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Biomedical Signals Analysis by DWT Signal De-Noising with Neural Networks

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Abstract: The core intention of this research is to investigate the wavelet function that is optimum in identifying and de-noising the various biomedical signals. Using traditional methods, it is difficult to recover the noises present in the signals. This study presents a detail analysis of Discrete Wavelet Transform (DWT) de-noising on various wavelet families and biomedical signals such as ECG, EMG and EEG. Researchers have developed a trained network in order to optimally denoise the signals by using a Back Propagation algorithm in the neural network. Initially noise is added to the original signal then the signal is decomposed using the Shift Invariant Method. After decomposition, the proposed Wavelet Based Method is used for noise removal. Then, the signal is reconstructed by using Wavelet Reconstruction Method. The denoised signals will be compressed by a hybrid wavelet shannon fano coding for reducing its storage size.

Key words: DWT, ECG, EEG, EMG, neural network, wavelet frequency thresholding

INTRODUCTION

A signal is a physical quantity which differs with respect to time, space and contains information from the source to the destination. The signal processing indicates any manual or mechanical operation which modifies, analyzes or else manipulates the information contained in a signal (Sifuzzaman *et al*., 2009). In the application of signal processing, wavelets provide a mathematical tool for the hierarchical decomposition of functions in signal and image processing (Garofalakis and Kumar, 2004). The noise is the most important problem in signal processing. Noise removal can be done by removal of small coefficients. The noise consists of high frequency components that are referred as linear de-noising. The corresponding scales of the wavelet transform are set to zero. Noise assumed by non-linear de-noising or wavelet shrinkage consists of low energy (Schremmer *et al*., 2001).

Signal de-noising refers to the process of removing the noise from a signal. It is the main problem in signal processing. To estimate a clean version of a given noisy signal is the main goal of natural signal de-noising (Levin and Nadler, 2011). The noise may corrupt the signal in many cases in a significant manner and it must be removed from the data in order to proceed with further data analysis. While de-noising the signal it will improve perceptual quality, compression effectiveness and the accuracy of signals (Varghese and Wang, 2008). Using simple filtering operations, it is very difficult to remove the noises (Rai and Trivedi, 2012) and by using the traditional methods it is difficult to recover the noises present in the signals.

An Electrocardiograph (ECG) is used to measure the rate and regularity of heartbeats. The nature of ECG signals is oscillatory and periodic. A complete ECG signal has a distinct and characteristic shape. In an ECG signal, the sources of noise may be either cardiac or extracardiac. Various noises contaminate the ECG signal (Kabir and Shahnaz, 2012). Electroencephalography (EEG) is used to diagnose the abnormal activity and functionality of the brain (Omerhodzic *et al*., 2010). Electromyography (EMG) is used to record the electrical impulses produced by the skeletal muscles. While recording EMG signals, it will consist of some noise signals as well. When researchers try to achieve good performance of EEG signals, the major problem that occurs here is the noisy EMG signals (Phinyomark *et al*., 2010). The biomedical signals may easily affected by noise and detecting these noises in the signals by using powerful and advance methodologies is becoming a very important requirement.

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In general, a wavelet is a wave like oscillation with the amplitude starting at zero, increasing and then decreasing to zero. The main function of wavelet transform is used to reduce the unwanted noise and blurring in the signals (Selesnick, 2007). Wavelet transform is the best technique used for de-noising the signals (Shankar and Duraiswamy, 2011). Wavelet transform achieves a correlation analysis; consequently the output is expected to be maximal when the input signal mostly resembles the mother wavelet. The Wavelet De-Noising Methods offer high quality and flexibility for the noise problem of signals and images. The Discrete Wavelet Transform (DWT) is based on performance in order to meet the mathematical criteria to obtain the discrete sequences as discrete wavelet functions. If the analyzing function is waveform adapted then DWT based de-noising can be performed better (German-Sallo and Ciufudean, 2012). Dividing the continuous time function into wavelets is referred to us Continuous Wavelet Transform (CWT).

In general, the term signal decomposition is referred to as the way of decomposing a given signal into a sum of simpler signals. To have a scale invariant interpretation of the image enables multi-resolution decomposition (Levin and Nadler, 2011). Decomposing a signal into scales with different time and frequency resolutions is done by a Multi-Resolution Analysis (MRA) algorithm. The small number of resulting coefficients is relatively good decomposition. To decompose a signal, wavelet transform uses some set of basic functions (German-Sallo and Ciufudean, 2012). By means of classification of spectral peaks, transient spectral peaks were detected and were investigated into distinction the between sinusoidal and noise components. The classification is based on descriptors derived from properties related to time-frequency distributions (Roebel *et al*., 2004).

There are many types of wavelets. The Daubechies wavelet is described by a maximal number of vanishing moments for some given support and the Haar wavelet is an order of rescaled square shaped function which together forms a wavelet family. Symlet wavelets are only an improved version of Daubechies wavelets with an increased symmetry while the Coiflet wavelets have scaling functions with vanishing moments. The Mathieu equation is a second order differential equation with periodic coefficients and the Legendre function has common applications in which spherical coordinate system are suitable.

Signal reconstruction generally means that the determination is of an original continuous signal from an order of equally spaced samples. Reconstructing the original sequence from the thresholded wavelet detail coefficients leads to a denoised (smoothed) version of the original sequence. Inverse Discrete Wavelet Transform (IDWT) is used to reconstruct the original signal. Calculate high resolution log-spectral features of the input noise signal to reconstruct the signal (Kristjansson and Hershey, 2003). Therefore, wavelet transform is a reliable and a much better technique than the Fourier transform technique. Biomedical signals require high storage to be stored and high bandwidth to be transmitted. Compression of the signals refers to reducing the redundancy in it. Signal compression techniques are limited to the amount of time required for compression and reconstruction, the noise embedded in the raw signal and the need for accurate reconstruction of the waves (Sayeed, 2013).

