# Development of Compensation Procedure for Systematic Errors of Coordinate Measuring Machines with Standard Tooling 

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#### Abstract

The study proposes, a new technique for Coordinate Measuring Machine (CMMs) calibrating to reduce the impact of systematic errors on the the detail accuracy measurement. The proposed methodology of the existing solutions difference is that the proposed method is based on the use of the standard equipment composed of a sphere, a cylinder and a plane. The compensation parameters of systematic errors are determined on the basis of these elements measurement with the improved and reference CMM. The technique may be used to test the existing CMM or to enhance the accuracy of new CMM.


Key words: Coordinate-measuring machines, measurement accuracy, method of compensation, systematic errors, cloud of points

## INTRODUCTION

In recent years, the role of Coordinate Measuring Machines (CMM) for the instrumental control of all production stages for various parts. The complex shape parts started to be used widely. Along with the increasing complexity of produced part shapes the accuracy requirements for their production were also increased and thus the measurement accuracy requirements for co-ordinate measuring machines were also increased. The CMM role is being increased along with the production control during the conduction of research. The qualitative research work is impossible without checking the results in practice and therefore, the accuracy of the measurement performance shows the quality of scientific research. Today, the scope of CMM use is wide enough: from astronomy to the machine-tool construction. The analysis of the alloy properties and the determination of the parts production characteristics is difficult to imagine without the obtaining of the accurate data concerning the fabricated parts geometry (Khaimovich and Balaykin, 2014).

The modern CMM is a complex device consisting of a set of components. The machine production quality directly depends on the detail measurement accuracy. To improve the measurement accuracy of the measuring equipment manufacturers are constantly engaged in CMM manufacturing process development. However, this process is associated with significant material costs.

Often, the further improvement of the CMM manufacturing process becomes an impractical one due to the significant increase of the measuring equipment cost. Mathematical methods are used for the further enhancement of the measurement accuracy.

Modern coordinate measuring machines may be divided into two types contact and contactless coordinate-measuring machines. Contactless CMM are based on the analysis of the light flux reflection from the component surface (Milroy et al., 1996). The operation principle of the contact measuring machines is based on the measurement of the workpiece touching points with a measuring probe and the detail surface reconstruction according to the obtained information (Shabliy and Dmitrieva, 2006). The proposed method is developed to work with the contact coordinate-measuring machines.

Different methods of systematic measurement error compensation are designed for these machines. These errors are caused by the imperfections during CMM manufacture. Dzhunkovsky (2007)'s study proposes the compensation technique for systematic errors based on the calibration plate measurement. Janeczko et al. (2000) consider CMM calibration technique with a laser interferometer. Researchers show that the use of a laser interferometer may significantly accelerate the process of calibration. One of the most common tools to determine the parameters of systematic error compensation is the end block. Kasparaitis and Shilyunas (1990) propose the compensation technique based on the measurement of the end blocks (Fig. 1).

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Fig. 1: CMM calibration tools; a) calibration plate; b) laser interferometer and c) end blocks set

The majority of existing methods have a similar deficiency: complex auxiliary tools are used for the compensation of errors. The aim of this research is to develop the procedure of systematic errors compensation using simpler auxiliary tools.

## MATERIALS AND METHODS

## Systematic errors compensation using standard tooling:

 The detail measurement result using the contact CMM is a set of surface contact points of the part and the probe. This set of points is represented by the point coordinates in the internal coordinate system of a measuring device. Typically, a Cartesian Coordinate System serves as the coordinate system of the device. The axes of this system are parallel to the axes of the measurement probe movement. The value of coordinates is the probedisplacement relative to a reference point. Due to CMM manufacturing error the location of axes may differ from the projected location, so the Internal Coordinate System of the device is different from the coordinate system associated with the measuring probe. When moving the probe along a working field the specified coordinate system may change the direction of axes and the center of the coordinate system. The measurement errors caused by the deviation of the actual coordinate system from the coordinate system of the device are systematic ones, so they may be compensated by mathematical methods.

