples (lines) associated with these binary pure strategies are $(0, G^1, G^N)$, $(J, G^1, 0)$ and $(J, 0, G^N)$ in the same space.

From the example when t = 2, same result would be obtained as when t = N, since this is a non-hierarchical command with semi-autonomous DM like divisional officers (i.e., DM2, DM3, ..., DMN have identical status). It was observed that the total processing activity and the workload of DM1 were not affected by the DMN's strategies whereas both the total processing activity and the workload of DMN were affected by the command or direct control input. This is so because the situation assessment Z^{1N} passed from DMN to DM1 was for information purposes (SITREP) which may or may not be acted upon but the command input v' from DM1 to DMN must be acted upon.

A similar surface was generated as in the case when t = 2 by considering all combinations of mixed strategy pairs of behavioral strategies and it was found that minimizing value of performance index which is obtained from a pure organizational strategy is the same as minimizing the workload by its (wrt) own strategies. Hence, minimum error strategy is a pure strategy without bounded rationality constraint. With this chain of events:

$$(\underline{X}) \text{ input} \rightarrow \underset{\text{strategy}}{\text{behavioral}} \rightarrow \underset{\text{strategy}}{\text{decision}} \rightarrow \underset{\text{strategy}}{\text{organizational}}$$

(stimulus effect responses overall response) $\rightarrow$ performanceindex(Y) $\rightarrow$ workload outcomes (efficiency or error judgment)

The input could be predetermined for the realization of specified target which is the essence of command, control, communication and intelligence (C^3I). NB: for two factors at 2 levels {(1, 2), (A, B)} all possible binary combinations are A1, A2, B1, B2 and if (1, 2) ≡ ($D_1^2 D_2^2$), (A, B) ≡ (D_1^1, D_2^1) then all possible binary combinations are (D_1^1, D_2^2), (D_1^1, D_2^2), (D_2^1, D_1^2) and (D_2^1, D_2^2).

The bounded rationality constraints were obtained in three dimensional space $R^3(J, G^1, G^N)$ in the forms of planes of constants G^t. The constraint to DMI is a plane parallel to the G^N axis and intersecting the G^1 axis at $G^1 = F^1$, τ with $G^1 \le G^2$ t = r a specified value.

G the total processing activities inequality implies that the DM rate of processing information cannot exceed the rate at which they arrive as inputs. The constraint of DMN is a plane that intersects G^N axis at:

$$G^N = F^N \tau \le J \tag{13}$$

For fixed values of F^t, the bounded rationality constraint is proportional to the tempo of operation, i.e., $G^t \propto \tau$. But as the tempo increases the G^t becomes smaller and fewer of the potential strategies are feasible, i.e., $G^t \propto \tau^{-1}$ or as $\tau \to \infty$, $G_\tau^t \to 0$ which implies activities slow down and there is hardly any variational observation in strategies. The solution of the satisfying problem or optimal solution to the stimulus is a subset of the solution of the feasible solution in the feasible space defined by:

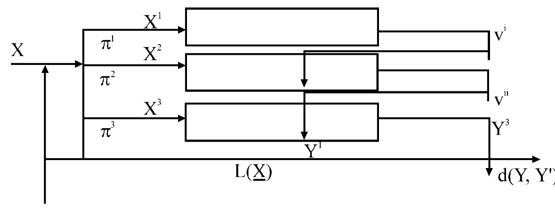

Fig. 5: Supreme commander (NB $Y^t = v^{t\prime}$)

$$J(\Delta) = \Sigma_{k,1} J_{kl} p_k p_l \le \bar{J} \tag{14}$$

where, $\bar{J}$ the threshold, is a constraint. Equation 13 is a plane (J, G^1, G^N) parallel to the (G^1, G^N) plane and intersects the j-axis at $\bar{J}$. The satisfying strategies are all points in the locus below the plane $\bar{J}$ that also satisfy the bounded rationality constraints.

C^3I hierarchical command (supreme commander) depicting chain of command with subordinates in a 3-team: See Fig. 1 and 2 and then consider the extension of Levis and Botcher Model, Fig. 5 is an extension and modification of Fig. 1 by this research.

This is the configuration of three interacting hierarchical decision makers with bounded rationality constraints.

The solutions space hence forth would be derived through theoretical game (because pdfs were used as strategies) approach and analytical geometry (because R^{N+1} dimensional space was employed where the N represents the number of DMs while the 1 is for their joint performance J).