Literature review: Rai and Trivedi (2012) utilized Haar, Daubechies and Symlets wavelet families for de-noising. For removing the noise from the ECG signals they used three different wavelet families (Haar, Daubechies and Symlets). The different noise structure (unscaled white noise, scaled white noise and non white noise) had been selected for ECG signals and then compared their statistical parameter to find out the best result.

German-Sallo and Ciufudean (2012) proposed the design of the waveform-adapted analyzing function to have a good wavelet decomposition of the analyzed signal. A good decomposition means a relatively small number of resulting coefficients and a good reconstruction of the signal from these. Their proposed procedure led to obtain discrete sequences as discrete wavelet functions to perform de-noising, these met certain mathematical criteria. Discrete Wavelet Transform (DWT) based de-noising was performed. A waveform-adapted wavelet transform based noise suppression procedure was introduced and the experimental result was obtained. Bradley (2003) has reviewed a number of approaches to reducing or removing the problem of shift variance in the Discrete Wavelet Transform (DWT). He has described a generalization of the critically sampled DWT and the fully sampled algorithm that provides approximate shift-invariance with an acceptable level of redundancy. They had proposed Over Complete DWT (OCDWT) is critically sub-sampled to a given level of the decomposition below which it is then fully sampled. This proposed algorithm illustrated in an edge detection context and directly compared to a number of other shift-invariant transforms in terms of complexity and redundancy.

Hawwar *et al.* (2000) presented the effectiveness of both soft and hard thresholding for desired detail levels. Filtering noise in real time has applications in speech and image processing. Considerable interest has arisen in recent years regarding filtering in the wavelet transform domain. This technique has been effective in noise removal with minimum side effects on important features such as image details and edges. An efficient hardware implementation based on the FPGA technology was proposed.

Reddy and Muralidhar (2012) introduced a new Wavelet Threshold Method namely Wavelet Threshold Method with Grey Incidence Degree (GID) (or the GID Threshold Method). It showed that the signal smoothness and similarity of the two signal criteria have been greatly improved by the GID Threshold Method compared with existing threshold methods. According to the characteristics of different ECG signals, the GID Threshold Method gets better results than it can adaptively deal with noise separation and details remaining of the two opposing signal problems so as to provide a better choice for Wavelet Threshold Methods of signal processing. Performance analysis was performed by evaluating the Mean Square Error (MSE), Signal to Noise Ratio (SNR) and visual inspection over the denoised signal from each algorithm. Their experimental result shows that the GID Hard Shrinkage Method with sub-band or level dependent thresholding gives the best de-noising performance on an ECG signal. The result shows that soft threshold will not always give better de-noising performance; it depended on which wavelet thresholding algorithm was chosen.

Mishali and Eldar (2009) described how to choose the parameters of the multi-coset sampling so that a unique multiband signal matches the given samples. To recover the signal the continuous reconstruction is replaced by a single finite-dimensional problem without the need for discretization. The resulting problem was studied within the framework of compressed sensing thus it can be solved efficiently using known tractable algorithms from this emerging area. They also developed a theoretical lower bound on the average sampling rate required for blind signal reconstruction which is twice the minimal rate of known-spectrum recovery. They had ensured a perfect reconstruction for a wide class of signals sampled at the minimal rate and provides a first systematic study of compressed sensing in a truly analog setting. Various experiments are presented by demonstrating blind sampling and reconstruction with minimal sampling rate. Omerhodzic *et al*. (2010) had implemented a Wavelet-based Neural Network (WNN) classifier for recognizing EEG signals and tested it under three sets of EEG signals (healthy subjects, patients with epilepsy and patients with epileptic syndrome during the seizure). First, the Discrete Wavelet Transform (DWT) with the Multi-Resolution Analysis (MRA) was applied to decompose an EEG signal at resolution levels of the components of the EEG signal (δ, θ, α, β and γ) and the Parseval's theorem was employed to extract the percentage distribution of energy features of the EEG signal at different resolution levels. Second, the Neural Network (NN) classifies these extracted features to identify the EEG's type according to the percentage distribution of energy features. The performance of the proposed algorithm had been evaluated using a total of 300 EEG signals. The results showed that the proposed classifier has the ability of recognizing and classifying the EEG signals efficiently.

Chavan *et al*. (2011) have proposed a hybrid compression technique comprised of Wavelet and Ridgelet Methods for compressing images. Initially, the image was denoised with a filter and then the denoised image was transformed utilizing Discrete Wavelet Transform (DWT). After that Finite Ridgelet Transform (FRT) was employed on the obtained wavelet coefficients and the compressed image of a reduced size was obtained. For decompression the Ω inverse FRT was employed subsequent to the inverse DWT process and the original image was obtained without any loss of data.

Main contributions: Signal de-noising is to estimate a clean version of a given noisy signal. In the existing system while de-noising the biomedical signals such as ECG, EEG and EMG, the trained system is unable to automatically detect the best wavelet suitable for de-noising. The Fourier transform analysis is inadequate and is localized only in the frequency band. The major drawback of Short Term Fourier Transform for signal de-noising is that the time frequency precision is not optimal. Digital Filters and Adaptive Methods can be applied only to signals whose statistical characteristics are stationary in many cases and can not be applied to non-stationary signals because of loss of information. To overcome these drawbacks wavelet transform is introduced and discrete wavelet transform is used for de-noising. The proposed method can be applied to both stationary and non-stationary signals. The wavelet transform analyzes the signal both in time and frequency domains. In this proposed method, a trained system automatically classifies which wavelet is best suitable for de-noising the three kinds of biomedical signals such as ECG, EEG and EMG signals. Thus, wavelet transform will be the better choice while comparing it to the other techniques.