The point coordinates in the coordinate system of the probe and the device coordinate system are related by Eq. 1:

$$
\left(\begin{array}{l}
x_{d}  \tag{1}\\
y_{d} \\
z_{d}
\end{array}\right)=R^{-1} \times\left(\begin{array}{l}
x_{p} \\
y_{p} \\
z_{p}
\end{array}\right)+T
$$

Where:
$\mathrm{X}_{\mathrm{d},} \mathrm{y}_{\mathrm{d}}, \mathrm{Z}_{\mathrm{d}}=$ The point coordinates in the coordinate system of the device
$\mathrm{X}_{\mathrm{d},} \mathrm{y}_{\mathrm{d}}, \mathrm{Z}_{\mathrm{d}}=$ The point coordinates in the coordinate system of the probe
$\mathrm{R} \quad=$ The transition matrix from the basis of the device probe coordinate system to the basis of the device coordinate system
T $\quad=$ Coordinate system transfer vector

The linear movement sensors provide the contact point coordinate values $\mathrm{x}, \mathrm{y}, \mathrm{z}$. Thus, it is necessary to define the transition matrix from the basis of the coordinate system of the measuring probe to the basis of the device coordinate system and determine the transfer vector of the device coordinate system to compensate the systematic errors and obtain the contact point coordinates in the coordinate system of the device. In order to determine the specified parameters one may use the information obtained by measuring of the reference tooling which is the plane, the cylinder and the sphere.

Let's measure a set of plane points and get the coordinates of the touch points in the coordinate system of the measuring probe ( $\mathrm{x}_{\mathrm{p} 1}, \mathrm{y}_{\mathrm{p} 1}, \mathrm{z}_{\mathrm{p} 1}, \ldots, \mathrm{x}_{\mathrm{pm}}, \mathrm{y}_{\mathrm{pn}}, \mathrm{z}_{\mathrm{pn}}$ ). Let's define the equation of the plane to reduce the influence of random component measurements by solving Eq. 2:

$$
\begin{array}{r}
\mathrm{A}_{\mathrm{p}} \mathrm{x}+\mathrm{B}_{\mathrm{p}} \mathrm{y}+\mathrm{C}_{\mathrm{p}} \mathrm{z}+\mathrm{D}=0 \\
\sum_{\mathrm{i}=1}^{\mathrm{n}}\left(\mathrm{z}-\mathrm{z}_{\mathrm{i}}\right)^{2} \rightarrow \min \tag{2}
\end{array}
$$

The solution of the optimization problem result allows to get the surface normal vector in the coordinate system of the probe $\left(A_{p}, B_{p}, C_{p}\right)$. Knowing the direction of the device coordinate system axes let's measure the tooling element for the plane with a coordinate measuring machine and obtain the surface normal vector in the coordinate system of the device ( $\mathrm{A}_{\mathrm{d}}, \mathrm{B}_{\mathrm{d}}, \mathrm{C}_{\mathrm{d}}$ ).

Similarly with the plane let's measure a cylindrical standard by a coordinate measuring machine whose characteristics are wanted to be improved with the reference CMM. In order to find the parameters of the cylinder it is necessary to find the minimum of the optimization function (Zheng et al., 2013):

$$
\begin{equation*}
\sum_{\mathrm{i}=1}^{\mathrm{m}}\left[\left(\mathrm{x}_{\mathrm{i}}-\mathrm{C}\right)^{\mathrm{T}}\left(\mathrm{I}-\mathrm{WW} W^{\mathrm{T}}\right)\left(\mathrm{x}_{\mathrm{i}}-\mathrm{C}\right)-\mathrm{r}^{2}\right]^{2} \rightarrow \min \tag{3}
\end{equation*}
$$

Where:
$\mathrm{X}_{\mathrm{i}}=$ The cylinder point coordinate
$\mathrm{C}=$ The point coordinates on the cylinder axis
$\mathrm{W}=$ The vector of the cylinder axis
$\mathrm{I}=$ Identity matrix
$\mathrm{r}=$ The cylinder radius
Solving this problem for points measured with the use of an improved CMM and a reference CMM we obtain the coordinates of the cylinder axis vectors in the coordinate system of the measuring device and the probe ( $D_{n}, E_{n}, F_{n}$ ).