DM 1:

$$\begin{aligned}
&G = T(x, z, zG_t^1) = T(x^1, z^{12}, z^{13}: z^{21}, z^{31}, v')\\
&G_b^1 = H(x^1, z^{12}, z^{13}) - G_t^1\ ;\ G_n^1 = H(u^1) + H_z^1(v^1)\\
&u^1:\ G_c^1 = \Sigma_{i=1}\{p_i\, g_c^i(p(x^1) + \alpha_I H(p_i)\} + H(z^1, z^{21}, z^{31}) + g_c^{A1}\{p(z^1)p(z^{12}), p(z^{13})\}\\
&v^1:\ \ +\Sigma_{j=1}[p_j g^j{}_c(p(z^{-1}|v') + \alpha_j H(p_j)] + H(v') + H(z^1) + H(z^{-1}) + Tz^1(x^1: z^{12}, z^{13})
\end{aligned} \tag{15}$$

($g_c^{B\prime}$ is omitted because no command input v' to DM1).

DM 2:

$$\begin{aligned}
&G_t^2 = T(x^2, z^{21}, z^{23}v': z^{12}, z^{23}, v'')\\
&G_b^2 = H(x^2, z^{21}, z^{23}, v') - G_t^2\\
&G_n^2 = H(u^2) + Hz^{-2}(v^2)\\
&u^2:\ G_c^2 = \sum_{i=1}[p_i g_c^i(p(x^2)) + \alpha_I H(p_i)] + H(z^2, z^{12}, z^{32}) + gA_c^2(p(z^2), p(z^{21}), p(z^{23}))\\
&v^2:\ +gB_c^2(p(z^2), p(v')) + \sum_{j=1}[p_j g_c^j(p\,(z^{-2}/v^{-2}) + \alpha_j H(p_j))] + H(v'') + H(z^{-2}) + H(z^{-2}) + H(z^{-2}, v^{-2}) +\\
&\qquad Tz^2(x^2: z^{21}, z^{23}) + Tz^2(x^2, z^{21}, z^{23}: v')
\end{aligned} \tag{16}$$

DM 3:

$$
\begin{aligned}
&G_t^3 = T(x^3, z^{32}, z^{31}, v^{"}: z^{13}, z^{23}, Y^3)\\
&G_b^3 = H(x^3, z^{32}, z^{31}, v^{"}) - G_t^3\\
&G_n^3 = H(u^3) + Hz^{-3}(v^3)\\
&u^3:\ \ G_c^3 = \Sigma_{i=1}[p_i g_c^i(p((x^3)) + \alpha_i H(p_i)] + H(z^3, z^{13}, z^{23}) +\\
&\qquad g_c A^3(p(z^3), p(z^{31}), p(z^{32})) + g_c B^3(p(z^{23}), p(v^{"}))\\
&v^3:\ \ +\Sigma_{j=1}[p_j g_c^j(p(zZ^{-3} \mid v^{-3})) + \alpha_j H(p_j)] + H(Y^3) + H(z^3) +\\
&\qquad H(z^{-3}) + H(z^3, v^3) + Tz^{-3}(x^3: z^{31}, z^{32}) + Tz^{-3}(x^3, z^{31}, z^{32}: v'')
\end{aligned}
\tag{17}
$$

Since, there are $n_1 = U^1V^1M^1$ and $n_2 = U^2V^2M^2$ possible decision strategies for DM1 and DM2, respectively there are $n_3 = U^3V^3M^3$ possible decision strategies for DM3 also. Since, internal decision strategies are a convex combination of pure strategies, let the respective strategies for DM1, DM2 and DM3 be:

$$
\begin{aligned}
n_1 D^1(p_{k1}) &= \Sigma_{k1=1} p_{k1} D_{k1}^2\\
n_2 D^2(p_{k2}) &= \Sigma_{k2=1} p_{k2} D_{k2}^2\\
n_3 D^3(p_{k3}) &= \Sigma_{k3=1} p_{k3} D_{k3}^2
\end{aligned}
\tag{18}
$$

where, p_{k1}, p_{k2}, p_{k3} are the corresponding probabilities. By the Partition Law of Information and total processing activities:

$$G^t = G_t^t + G_b^t + G_n^t + G_c^t \le F^t t,\ t = 1, 2, 3 \tag{19}$$

Also:

$$G^1 = G^1(\Delta_b), G^2 = G^2(\Delta_b) \text{ and } G^3 = G^3(\Delta_b) \tag{20}$$