MATERIALS AND METHODS

The original signal is assumed as o(n) and the white Gaussian noise is assumed as e(n). By accumulating these signals the noisy signal s(n) will be attained. It is typically of the form:

$$s(n) = o(n)+e(n) \tag{1}$$

The general wavelet based de-noising procedures are composed of these steps:

- Decomposition: a wavelet function is chosen and decomposed upto level l
- De-noising wavelet detail coefficients: For every level of decomposition, select a threshold value and apply thresholding to the detail coefficients
- Reconstruction: calculate the reconstruction based on the approximation coefficients of level l and the modified detail coefficients of levels from 1 to l
- Classifier: a classifier achieves its objective by making a classification decision based on some characteristics
- Compression: reducing the amount of bits essential to define a signal to an approved accuracy

Wavelet transform is a mathematical tool for processing one-dimensional or multidimensional signals (Sindelarova *et al*., 2000). The scalar wavelet transform has been widely used in many applications like signal de-noising, image compression and in medical applications as well. In the preceding two decades, wavelet transform based de-noising was the best alternative for Fourier transform based signal de-noising (Mota and Vasconcelos, 2005). The first step of the De-noising procedure using wavelet transform is selection of the mother wavelet $-\psi_{m,n}(t)$ which forms a set of functions (family of wavelets), by compression or stretching or translation. The next step is the decomposition level. An example for a mother wavelet equation is as follows:

$$\psi_{m,n}(t) = 2^{\frac{m}{2}}\psi(2^{-m}t-n) \tag{2}$$

Where:
n = Coefficient of time translation
m = Coefficient of scale (compression)

Noise reduction plays a major role in signal processing. The various noise structures in different biomedical signals are unscaled white noise, scaled white noise, non white noise white Gaussian noise, etc. Wavelet thresholding, wavelet shrinkage and non-linear shrinkage are widely used terms for wavelet domain de-noising. The wavelet which is similar to the signal is selected for signal de-noising. The proposed technique is opted to implement using MATLAB 7.10 tool and will be evaluated using the database signals such as MIT-BIH arrhythmias and Sleep-EDF databases.

Wavelet transform: An indispensable tool for a variety of applications such as classification, compression and estimation is wavelet transform. Signal information in wavelet development is conveyed by a comparatively small number of large coefficients. This property of the wavelet transform makes the use of wavelets mainly ideal in signal estimation. It has been revealed that wavelets can eliminate noise more effectively than previously used methods. Wavelet transforms can decompose a signal into numerous scales that represent dissimilar frequency bands and at each scale, the position of the signal's instantaneous structures can be determined approximately. The purpose is to satisfy certain mathematical necessities and is used in representing data or other functions of a wavelet. The wavelet transform provides a time-frequency illustration of the signal. The signals are examined and expressed as a linear combination of the sum of the product of the wavelet coefficients and a mother wavelet by wavelet transform (Kabir and Shahnaz, 2012). The original signal is transformed using predefined wavelets in wavelet transform. The wavelets are orthogonal, biorthogonal and multiwavelets. By calculating signal to noise ratio of the signal the accuracy of the wavelet transform is determined after reconstruction of a signal. The wavelet transform is given by:

$$X_w(c,d) = \frac{1}{\sqrt{c}}\int_{-\infty}^{\infty} K\times\left(\frac{t-d}{c}\right)x(t)dt \tag{3}$$

Where:
c and d = Wavelet function parameters
x(t) = The signal to be transformed

Some of the applications identify pure frequencies, de-noising signals, compressing images, detecting discontinuities, breakdown points and self-similarity.

Discrete wavelet transform: Discrete wavelet transform is the same as filtering, it by a bank of filters of non-overlapping bandwidths which vary by an octave. It is based on sub-band coding which is found to yield a fast calculation of wavelet transform. It is easy to implement and diminish the calculation time and resources required. A set of dilations and translations $\psi_{i,j}(t)$ of a preferred mother wavelet $\psi(t)$ is used for signal analysis. It is important to know the behavior of these filters with these wavelet coefficients. According to the mother wavelet design the coefficients of these filter banks are determined. A common equation for the discrete wavelet transform signal is written as:

$$X[e,f] = \sum_{m=-\infty}^{\infty} x[m]\varphi_{e,f}[m] \quad (4)$$

Where:
$\varphi[m]$ = The window of finite length
f = A real number known as window translation parameter
e = A positive real number named as contraction parameter

Continuous wavelet transform: The Continuous Wavelet Transform (CWT) converts a continuous signal into extremely redundant signal of dual continuous variables: translation and scale. The resulting changed signal is easy to interpret and valuable for time-frequency analysis (Zakaria, 2010):

$$g(c,\ d) = |d|^{1/2} \int_{-\infty}^{\infty} g(t)\varphi\left(\frac{t-c}{d}\right)dt \quad (5)$$

where, c, d∈R, c≠0 and they are dilating and translating coefficients, respectively.

Wavelet filters: Discrete wavelet transform is performed by repeated filtering of the input signal using two filters. The filters are a Low Pass Filter (LPF) and a High Pass Filter (HPF) to decompose the signal into different scales. The output coefficient gained by the low pass filter is the approximation coefficient. The scaling function output is in the form of:

$$\varphi(t) = 2\sum_{q=0}^{M} h(q)\varphi(2t-q) \quad (6)$$

The output of the high pass filter is the detailed coefficient. The wavelet function output is in the form of:

$$w(t) = 2\sum_{q=0}^{M} g(q)\varphi(2t-q) \quad (7)$$

The approximation coefficient is consequently divided into new approximation and detailed coefficients. By choosing the mother wavelet the coefficients of such filter banks are calculated. This decomposition process is repeated until the required frequency response is achieved from the given input signal (Karthikeyan and Yaacob, 2012; Gokhale, 2012).

Wavelet families: Wavelength families are in Table 1 with respect to wavelets.

Daubachies: Commonly Daubechies family wavelets are signed dbN (N is the order). This wavelet belongs to the orthogonal wavelets.