The transition matrix from the coordinate system basis of the device probe to the basis of the Device Coordinate System may be represented as the product of three transition matrices along the coordinate axes $\mathrm{x}, \mathrm{y}, \mathrm{z}$ :

$$
\begin{gather*}
\mathrm{R}=\mathrm{R}_{\mathrm{z}}(\alpha) \mathrm{R}_{\mathrm{y}}(\beta) \mathrm{R}_{\mathrm{z}}(\gamma)  \tag{4}\\
\mathrm{R}_{\mathrm{z}}(\alpha)=\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos (\alpha) & \sin (\alpha) \\
0 & -\sin (\alpha) & \cos (\alpha)
\end{array}\right)  \tag{5}\\
\mathrm{R}_{\mathrm{y}}(\beta)=\left(\begin{array}{ccc}
\cos (\beta) & 0 & \sin (\beta) \\
0 & 1 & 0 \\
-\sin (\beta) & 0 & \cos (\beta)
\end{array}\right)  \tag{6}\\
\mathrm{R}_{\mathrm{z}}(\gamma)=\left(\begin{array}{ccc}
\cos (\gamma) & \sin (\gamma) & 0 \\
-\sin (\gamma) & \cos (\gamma) & 0 \\
0 & 0 & 1
\end{array}\right) \tag{7}
\end{gather*}
$$

The angles between the vector normal to the plane, the cylinder axis vector and the reference vectors of the device coordinate system do not depend on the coordinate system, so we may put down Eq. 8:

$$
\begin{align*}
& \frac{\overrightarrow{\mathrm{I}_{\mathrm{d}}} \times \overrightarrow{\mathrm{n}_{1} \mathrm{~d}}}{\left|\overrightarrow{\mathrm{n}_{\mathrm{d}}}\right| \overrightarrow{\mathrm{n}_{1} \mathrm{~d}} \mid}=\frac{\overrightarrow{\mathrm{R} 1_{\mathrm{p}}} \times \overrightarrow{\mathrm{n}_{1} \mathrm{p}}}{\left|\mathrm{R} \overrightarrow{\mathrm{l}_{\mathrm{p}}} \times\left|\overrightarrow{\mathrm{n}_{\mathrm{n}}}\right|\right|} \\
& \frac{\overline{\mathrm{jd}} \times \overline{\mathrm{n}_{1} \mathrm{~d}}}{|\overrightarrow{\mathrm{jd}}|\left|\overrightarrow{\mathrm{n}_{1 \mathrm{~d}}}\right|}=\frac{R \overline{\mathrm{j}_{\mathrm{p}}} \times \overrightarrow{n_{1} \mathrm{p}}}{\left|R \overrightarrow{\mathrm{j}_{\mathrm{p}}}\right| \times\left|\overrightarrow{n_{1 \mathrm{p}}}\right|}  \tag{8}\\
& \frac{\overline{\mathrm{kd}} \times \overline{\mathrm{n}_{2} \mathrm{~d}}}{|\overrightarrow{\mathrm{kd}}|\left|\overrightarrow{\mathrm{n}_{2 \mathrm{~d}}}\right|}=\frac{\mathrm{R} \overrightarrow{\mathrm{k}_{\mathrm{p}}} \times \overrightarrow{\mathrm{n}_{2} \mathrm{p}}}{\left|\mathrm{R} \mathrm{\bar{k}_{p}}\right|\left|\times\left|\overrightarrow{\mathrm{n}_{2 \mathrm{p}}}\right|\right.}
\end{align*}
$$

$\overrightarrow{\mathrm{i}_{\mathrm{d}}}, \overrightarrow{\mathrm{j}_{\mathrm{d}}}, \overrightarrow{\mathrm{k}_{\mathrm{d}}}=$ The basic vectors of the CMM coordinate system
$\overrightarrow{\mathrm{i}}_{\mathrm{p}}, \overrightarrow{\mathrm{j}}_{\mathrm{p}} \overrightarrow{\mathrm{k}}_{\mathrm{p}}=$ The basic vectors of the measuring probe coordinate system
$\overline{\mathrm{n}}_{1 \mathrm{~d}} \quad=$ Normal vector to the plane within CMM coordinate system
$\overline{n_{1 p}} \quad=$ Normal vector to the plane within a measuring probe coordinate system
$\overline{n_{2 \mathrm{~d}}} \quad=$ Cylinder axis vector in the CMM coordinate system
$\overline{n_{2 p}} \quad=$ Cylinder axis vector in the measuring probe coordinate system

Solving this system relative to $\alpha, \beta, \gamma$ variables we obtain the transformation matrix of the R coordinate system.