To obtain the minimum value of J, the locus of all admissible ((J), G^1, G^2, G^3) quadruples would be considered. First construct all binary variations between pure strategies; each of which produces a plane in the quadruple dimensional space R^4 (J, G_3^1, G^2, G^3) from pure strategies (D_1^1, D_2^1, D_3^1), (D_1^2, D_2^2, D_3^2), (D_1^3, D_2^3, D_3^3) for DM1, DM2, DM3, respectively. Since, there are three decision makers each with three pure strategies, there are altogether $3^3 = 27$, twenty-seven pure strategies (3 factors at 3 levels) with an associated triple variations: Consider three factors at 3 levels: (A, B, C), (1, 2, 3) all possible binary combinations would be: A1 A2 A3, B1, B2, B3, C1, C2, C3 while all possible triple combination are: {A1A A2A A3A, A1B A2B A3B A1, A1C A2C A3C, B1A B2A B3A, B1B B2B B3B, B1C B2C B3C, C1A C2A C3A, C1B C2B C3B, C1C C2C C3C}. So that between DMI×DM2, researchers have: (D_1^1, D_1^2), (D_1^1, D_2^2), (D_1^1, D_3^2), (D_2^1, D_1^2), (D_2^1, D_2^2), (D_2^1, D_3^2), (D_3^1, D_1^2), (D_3^1, D_2^2), (D_3^1, D_3^2); between DMI×DM3, researchers have: (D_1^1, D_1^3), (D_1^1, D_2^3), (D_1^1, D_3^3), (D_2^1, D_1^3), (D_2^1, D_2^3), (D_2^1, D_3^3), (D_3^1, D_1^3), (D_3^1, D_2^3), (D_3^1, D_3^3); between DM2×DM3, researchers have (D_1^2, D_1^3), (D_1^2, D_2^3), (D_1^2, D_3^3), (D_2^2, D_1^3), (D_2^2, D_2^3), (D_2^2, D_3^3), (D_3^2, D_1^3), (D_3^2, D_2^3), $(D_3^2\ D_3^3)$. As all the possible binary combinations.

All possible triple combinations would now be obtained from the binary combinations, thus, between (DMI, DM2)×(DM3): (D_1^1, D_1^2, D_1^3), (D_1^1, D_2^2, D_2^3), (D_1^1, D_3^2, D_3^3), (D_2^1, D_1^2, D_1^3), (D_2^1, D_2^2, D_2^3), (D_2^1, D_3^2, D_3^3), (D_3^1, D_1^2, D_1^3), (D_3^1, D_2^2, D_2^3), (D_3^1, D_3^2, D_3^3); between (DMI, DM3)×(DM2): (D_1^1, D_1^3, D_1^2), (D_1^1, D_2^3, D_2^2), (D_1^1, D_3^3, D_3^2), (D_2^1, D_1^3, D_1^2), (D_2^1, D_2^3, D_2^2), (D_2^1, D_3^3, D_3^2), (D_3^1, D_1^3, D_1^2), (D_3^1, D_2^3, D_2^2), (D_3^1, D_3^3, D_3^2); between (DM2, DM3)×(DM1): (D_1^2, D_1^3, D_1^1), (D_1^2, D_2^3, D_2^1), (D_1^2, D_3^3, D_3^1), (D_2^2, D_1^3), D_1^1, (D_2^2, D_2^3, D_2^1), (D_2^2, D_3^3, D_3^1), (D_3^2, D_1^3, D_1^1), (D_3^2, D_2^3, D_2^1), (D_3^2, D_3^3, D_3^1).

The quadruples associated with these triple pure strategies are (0, G^1, G^2, G^3), (J, O, G^2, G^3), (J, G^1, O, G^3), (J, G^1, G^2, O), (J, G^1, G^2, G^3) in the space R^4(J, G^1, G^2, G^3).

In this Hierarchical Model, the performance of the DM3 would be investigated since previous example has demonstrated the influence of DMI on DM2 using the same approach of Stochastic parametricity in the feasible R^4 space containing the locus of all admissible triple pure strategies. The location of these points may prove difficult graphically. If the satisfying locus is established the bounded rationality constants would now be obtained from the R^4 space (J, G^1, G^2, G^3) in the form of surfaces of constraints G^t, t = 1, 2, 3. It would be observed that the bounded rationality constraint on DM3 are many due to command inputs and encompassing thereby adversely affecting the performance index and consequently increasing the workload or reducing the efficiency (because processing rate dwindles while duration increases; $F \to 0$ and $T \to \infty$). The performance, J which is a function of pure strategies is diminished when the characteristics of the decision makers are hampered with the increase in the number of constraints. The solution as usual would be subspace of the feasible space defined by:

$$J(\Delta) = \Sigma_{k,1,m} J_{klm} p_k p_l p_m \le \bar{J} \tag{21}$$

Table 1: Bounded constraints

N	N^2	N^3	N^4	N^5	N^6	N^7	N^8	N^9	N^{10}
0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256	512	1024
3	9	27	81	243	729	2187	6561	19683	59049
4	16	64	256	1024	4096	16384	65536	262144	1048576
5	25	125	625	3125	15625	78125	390625	1953125	9765625
6	36	216	1296	7776	46656	279936	1679616	10077696	60466176
7	49	343	2401	16807	117649	823543	5764801	40353607	282475249
8	64	512	4096	32768	262144	2097152	16777216	134217728	1073741824
9	81	729	6561	59049	531441	4782969	43046721	387420489	3486784401
10	100	1000	10000	100000	1000000	10000000	100000000	1000000000	10000000000