Table 1: Wavelet families

Wavelet families	Wavelets
Daubechies	'db1' or 'haar', 'db2',...,'db10',...,'db45'
Coiflets	'coif1', ...,'coif5'
Symlets	'sym2',...,'sym8',...,'sym45'
Discrete meyer	'dmey'
Biorthogonal	'bior1.1', 'bior1.3', 'bior1.5'
	'bior2.2', 'bior2.4', 'bior2.6', 'bior2.8'
	'bior3.1', 'bior3.3', 'bior3.5', 'bior3.7'
	'bior3.9', 'bior4.4', 'bior5.5', 'bior6.8'

Coiflets: A discrete wavelet was designed by Ingrid Daubechies to have a scaling function with vanishing moments. The scaling function and the wavelet function must be normalized by a common factor. By retrogressive the order of the scaling task coefficients and then reversing the symbol of every second one is the wavelet coefficients.

Symlets: The symlet family wavelets are signed symN (N is the order). The symlets are nearly symmetrical, orthogonal and biorthogonal wavelets suggested by Daubechies as modifications to the db family. The properties of the two wavelet families are similar (Chavan *et al*., 2011).

Biorthogonal: Biorthogonal filters state a superset of orthogonal wavelet filters and have found their use virtually in all areas where wavelets are used. The biorthogonal family wavelets are signed as bior. Biorthogonal wavelet transform has frequently been used in numerous image processing applications because it makes multi-resolution analysis possible and does not produce redundant information.

Wavelet decomposition: In wavelet decomposition, it is possible to alter the resulting coefficient previously for a signal and signal reconstruction is to eliminate undesirable signal components. Choose a wavelet function with level N. Calculate the wavelet decomposition of the signals at level N (Rai and Trivedi, 2012).

Shift Invariant Method: The commonly referred algorithm trous which is to make the DWT shift-invariant is not to do any subsampling at all. The a Trous algorithm is shift-invariant and it can be used with some of the mother wavelets usually used with the DWT (Bradley, 2003). Only the mother wavelet has to be expanded there is no sub-sampling of data performed at each level of the transform.

Empirical mode decomposition: Emprical mode decomposition is especially related for non-linear and non-stationary signals. The calculation of EMD does not require any previously known value of the signal

(Kabir and Shahnaz, 2012). Emprical mode decomposition adaptively decomposes a multicomponent signal into Intrinsic Mode Functions (IMF). The fact that the functions into which a signal is decomposed are all in the time domain and of the same length as the original signal permits for variable frequency in time to be conserved. By gaining IMF from real world signals it is essential because natural processes often have multiple causes. And every cause may happen at specific time intervals. In EMD analysis this kind of data is evident but quite secreted in the Fourier domain. This technique is faced with the difficulty of being fundamentally defined by an algorithm and therefore of not acknowledging an analytical formulation which would permit a theoretical investigation and performance estimation. Some initial elements of investigational performance evaluation will also be provided for giving an essence of the efficiency of the decomposition as well as of the trouble of its interpretation.

Multi-resolution analysis: In any discrete wavelet transform, there is only a limited number of wavelet coefficients for each bounded rectangular region in the upper halfplane. Each coefficient needs the evaluation of an integral. A multiresolution representation presents an uncomplicated hierarchical framework for interpretating the image information. This method is performed on the wavelet coefficients directly.

Wavelet thresholding: Choice of a suitable wavelet function, thresholding methods and the thresholding rule play a vital role in signal de-noising (Karthikeyan and Yaacob, 2012). Thresholding methods used with discrete wavelet transform based filtering are to modify the obtained coefficients. By using wavelet thresholding the noise in the signals can be removed. There are several methods to choose the best analyzing function, the type of thresholding and the threshold values (German-Sallo and Ciufudean, 2012). Threshold which is proportional to the standard deviation of the noise is the universal threshold T and is defined as:

$$T = \sigma\sqrt{2\text{In}M} \tag{8}$$

Where:

M = Signal size

σ^2 = The noise variance and it is given by:

$$\sigma^2 = \left[\frac{\left(\text{median}\left(|X_i|\right)\right)}{0.6745}\right] \tag{9}$$

where, ($|X_i|$) represents the median value of the absolute values of wavelet coefficients X_i. Hard thresholding:

$$x_i = \begin{cases} x_i & |x_i| \geq 1.414\sigma\left[\ln(L)\right]^{0.5} \\ 0 & |x_i| \leq 1.414\sigma\left[\ln(L)\right]^{0.5} \end{cases} \tag{10}$$

where, $1.414\sigma[\text{In}(L)]^{0.5}$ is the threshold value. Soft thresholding:

$$x_i = \begin{cases} \frac{x_i}{|x_i|}\left(|x_i| - 1.414\sigma\left[\ln(L)\right]^{0.5}\right) & |x_i| \geq 1.414\sigma\left[\ln(L)\right]^{0.5} \\ 0 & |x_i| \leq 1.414\sigma\left[\ln(L)\right]^{0.5} \end{cases} \tag{11}$$

Wavelet frequency thresholding: The basic idea of wavelet frequency thresholding is based on judging the extent of their relation from the similarity to orders of geometric curve shapes. The closer those are the greater frequency thresholding of the corresponding order is and vice versa. According to this, researchers can compute the wavelet frequency analysis of the wavelet coefficients according to the relationship among the approximate time sequences. This method not only can filter most of the noise but it can commendably also hold signal details. More than that this technique can commendably compromise the problem of signal details remaining and noise suppression as well. For this reason, the signal processed by wavelet frequency thresholding has a better smoothness and similarity (Reddy and Muralidhar, 2012). Generally wavelet frequency thresholding is represented by the given equation:

$$\delta = \frac{1}{m}\sum_{i=1}^{m}\delta_i(t) \tag{12}$$

EMD-thresholding: EMD achieves a sub band like filtering resulting in fundamentally uncorrelated IMFs. Although, the corresponding filter-bank arrangement is by no means pre-determined and fixed as in wavelet decomposition, one can accomplish thresholding in each IMF in order to nearly ignore low energy IMF parts which are expected to be significantly despoiled by noise (Kopsinis and McLaughlin, 2008). The empirical mode decomposition is an adaptive data-driven technique which is used for effective decomposition of a noisy signal into its functional components.