In order to find the transfer vector let's use the information obtained during the measurement of a sphere tooling element. Let's solve Eq. 9 in the Device Coordinate System and the measuring probe coordinate system and obtain the coordinates of a sphere center in both coordinate systems $\mathrm{X}_{\mathrm{d} 0}, \mathrm{X}_{\mathrm{p} 0}$ :

$$
\begin{gather*}
\left(x-x_{0}\right)^{2}+\left(y-y_{0}\right)^{2}+\left(z-z_{0}\right)^{2}=R^{2} \\
\sum_{i=1}^{n}\left(z-z_{i}\right)^{2} \rightarrow \min \tag{9}
\end{gather*}
$$

Where:
$\mathrm{x}_{0}, \mathrm{y}_{0}, \mathrm{z}_{0}=$ Sphere center coordinates
$\mathrm{R} \quad=$ Sphere radius
$\mathrm{x}_{\mathrm{i}}, \mathrm{y}_{\mathrm{i}}, \mathrm{z}_{\mathrm{i}}=$ Measured sphere point coordinates
Substituting the obtained coordinates into (Eq. 1) we obtain the transfer vector of CMM coordinate system:

$$
\begin{equation*}
\mathrm{T}=\mathrm{X}_{\mathrm{n} 0}-\mathrm{R}^{-1} \mathrm{X}_{\mathrm{w} 0} \tag{10}
\end{equation*}
$$

The deviation of the coordinate measuring machine geometric parameters from its design parameters is not a constant value throughout the CMM workspace (Izmailov and Solodov, 2005). Therefore, the systematic bias compensation parameters ( R and T ) will depend on the probe position: $\mathrm{R}=\mathrm{R}(\mathrm{x}, \mathrm{y}, \mathrm{z}), \mathrm{T}=\mathrm{T}(\mathrm{x}, \mathrm{y}, \mathrm{z})$. It is
necessary to calculate compensation parameters for a set of points and approximate them for the intermediate position of a measuring probe to compensate the CMM systematic errors throughout the work area.

Modified compensation method of systematic errors: The coordinate system axes of a measuring probe are usually directed parallel to the probe movement axes. CMM is designed in such a way that the specified system of coordinates is the Cartesian Coordinate System. Due to CMM manufacturing error the coordinate system associated with the measuring probe may be non orthogonal one which causes the occurrence of measurement errors. To compensate the systematic errors according to the method described above, you must first compensate the non orthogonality of the probe coordinate system axes.

The coordinates of points in a non-orthogonal coordinate system and orthogonal coordinate system with the general starting point and $i$ axis may be related as follows:

$$
\begin{align*}
& x=x^{\prime}+\cos \left(\alpha_{12}\right) y^{\prime}+\cos \left(\alpha_{13}\right) z^{\prime} \\
& y=\sin \left(\alpha_{12}\right) y^{\prime}+\cos \left(\alpha_{23}\right) z^{\prime}  \tag{11}\\
& z=\sqrt{1-\cos ^{2}\left(\alpha_{13}\right)-\cos ^{2}\left(\alpha_{23}\right) z^{\prime}}
\end{align*}
$$

Where:
$\mathrm{x}, \mathrm{y}, \mathrm{z}=$ The coordinates within the orthogonal coordinate system
$x^{\prime}, y^{\prime}, z^{\prime}=$ The coordinates within the non orthogonal coordinate system
$a_{m n} \quad=$ The angle between basic vectors $\overline{\mathbf{e}}_{\mathrm{m}}^{\prime}$ and $\overline{\mathbf{e}}_{n}^{\prime}$
Let's measure the sphere in this system of coordinates in order to determine the angles between the basic vectors of the non-orthogonal coordinate system (Izmailov and Solodov, 2005). The vector length square is equal to the scalar multiplication of a vector by itself, hence:

$$
\begin{gather*}
\left(\left(x^{\prime}-\mathrm{x}_{0}^{\prime}\right) \overrightarrow{\mathrm{e}_{1}^{\prime}}+\left(\mathrm{y}^{\prime}-\mathrm{y}_{0}^{\prime}\right) \overrightarrow{\mathrm{e}_{2}^{\prime}}+\left(\mathrm{z}^{\prime}-\mathrm{z}_{0}^{\prime}\right) \overrightarrow{\mathrm{e}_{3}^{\prime}}\right)^{2}= \\
\left(\mathrm{x}^{\prime}-\mathrm{x}_{0}^{\prime}\right)^{2} \overrightarrow{\mathrm{e}_{1}^{\prime}} \overrightarrow{\mathrm{e}_{1}^{\prime}}+\left(\mathrm{y}^{\prime}-\mathrm{y}_{0}^{\prime}\right)^{2} \overrightarrow{\mathrm{e}_{2}^{\prime}} \overrightarrow{\mathrm{e}_{2}^{\prime}}+  \tag{12}\\
\left(\mathrm{z}^{\prime}-\mathrm{z}_{0}^{\prime}\right)^{2} \overrightarrow{\mathrm{e}_{3}^{\prime}} \overrightarrow{\mathrm{e}_{3}^{\prime}}+2\left(\mathrm{x}^{\prime}-\mathrm{x}_{0}^{\prime}\right)\left(\mathrm{y}^{\prime}-\mathrm{y}_{0}^{\prime}\right) \overrightarrow{\mathrm{e}_{1}^{\prime} \mathrm{e}_{2}^{\prime}} \\
\sum_{\mathrm{i}=1}^{\mathrm{n}}\left(\mathrm{z}^{\prime}-\mathrm{z}_{\mathrm{i}}^{\prime}\right)^{2} \rightarrow \min \tag{13}
\end{gather*}
$$

Solving the minimization problem Eq. 12 and 13, we define the angle values between the basis vectors. With the help of the obtained angles and Eq. 11 we obtain the coordinates of points in an Orthogonal System tied to the measuring probe.

After the compensation of non-orthogonality let's determine the parameters of compensation for systematic errors according to Eq. 1-10.

## RESULTS AND DISCUSSION

The proposed procedure was used to compensate the systematic device errors in order to measure the blade airfoil geometry of a Gas Turbine Engine (GTE). This item makes a significant impact on the GTE performance, so there are high demands for its production quality (Fig. 2).

The device has a fixed base 1 on which the movable platform 2 is set. The platform has the accessories to fix the blade 3. The U-shaped plate 5 containing two measuring probes 6 is attached to the frame 4 , located on a fixed base. Platform 2 and U-shaped plate 5 are driven motion by step servodrives and their position is tracked by linear movement sensors. The information from the linear movement sensors and measuring probes is transferred to the computer to control the movement and measure the profile of a blade airfoil.

In accordance with the method proposed this device measured the tooling: a sphere, a plane and a cylinder. These elements were also measured using the reference CMM . In order to check the compensation procedure with this instrument the measurements of GTE compressor blade were performed and then the comparison of the blade measured points deflection from its nominal value was performed in the case of the method use and without the use of it. The comparison results are displayed by Fig. 3.

The results of produced blade deviation comparison from the nominal profile show that the proposed method improves the detail measurement accuracy. The analysis of the measured cloud of points showed that the various compensation parameters for different parts of the working area differ. It is related to the rail fabrication


Fig. 2: Diagram of the device measuring the GTE compressor blades; a) Side and b) top view


Fig. 3: Deviation of an airfoil shape from a nominal profile; a) without compensation and b) with compensation
errors. These rails serve as the support for the movable platform 2. Therefore, it is necessary to select the processing area in such a way to reduce the influence of random components on the one hand and not to lose the essential features of the device kinematic part on the other hand and the systematic errors associated with it.

## CONCLUSION

The measurement accuracy performed by the coordinate measuring machine is influenced by many factors: temperature, vibration, the nonorthogonality and deformation of axes, axis turns, measuring probe errors, contact deformations and other factors. The proposed method of systematic errors compensation for coordinate
measuring machines with the standard tooling allows to compensate a number of factors and improve the accuracy of measurements. The advantage of this technique is the use of standard tooling which is easier and cheaper for production than such tools as calibration plates or laser interferometers. This technique allows to compensate the systematic measurement errors associated with the CMM manufacture precision but is not able to compensate errors caused by other factors such as the bending of a probe or the error of a measured detail fixation. To further improve the accuracy. It is necessary to perform the work determining the error compensation methods caused by these factors.

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