NB: $G^t = F^t.T \leq J$ with $G^t \leq G^{t+1}$

Table 2: G^2 versus J_2

G^2	$F^2T = 2$	J_2	G^3	J_3	G^N	J_N
13.8	-	240.0	-	-	-	-
14.5	-	241.0	-	-	-	-
14.9	-	242.0	-	-	-	-
15.0	-	242.7	-	-	-	-
14.2	-	243.6	-	-	-	-
14.3	-	241.4	-	-	-	-

G^t is bounded rationality; F^t is rate of processing; T is temp of events

Table 3: Regression values for $G^2 = aJ_2+b$

Y	x	x^2	xy	$\log_y = z$	$\log_x = w$
13.8	240.0	57600.00	3312.00	1.139879	2.380211
14.5	241.0	58081.00	3494.50	1.161368	2.382017
14.9	242.0	58564.00	3605.80	1.173186	2.383815
15.0	242.7	58903.29	3640.50	1.176091	2.385070
14.2	243.6	59340.96	3459.12	1.152288	2.386677
14.3	241.4	58273.96	3452.02	1.155336	2.382737
86.7	1450.7	350763.20	20963.94	6.958149	14.300530

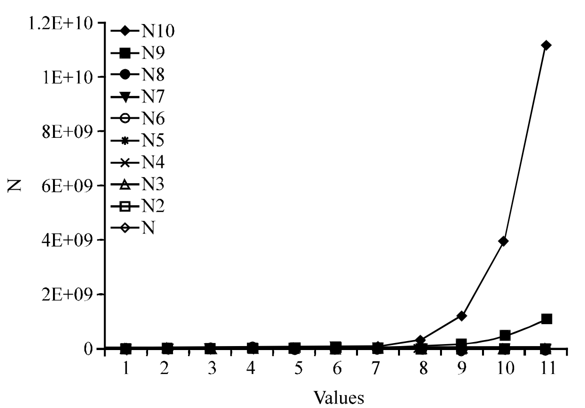

Fig. 6: The graph of Table 1 of bounded constraints

J being the threshold. Equation 19 is surface in the space (J, G^1, G^2, G^3) which intersects the J-axis at J. The satisfying strategies are all points in the locus below the planes that also satisfy the bounded rationality constraints. NB: The boundry relationality is:

$$G^t = J_t \text{ for } t = 1, 2, 3 \qquad (22)$$

Oladejo is an application of C^3I model related to this research. Now considering a more general case when t = N (Fig. 6). Table 1 gives the values of bounded rationality constraints with associated decision makers. Figure is the graph of values on Table 6. The values in this table were obtained from the Fig. 1 of admissible (J, G^2) pairs []. Let the steady state relationship between G^2 and J_2 be linear (y = ax+b). Then, $G^2 = aJ_2+b$. Table 2 gives the values of the throughput G and the threshold J of 2 interacting decision makers with bounded rationality constrains. Table 3 gives the values for obtaining regression equation for 2 interacting decision makers with bounded rationality constraints. Figure 7 is the graph obtained from Table 3. For y = ax+b:

$$\sum y = a\sum x + nb$$

$$\sum xy = a\sum x^2 + b\sum x$$

$$b = \bar{y} - a\bar{x}$$

$$a = \frac{n\sum xy - \left(\sum x\right)^2}{n\sum x^2 - \left(\sum x\right)^2} = \frac{6(20963.94) - (1450.7)^2}{6(350763.21) - (1450.7)^2}$$

$$= \frac{125783.64 - 2104530.49}{2104579.26 - 2104530.49}$$

$$= \frac{-1978746.85}{48.77} = -40573.03$$

$$b = \frac{86.7}{6} - (-40573.03)\frac{1450.7}{6}$$

$$= 14.45 + 9809883.31 = 980.9897.76$$

By least square a = -40573.03, b = 9809897.76. Hence:

$$y = -40573.03x + 9809897.76$$

$$G_1^2 = -40573.03J_2 + 9809897.76$$

Then:

$$G_1^t = -40573.03J_t + 9809897.76$$

So that:

$$G_1^3 = -40573.03J_3 + 9809897.76$$

Let J_3 have arbitrary values 245, 246, 247, 248, 249, 250. Then, Table 4 gives the values of the throughput G