Wavelet based thresholding: Wavelet based non-linear thresholding is effective for noise reduction only to the extent to which the wavelet representation of the noise-free signal is sparse. An efficient method of noise reduction only to the extent to which the wavelet representation of the noise free signal is sparse is wavelet based thresholding (Selesnick, 2007). In this method, the process of each coefficient from the detail sub bands with thresholding function is applied to obtain the output.

Wavelet reconstruction: The original signal is produced from the wavelet coefficients in most of the wavelet transform applications. The analysis and synthesis filters have to fulfill certain criteria to achieve a perfect reconstruction. Using the wavelet coefficients the original signal was reconstructed by applying inverse wavelet transform. The procedure of wavelet reconstruction comprises of up sampling by inserting zeros among the samples and filtering to expansion of the signal. A global reconstruction of the denoised signal is given by:

$$\hat{y}(t) = \sum_{j=N_1}^{N_2} \tilde{K}^{(j)}(t) + \sum_{j=N_{2+1}}^{P} K^{(j)}(t) \tag{13}$$

Neural network classifier: In general, a neural network comprises of units, i.e., neurons arranged in layers which translate an input vector into some output. Every unit takes an input, puts on a function to it and then passes the output on to the following layer. Normally the networks are defined to be feed-forward in which a unit feeds its output to all the units on the following layer but there is no feedback to the previous layer. Feed-forward ANNs allow signals to travel one way only from input to output.

There is no feedback (loops), i.e., the output of any layer does not affect that same layer. Feed-forward ANNs tend to be frank networks that associate inputs with outputs. They are sketchily used in pattern recognition. This category of group is also referred to as bottom up or top down. In Fig. 1, neural network classifier debited.

Weightings are applied to the signals transient from one unit to another and it is these weightings which are adjusted in the training phase to adapt a neural network to the particular problem at hand, this is the learning phase. These range from role illustrations to pattern recognitions which is what researchers will consider. For example, consider a finite sequence of input signals as $X = (X_1, X_2, ..., X_n)$, the signal P_n, where, $n = 1, 2, ..., N$, may be real numbers or vectors. By introducing a transfer function that depends on a certain parameter vector $H = (H_1, H_2, ..., H_m)$. Consequently, Y will be elected according to a chosen optimality criterion. The optimization is typically carried out in such a way that the sequence of model output signals $Y = (Y_1, Y_2, ..., Y_n)$ generated by the function $f = f(H, P)$ is as close as possible to the corresponding set of observations $P = (P_1, P_2, ..., P_n)$. Feedback networks are used to implement optimization problems. Feedback paths with these networks are interconnected with the neurons. Neurons express the differential equation given as:

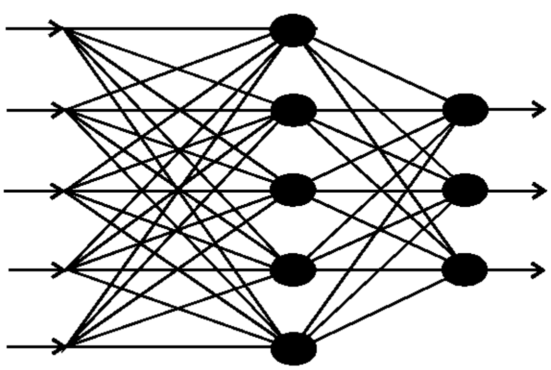

Fig. 1: Neural network

$$\frac{du_j}{dt} = -\frac{u_j}{\phi_j} + \sum_{i=1}^{m} R_{ji} x_i + J_j \tag{14}$$

Where:
x_i = $h(u_i)$
i = 1, 2, ..., m
h( ) = The sigmoid activation function

Back propagation neural network: Backpropagation neural network is the occurrence in which the action potential of aneuron produces a voltage spike mutually at the end of the axon (normal propagation) and back through to the dendritic arbor ordendrites from which of the original input current initiated. It has been revealed that this uncomplicated process can be used in a manner similar to the back propagation algorithm used in multilayer perceptrons, a type of artificial neural network. An action potential extends down the axon because of the gating properties of voltage-gated sodium channels and voltage-gated potassium channels.

Signal compression: Signal compression is essential because the sum of data that has to be transmitted may exceeds the carrying volume of most networks. Data rate is measured in bits per second and the data rate is determined by the volume of the carrier medium which is also measured in bps. In discrete wavelet transform, the most prominent information in the signal emerges in high amplitudes and not much of prominent information emerges in very low amplitudes. Data compression can be achieved by removing these smaller amplitudes. High compression ratio with good quality of reconstruction is achieved by wavelet transform. By using wavelet transform high compression ratio with excellent reconstruction is achieved.

The long-lasting proliferation of computerized biomedical signal processing systems along with the increased characteristic performance requirements and

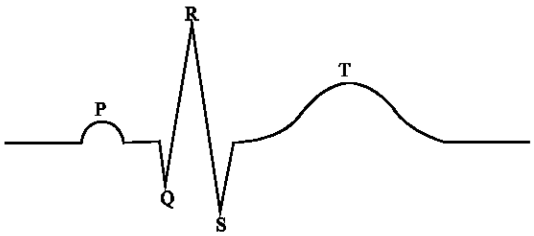

Fig. 2: Important features of ECG signal

demand for lesser cost medical care have mandated reliable, accurate and more efficient biomedical data compression techniques (Pathak and Wadhwani, 2012).

EBP-ANN based data compression: Initially the network is trained for the signal to be compressed. After successful training the trained weights between the output and hidden layer and the activation levels of the hidden layer units are stored during compression. The stored information needs less space and is used for reconstruction of the original signal.

QRS-complex detection and estimation: Since, the QRS complexes have a time-varying morphology they are not constantly the strongest signal elements in an ECG signal. Therefore, T-waves with features comparable to that of the QRS complex as well as spikes from high-frequency pacemakers can cooperate with the detection of the QRS complex as shown in Fig. 2. In addition, there are numerous sources of noise in a clinical atmosphere that can damage the ECG signal.

Shannon-Fano algorithm: Every signal has a dissimilar number of bits. According to probability, these are arranged in descending order. The lower occurrence probability is stored by a larger number of bits and the higher occurrence probability is stored by a lesser number of bits. For the decompression algorithm, the signals of different length are not a barrier.

Mathematical model: The original signal is assumed as o(n) and the white Gaussian noise is assumed as e(n). By accumulating these signals the noisy signal s(n) can be attained. It is typically of the form:

$$s(n) = o(n)+e(n)$$

The discrete wavelet transform is given as a function f(x) which can be represented by the super position of daughters $\psi_{c,d}(t)$ of a mother wavelet $\psi(t)$, where $\psi_{c,d}(t)$ can be expressed by:

$$\psi_{c,d}(t) = \frac{1}{\sqrt{c}}\psi\left(\frac{t-c}{d}\right)$$

Discrete Wavelet Transform (DWT) is performed by repeated filtering of the input signal using two filters. The filters are a Low Pass Filter (LPF) and a High Pass Filter (HPF) to decompose the signal into different scales. The output coefficient gained by the low pass filter is the approximation coefficient. This is expressed as:

$$a_{i+1}[n] = \begin{cases} a_i \times \bar{k}[2n] & i < N \\ a_i \times \bar{k}_i\text{-}N[n] & \text{otherwise} \end{cases}$$

The output coefficient of the high pass filter is detailed coefficient. This is expressed as:

$$y_{i+1}[n] = \begin{cases} a_i \times \bar{f}[2n] & i < N \\ a_i \times \bar{f}_i\text{-}N[n] & \text{otherwise} \end{cases}$$

The approximation coefficient is then divided into detail and approximation coefficients. This is expressed as given:

$$a_i[n] = \begin{cases} \breve{a}_{i+1} \times \hat{k}[n] + \breve{y}_{j+1} \times \hat{f}[2n] & i < N \\ \frac{1}{2}\left(a_{i+1} \times \hat{k}_{i-N}[n] + y_{i+1} \times \hat{f}_{i-N}[n]\right) & \text{otherwise} \end{cases}$$

and the process is repeated to N levels. The next step is to calculate the noise intensity for various levels:

$$\alpha_j = \left(\frac{\text{median}\left(\left|d_j\right|\right)}{0.6745}\right)$$

And:

$$\eta_j = \alpha\sqrt{2\text{In}(L)}$$

The output of the wavelet decomposition is the approximation and the detail coefficients are obtained in every level of decomposition. Then, calculate the wavelet frequency thresholding between the detail coefficient and the approximation coefficient by the following steps:

- Calculate the mappings of the initial values for various sequences. Let: $Y_j' = Y_j / y_j = (y_j'(1), y_j'(2), ..., y_j'(n))$ where, j = 0, 1, ..., M
- Calculate the difference of the mappings. Let $\Omega_j = (\Omega_j(1), \Omega_j(2), ..., \Omega_j(n)$ where, $\Omega_j(g) = \left|x_0'(g) - x_j'(g)\right|$ and j = 0, 1, 2, ..., N
- Calculate the biggest and smallest difference of $\Omega_j(g)$. Let $R = \max_j \max_g \Omega_j(g)$ and $r = \min_j \min_g \Omega_j(g)$

- Calculate the incident co-efficients $\gamma_j(g) = r+\vartheta R/\Omega_j(g)+\vartheta R$. Where, $\vartheta \in (0, 1)$; $y = 1, 2, ..., n$ and $j = 0, 1, 2, ..., N$
- Calculate the wavelet frequency thresholding. So, $\gamma = 1/n\sum^{n}_{g=1}\gamma_j(g)$ where, $j = 0, 1, 2, \ldots, N$:

$$\gamma_{0j}(g) = \gamma\left(x_0(g), x_j(g)\right) = \frac{\min_j \min_g \left|x_o(g)\text{-}x_j(g)\right| + \vartheta \max_i \max_g \left|x_0(g)\text{-}x_j(g)\right|}{\left|x_o(g)\text{-}x_j(g)\right| + \vartheta \max_j \max_g \left|x_o\text{-}x_j(g)\right|}$$

According to the mentioned method calculate the threshold $\eta_{THR} = \eta_j.\gamma_j$ then maintain the original value when a position wavelet transform coefficient value is larger than the threshold, otherwise let the value be zero. Calculate the threshold by using the formula THR = S[n]/24 where, S[n] is the original signal. After that the reconstruction can be done to produce the original signal.

This is an inverse process of decomposition and the denoised signal will be obtained. In order to find the optimized wavelet the signal is trained in the neural network with the features present in the signal. The inputs are labeled as $x_1, x_2, ..., x_m$ and the weights are labeled as $w_1, w_2, ..., w_m$. The weighted sum of input values are represented as:

$$\omega = \sum_{j=1}^{m} w_j x_j$$

After that the signals are compressed through the probabilities of the source produce symbols. The probabilities are $q_1, q_2, ..., q_n$ and the entropy is given by:

$$V = -\sum_{j=0}^{m} q_j \log_2 q_j$$

After the compression has been successfully completed the signals are stored in a database.

Flow chart: Figure 3 shows flow chart about optimized wavelet. Figure 4 shows signal de-noising and compression. The signal is decomposed and then reconstructed upto level 3. For example, consider the signal is denoted as S_{j+1}. In the synthesis filter the signal is then decomposed using a high pass filter which is denoted as $H_0(n)$ and a low pass filter which is denoted as $H_1(n)$ and as a result the detail coefficient d_j and approximation coefficient a_j are gained.