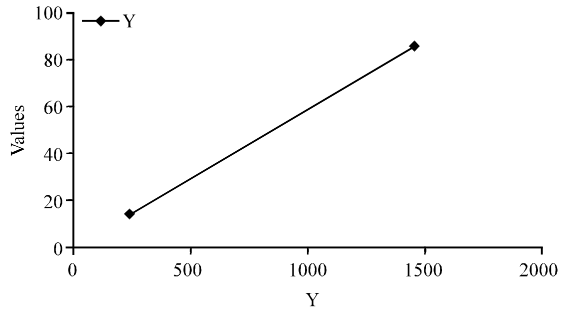

Fig. 7: Linear regression of admissible (J_2, G^2) pairs

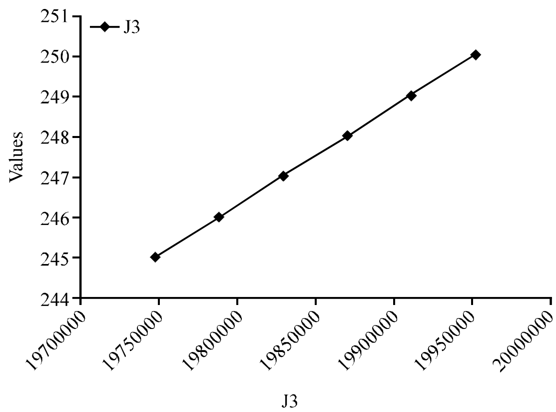

Fig. 8: Graph of admissible (J_3, G^3) pairs

Table 4: G_1^3 versus J_3

G_1^3	J_3
19750290	245
19790863	246
19831436	247
19872009	248
19912582	249
19953155	250

Table 5: Regression values for $G_e^t = (J_t)^a.b$

Z	W	W²	WZ
1.14	2.380	5.664400	2.713200
1.161	2.382	5.673924	2.765502
1.173	2.384	5.683456	2.796432
1.176	2.385	5.688225	2.804760
1.152	2.387	5.697769	2.749824
1.155	2.383	5.678689	2.752365
6.957	14.301	34.086460	16.582080

and the threshold J of 3 hierarchical interacting decision makers with bounded rationality constrains (Fig. 8). Figure 8 is the graph of values on Table 4.

Let the steady state relationship between G^2 and J_2 be exponential ($y = x^a b$ or $\log y = a \log x+\log b$). Then, similarly, $G_e^t = (J_t)^a.b$ or $\log G_e^t = a \log J_t+\log b$. For $y = x^a b$ or $\log y = a \log x+\log b$ or $z = aw+u$ where $z = \log y$, $w = \log x$, $u = \log b$.

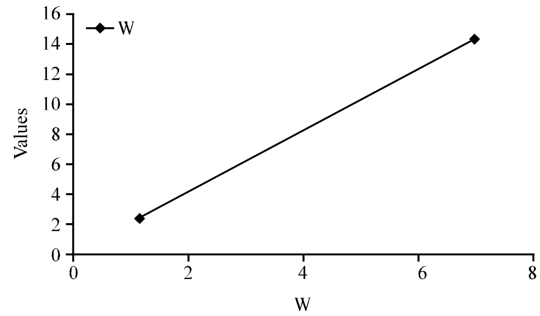

Fig. 9: Graph of admissible (J_4, G^4) pairs

Table 6: G_1^3 versus J_3

G_1^3	J_3
0	0.0
2.3788E-29633	0.2
8.3877E-12763	0.5
1	1.0
6.7681E1764	1.1
7.3311E3356	1.2

Table 5 gives the values for obtaining regression equation for 3 hierarchical interacting decision makers with bounded rationality constraints. Figure 9 is the graph of values on Table 5:

$$u = \bar{z} - a\bar{w} = 1.1595 - a(2.3835)$$

$$a = \frac{n\sum wz-\left(\sum w\right)^2}{n\sum w^2-\left(\sum w\right)^2} = \frac{6(16.5821)-(14.301)^2}{6(34.086)-(14.301)^2}$$

$$= \frac{-105.0262}{-0.0026} = 40394.6923$$

$u = -96279.5896$, $b = \text{antilog } u = \text{antilog } (-96279.5896)$. Hence, $y = x^{42394.69231}$. b. Similarly, $G_e^t = J_t^a.b = J_t^{42394.69231}$ so that $G_e^3 = J_3^a.b = J_t^{42394.69231}$.

By least square method $a = 42394.69231$, $b = \text{antilog}$ (-96279.5896). Hence, $y = x^a.b = x^{42394.69231}$, $G_e^t = J_t^a.b = J_t^{42394.69231}$. So that $G_e^3 = J_3^a.b = J_3^{42394.69231}$.

Using the same arbitrary values of J_3 above, this gave astronomical values that can not be represented. Now choosing lower the result is as shown in Table 6.

Table 6 is values of throughput G and the threshold J of 3 hierarchical interacting decision makers with bounded rationality constraints. This is the graph of Table 6 which is non-linear. Figure 10 is the graph of values on Table 6.

For $y = a^x b$, a geometric steady state relationship. Then, $\log y = x \log a+\log b$ or $t = px+q$ where $t = \log y$, $p = \log a$, $q = \log b$. For $y = a^x b$ or $\log y = x \log a+\log b$:

$$t = px + q$$
$$q = \bar{t} - p\bar{x}$$

$$p = \frac{n\sum tx - \left(\sum x\right)^2}{n\sum x^2 - \left(\sum x\right)^2} = \frac{-2094437.732}{47.51} = -44084.1451$$

Table 7 gives the values for obtaining regressionequation for 3 hierarchical interacting decision makers with bounded rationality constraints:

$$\begin{aligned} q &= \bar{t} - px \\ &= 1.1595 - (-44084.1451)(241.7833) \\ &= 10658811.24 = 1.0659\times 10^7 \end{aligned}$$

$\therefore$b = antilog q, a = antilog p = antilog (-44084.145) = does not exist. Hence, y = $a^x b$ and similarly, $G_g^t = a^{J_t}.b$ so that $G_g^3 = a^{J_3}.b$.