Then, in the next level of decomposition the approximation coefficients are again decomposed and the process is repeated upto N levels. By using the coefficients gained at every level calculate the threshold value by using the Wavelet Frequency Thresholding

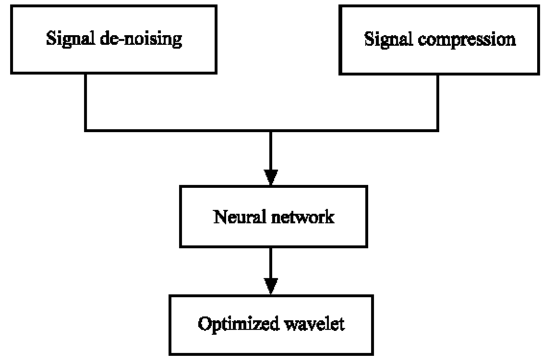

Fig. 3: Flow chart

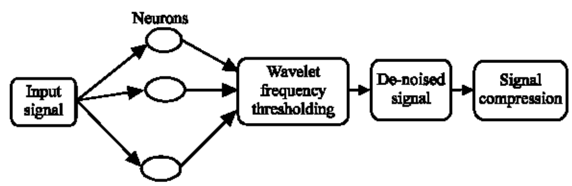

Fig. 4: Neural network de-noising and compression

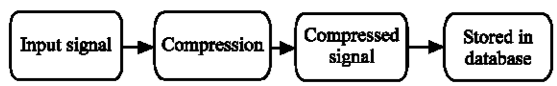

Fig. 5: Signal compression

Method. After the calculation of the threshold value, the coefficients above the threshold value remain constant and the coefficients which are below the threshold value are changed as zero. In the reconstruction stage the coefficients are fed as an input to the analysis filter. Here, the reverse process of decomposition will be done and the output will be the denoised signal S_{j+1}.

The signals are compressed using compression techniques. The signals are then fed as the input when the compressions have been over the compressed signal will be attained as the output and these signals are stored in a database as per shown in Fig. 5. The input of the neural network will be the samples from numerous channels which is equivalent to the time segments with a constant length. The target channel equivalent to the time segment used in the input will be the output (Rodrigues and Couto, 2012). The preprocessed signals will be trained in the neural network to automatically predict which wavelet is best suitable for de-noising and compression.

The biomedical signals are collected from the databases and the databases enclose the signals without noise. Subsequently, the noise is added to those signals by using a MATLAB Program and then the number of samples is chosen for further processing. In this experiment 40,000 samples of the signals are chosen and calculated are a variety of statistical parameters of the noisy biomedical signals.

The noisy signals are decomposed at level three using various wavelet families. De-noising of the signals has been performed based on the selected threshold value. By using the parameters select the optimized wavelet transform which is best suitable for de-noising using the neural network classifier and then these signals are compressed.

Signal to noise ratio: The signal to noise ratio is the ratio of the true signal amplitude to the standard deviation of the noise. The quality of the signal is termed as the signal to noise ratio which is expressed as:

$$SNR = 10\log\frac{S_{original}}{S_{noise}}$$

Where:

$S_{original}$ = The original signal without noise
S_{noise} = The noisy signal

Mean square error: The difference between the denoisied signal and the original signal is given by the following equation:

$$MSE = \frac{1}{M}\sum_{j=1}^{M}\left(y(j)-\overline{y(j)}\right)^2$$

Where:

$y(j)$ = The original signal
$\overline{y(j)}$ = The denoised signal
M = The length of the signal

Compression ratio:

$$\text{Compression ratio} = \frac{\text{Size of original signal} - \text{Size of compressed signal}}{\text{Original signal}}\times 100$$

Compression factor:

$$\text{Compression factor} = \frac{\text{Size of the signal before compression}}{\text{Size of the signal after compression}}$$

RESULTS AND DISCUSSION

Researchers have experimented the proposed Wavelet De-Noising Method in various biomedical signals such as ECG, EEG and EMG signals taken from the MIT-BIH database. By using wavelet toolbox from MATLAB environment they have experimented the de-noising approach. Researchers have trained the neural network classifier to select the best suitable wavelet type for each kind of signal. In the training stage they have denoised the signals by all wavelets and the best resulting wavelet is selected based on the results obtained. The conventional parameters such as PSNR and PRD are compared to find the optimal de-noising wavelet. Table 2-7 show the de-noising output of ECG, EEG and EMG signals, respectively.

Table 2: De-noising results of different wavelets for ECG signal 1

Wavelet name	Mean square error	Signal noise ratio	PRD
haar	2.5234	192.4924	0.0066082
db1	2.5234	192.4924	0.0066082
db2	2.5086	192.5514	0.0065887
db3	2.4922	192.6170	0.0065671
sym2	2.5008	192.5826	0.0065784
sym3	2.4932	192.6108	0.0065692
sym4	2.5047	192.5670	0.0065836
coif1	2.5063	192.5608	0.0065856
coif2	2.5070	192.5577	0.0065866
coif3	2.5063	192.5608	0.0065856
bior1.1	2.5125	192.5359	0.0065938
bior1.3	2.5055	192.5639	0.0065846
bior1.5	2.5125	192.5359	0.0065938
rbio1.1	2.5133	192.5328	0.0065948
rbio1.3	2.5438	192.4123	0.0066347
rbio1.5	2.5297	192.4677	0.0066163

Table 3: De-noising results of different wavelets for ECG signal 2

Wavelet name	Mean square error	Signal noise ratio	PRD
haar	2.4453	192.8073	0.00650490
db1	2.4453	192.8073	0.00650490
db2	2.5070	192.5581	0.00665865
db3	2.5031	192.5737	0.00658140
sym2	2.5039	192.5705	0.00658240
sym3	2.4938	192.6112	0.00656900
sym4	2.5078	192.5550	0.00658750
coif1	2.4953	192.6049	0.00657110
coif2	2.5047	192.5674	0.00658340
coif3	2.5148	192.5270	0.00659680
bior1.1	2.4461	192.8041	0.00650600
bior1.3	2.4953	192.6049	0.00657110
bior1.5	2.4969	192.5987	0.00657310
rbio1.1	2.4445	192.8105	0.00650390
rbio1.3	2.4469	192.8009	0.00650700
rbio1.5	2.4508	192.7850	0.00651220

Training stage de-noising results: Researchers have analyzed the de-noising outputs and the optimal de-noising wavelets for the three kinds of bio-medical signals. It also demonstrates the optimal de-noising wavelets for ECG, EEG and EMG signals.