Since, the antilog of a negative number does not exist this model is invisible for the data. Since, the solution is infeasible there is no regression equation.

The main contribution of this reserch: For t = N$\in$II$^+$ (a finite positive integer), the N-team of Hierarchical organisation would be developed using games theory, combinatorics, induction and analytical geometry, respectively.

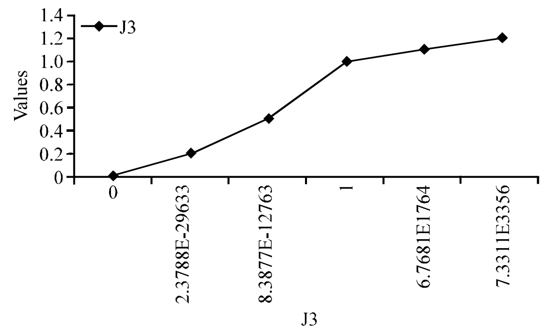

Fig. 10: Graph of admissible (J_3, G^3) pairs

Table 7: Regression values for $G_g^t = J_t^a.b$

t	x	x^2	tx
1.14	240.0	57600.00	273.6000
1.161	241.0	58081.00	279.8010
1.173	242.0	58564.00	283.8660
1.176	242.7	58903.29	285.4152
1.152	243.6	59340.96	280.6272
1.155	241.4	58273.96	278.8170
6.957	1450.7	350763.20	1682.1260

Theoretical consideration: The internal decision strategies which are convex combination of pure strategies are:

$$n_t:\ D^t(p_{kt}) = \Sigma_{kt}\, p_{kt} D^t_{kt},\ t = 1,\ldots,N$$

The total processing activity:

$$G^t = G_t^t + G_b^t + G_n^t + G_c^t \le F^t.t,\ t = 1,\ldots,N \tag{23}$$

For a team of N DM's (DM t (t = 1, 2, ..., N)):

$$\begin{aligned} G^t = {} & T(X^t, z^{t(N-1)},\ldots, z^{t1}, V^{(N-1)1}: z^{1t},\ldots, z^{(N-1)t}, Y^t) + H(X^t, z^{t(N-1)},\ldots, z^{t1}, V^{N-1)'}) - G_t^t + H(u^t) + \\ & Hz^{-t}(v^t) + \Sigma_{i=1}[p_i g_c^i(p(X^t) + \alpha_i H(p_i))] + H(z^t, z^{1t},\ldots, z^{(N-1)t}) + g_c^{At}(p(z^t), p(z^{t1}),\ldots, p(z^{t(N-1)})) + \\ & g_c^{Bt}(P(z^t), p(V^{(N-1)'})) + \sum_{j=1}^{vt}[p_j g^j{}_c(p(z^{-t} \mid v^{-t})) + \alpha_j H(p_j)] + H(Y^t) + H(z^t) + H(z^{-t}) + H(z^{-t}, v^{-t}) + \\ & Tz^{-t}(X^t: z^{t1},\ldots, z^{-t(N-1)}) + Tz^{-t}(X^t, z^{t1},\ldots, z^{t(N-1)}: V^{(N-1)'}) \end{aligned} \tag{24}$$

DM1:

$$\begin{aligned} G^1 = {} & T(X^1, z^{1(N-1)},\ldots, z^1, V^{(N-1)1}: z^1,\ldots, z^{(N-1)1}, Y^1) + H(X^1, z^{1(N-1)}, z^1, v^{N-1)'}) - \\ & G_t^1 + H(u^1) + H_z(v^1) + V^1 \Sigma_{i=1}[p_i g_c^i(p(X^1) + \alpha_i H(p_i)] + H(z^1, z^1,\ldots, z^{(N-1)1}) + \\ & g_c^{A1}(p(z^1), p(z^1),\ldots, p(z^{1(N-1)})) + V^1 \Sigma_{j=1}[p_j g_c^j(p(z^1 \mid V^1)) + \alpha_j H(p_j)] + H(Y^1) + \\ & H(z^1) + H(z^1) + H(z^1, V^1) + T(X^1: z^1,\ldots, z^{1(N-1)}) + T_Z^{-1}(X^1, z^1,\ldots, z^{1(N-1)}: V^{(N-1)'}) \end{aligned} \tag{25}$$

NB: $Y^t = V^{(t)'}$. The DM N:

$$\begin{aligned} G^N = {} & T(X^N, z^{N(N-1)},\ldots, z^{N1}, v^{(N-1)'}: z^{1N},\ldots, z^{(N-1)N}, Y^N) + H(X^N, z^{N(N-1)},\ldots, z^{N1}, v^{(N-1)'}) - \\ & G_t^N + H(U^N) + H_z^N(v^N) + U^N \Sigma_{i=1}[p_i g_c^i(p(x^N)) + \alpha_i \mid + p_i] + H(z^N, z^{iN},\ldots, z^{(N-1)N}) V^N + \\ & g_c^{AN}(p(z^N), p(z^{N1}),\ldots, p(z^{N(N-1)}) + g_c^{BN}(p(z^N), p(v^{(N-1)1})) + \Sigma_{j=1}[p_j g_c^j(p(z^N = v^N)) + \\ & \alpha_j K \mid + p_j] + H(Y^N) + H(Z^N) + H(\overline{N}) + H(z^{\overline{N}}, v^N) + T(x^N: z^{N1},\ldots, z^{N(N-1)})) + \\ & T_z^{\overline{N}}(x^N, z^{N1},\ldots, z^{N(N-1)}: v^{(N-1)'}) \end{aligned}$$

Also, $G^t = G^t(\Delta_b)$, t = 1, ..., N, Δ_b is functional behavioural strategies.

Combinatorics consideration

N^N (Pure strategies) $(A_1, A_2, ..., A_N)\times(1, 2, ..., N)$: Axiom 1: N factors at N levels have N^N sample points.

Proof: Binary combinations:

$$\begin{array}{l} A_1 1, A_1 2, ..., A_1 N \\ A_2 1, A_2 2, ..., A_3 N \\ \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \\ A_N 1, A_N 2, ..., A_N N \text{ or } (N\times N = N^2) \end{array}$$

Tertiary combinations:

$$\begin{array}{l} A_1 1A_1, A_1 2A_1, ..., A_1 NA_1 \\ A_2 1A_1, A_2 1A_2, ..., A_2 1A_N \\ \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \\ A_N 1A_1, A_N 2A_1, ..., A_N NA_1 \end{array}$$

$$\begin{array}{l} A_1 1A_2, A_1 2A_2, ..., A_1 NA_2 \\ A_2 1A_2, A_2 2A_2, ..., A_3 NA_2 \\ \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \\ A_N 1A_2, A_N 2A_2, ..., A_N NA_2 \\ \vdots \\ \vdots \\ \vdots \qquad \text{or } (N\times N)N = N^2.N^{3-2} = N^3) \\ \vdots \\ \vdots \\ \vdots \\ \vdots \end{array}$$

$$\begin{array}{l} A_1 1A_N, A_1 2A_N, ..., A_1 NA_N \\ A_2 1A_N, A_2 2A_N, ..., A_3 NA_N \\ \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \\ A_N 1A_N, A_N 2A_N, ..., A_N NA_N \end{array}$$

Induction consideration

N-tuple combinations: By induction, from tertiary combinations then:

$$(N\times N)\prod^{N-2} N = N^2.N^{N-2} = N^N$$

Hence, there are N^N Sample points QED. Axiom 2: N-factors at N-levels at N-levels,... at N-levels combinations (i.e., N(N-levels)) has sample points in the order of:

$$N^{N^{N^{N^{\cdot^{\cdot^{N}}}}}} \quad \text{\} Each level raised to power N, for all the N levels}$$

That is N-factors at N^*N-levels combinations.

Proof:

N-factors at N-levels give N^N sample points. N-factors at N^N-levels gives N^{N**N} points. N-factors at N^{N**N}-levels gives $(N^{N**N})^{**N.}$ where $N^{**}N = N^N$. By induction N-factors at N^*(N-levels) yields the result obtained.

Analytic geometry consideration: There are N^N pure strategies associated with the N-dimensional variation in the space R^{N+1}. The locus of all admissible $(J, G^1,...,G^N)$, (N+1) tuples or R^{N+1} dimensional space would be considered or searched for minimum J. All the N^N possible N-dimensional combinations would prove impossible to obtain graphically. In the event that they are obtained, the associated (N +1) topples with these pure strategies are $(O, G^1, ..., G^N)$, $(J, O, G^2, ..., G^N)$, $(J, O, G^1, G^3, ..., G^N)$, ... $(J, G^1, ..., G^N)$ in the space $(J, G^1, \ldots, G^N)$, i.e., R^{N+1}.

If the locus of all satisfying admissible N-tuples is established the bounded rationality constraints would be obtained from the R^{N+1} space $(J, G^1, ..., G^N)$ in the form of surfaces of constants G^t where, t =1, ..., N which are the bounding rationality constraints. The solution as usual would be the subspace of the feasible space defined by:

$$J(\Delta) = \sum_{K1,K2,...,KN} J_{K1\,K2...KN} P_{K1} P_{K2} ... P_{KN} \leq \bar{J} \qquad (26)$$

Equation 24 is the surface in the space $(J, G^1, ..., G^N)$ which intersects the J-axis at J. The satisfying strategies are all points in the locus below the surfaces that also satisfy the bounded rationality constraints.

The performance of DM N would be greatly hindered due to the effects of the colossal amount of bounded rationality constraints imposed by command inputs. The following results were obtained concerning F, the rate of processing activities and their tempo:

- As $F\to\infty$, the efficiency, min $d(Y,Y')$ diminishes or is non-optimal for a fixed tempo (τ = constant)
- As $\tau\to\infty$ for F = constant, the min $d(Y,Y')$ diminishes or is non-optimal
- As $\tau \propto F$, i.e., direct proportionality, min $d(Y,Y')$ = constant

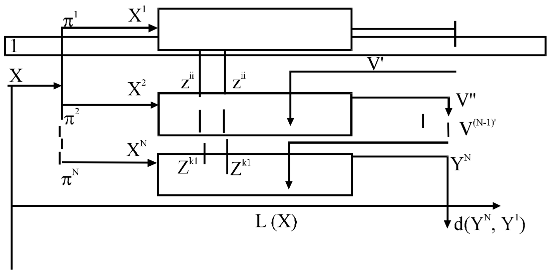

Fig. 11: N-teams of interacting decision makers with bounded rationality

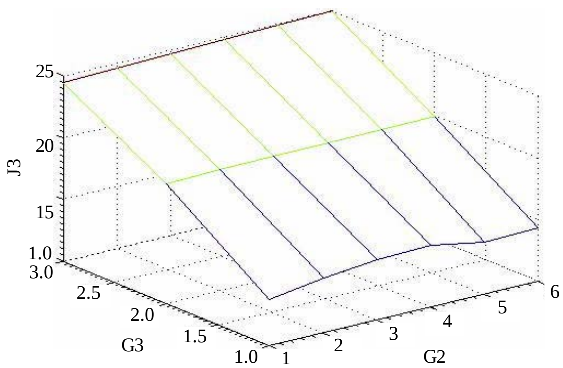

Fig. 12: Region of admissible (J_t, G^t, G^N) triples

- When $\tau \propto F^{-1}$, i.e., inverse proportionality and since τ, $F \geq 0$ as τ increases for very low F ($F \to 0$) values, min $d(Y,Y')$ is worst as F increases for very low τ (i.e., rate of occurrence of activities is very small or negligible), ($\tau \to 0$), min $d(Y,Y')$ is indeterminate if there is nothing to process

It could be rightly summarized that the hierarchical command diminishes efficiency or increases workload of DM N due to higher processing rates. When ever the number of constraint are finite but large, the computation for $J \leq J$ is the same as in the case when t = 2, 3 in the corresponding spaces (Fig. 1, 2 and 5).

Figure 11 is the developed organogram of the derived generalized model of N-team hierarchical interacting decision makers with bounded rationality constraints. z^{ij}, z^{lk} are SITREP'S where i, j, k = 1, ..., N. For t = 2, 3, ..., N or $0<t\leq 2, 3, ..., N$.

The region of admissible solution of the triple (J_t, G^t, G^N) is shown in Fig. 12. Figure 12 is the admissible region of feasible solutions having coordinates (J_t, G^t, G^N) for t = 3, 4, ..., N.

Algorithm:

An algorithm is hereby provided because this work is a theoretical approach of modeling a command, control, communication and intelligence system.

1. N = n, t = 0, Y(t) = 0, assign initial values to all strategies, scalars and entropies
2. t = t+1
3. Compute G_t^t, G_n^t, G_b^t, G_c^t (using subroutines)
4. $G^t = G_t^t+G_n^t+G_b^t+G_c^t$
5. $Y(t+1) = Y(t)+G^t$
6. If t<N GOTO 2
7. Print Y(t)
8. End

CONCLUSION

This research extended the research of Levis and Bocttcher by modelling the N-team interacting decision makers with bounded rationality constraints. It also deprived the Threshold (performance) Model. The performance of DMN was greatly hindered due to the effects of colossal amount of bounded rationality constraints of command inputs. Hence, hierarchical command diminishes efficiency of $C^4I^2S^2M$ (Computer Command and Control Communication, Information Intelligence and Surveillance Management System) and thereby increases work load of subordinate DM. Decision Centres are recommended for adaptation in order to overcome low performance arising from the effects of the imposed enormous constraints. The cases when $t\geq 4$, ..., N are considered for analytical purposes, even though cumbersome, they are unusual. The system with N DMs should be broken into decision centres that form non-hierarchical model of 2 DM per cell to ensure optimality. Since, every pair of DM form optimal team. This is necessary in order to assess the workload or efficiency of any DM. This model is adaptive to any system (control systems) where specific output is required and feed back is an integral factor to enhance optimization. Further reseraches may consider this generalized model with a view to transform it to deferential equation equivalent. Empirical consideration may be done to establish the values of none parameters.

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