The comparison results for de-noising are presented in the Table 8. For ECG signal 1, the wavelet db3 provides better results in terms of all parameters MSE, SNR and PRD. The rbio1.1 wavelet produces the lowest MSE and PRD values and a highest value of SNR for the second ECG signal. For the first EEG signal, both db2 and bior1.3

Table 4: De-noising results of different wavelets for EEG signal 1

Wavelet name	Mean square error	Signal noise ratio	PRD
haar	1.1466	161.8964	0.030512
db1	1.1508	161.8595	0.030568
db2	1.1459	161.9025	0.030502
db3	1.1515	161.8533	0.030577
sym2	1.1480	161.8840	0.030530
sym3	1.1529	161.8411	0.030596
sym4	1.1487	161.8779	0.030540
coif1	1.1484	161.8810	0.030535
coif2	1.1462	161.8994	0.030507
coif3	1.1473	161.8902	0.030521
bior1.1	1.1522	161.8472	0.030587
bior1.3	1.1459	161.9025	0.030502
bior1.5	1.1466	161.8964	0.030512
rbio1.1	1.1480	161.8840	0.030530
rbio1.3	1.1515	161.8533	0.030577
rbio1.5	1.1494	161.8717	0.030549

Table 5: De-noising results of different wavelets for EEG signal 2

Wavelet name	Mean square error	Signal noise ratio	PRD
haar	1.1382	161.8694	0.030553
db1	1.1378	161.8725	0.030548
db2	1.1347	161.9002	0.030506
db3	1.1343	161.9033	0.030501
sym2	1.1340	161.9064	0.030496
sym3	1.1305	161.9373	0.030449
sym4	1.1329	161.9156	0.030482
coif1	1.1315	161.9280	0.030463
coif2	1.1336	161.9095	0.030492
coif3	1.1322	161.9218	0.030473
bior1.1	1.1371	161.8786	0.030539
bior1.3	1.1354	161.8940	0.030515
bior1.5	1.1312	161.9311	0.030459
rbio1.1	1.1357	161.8910	0.030520
rbio1.3	1.1389	161.8633	0.030562
rbio1.5	1.1350	161.8971	0.030510

Table 6: De-noising results of different wavelets for EMG signal 1

Wavelet name	Mean square error	Signal noise ratio	PRD
haar	0.071475	160.5723	0.032600
db1	0.074775	160.1209	0.033344
db2	0.059875	162.3432	0.029838
db3	0.058200	162.6269	0.029417
sym2	0.058650	162.5499	0.029531
sym3	0.058875	162.5116	0.029587
sym4	0.060525	162.2352	0.029999
coif1	0.061675	162.0470	0.030283
coif2	0.062500	161.9141	0.030485
coif3	0.061950	162.0025	0.030350
bior1.1	0.072925	160.3715	0.032929
bior1.3	0.058250	162.6183	0.029430
bior1.5	0.055600	163.0839	0.028753
rbio1.1	0.074425	160.1679	0.033266
rbio1.3	0.079925	159.4549	0.034473
rbio1.5	0.083000	159.0774	0.035130

produce the highest value for SNR and the lowest value for MSE and PRD. The sym3 wavelet generates better results for EEG signal 2. For both the EMG signals 1 and 2, better results are given by the wavelet bior 1.5. The wavelet coif 3 provides a better compression ratio for a signal where the corresponding PRD value is not the least one. The least PRD value is given by the wavelet rbio1.3. Table 9 and 10 depict the best results produced for each signal with their values for each parameter. From the de-noising results, it is obvious that for an EMG signal, the wavelet bior1.5 produces accurate results. For an EEG signal, sym3 wavelet produces the best results. For an ECG signal, rbio1.1 offers the best values of MSE, SNR and PRD.

Table 7: De-noising results of different wavelets for EMG signal 2

Wavelet name	Mean square error	Signal noise ratio	PRD
haar	0.077650	162.9512	0.028944
db1	0.075850	163.1858	0.028607
db2	0.063825	164.9119	0.026241
db3	0.060725	165.4098	0.025596
sym2	0.062650	165.0977	0.025999
sym3	0.060325	165.4759	0.025512
sym4	0.059875	165.5508	0.025416
coif1	0.061900	165.2182	0.025842
coif2	0.059325	165.6431	0.025299
coif3	0.061350	165.3074	0.025727
bior1.1	0.077075	163.0256	0.028837
bior1.3	0.057825	165.8991	0.024977
bior1.5	0.056300	166.1664	0.024646
rbio1.1	0.075850	163.1858	0.028607
rbio1.3	0.080850	162.5474	0.029534
rbio1.5	0.086425	161.8806	0.030536

Table 8: Compression results of different wavelets for a sample signal

Wavelet name	PRD value	Compression ratio
haar	0.120290	62.6984
db1	0.120290	62.6984
db2	0.096617	70.6349
db3	0.112870	71.4286
sym2	0.096617	70.6349
sym3	0.112870	71.4286
sym4	0.106310	72.6190
coif1	0.106320	71.4286
coif2	0.110800	73.4127
coif3	0.112020	74.6032
bior1.1	0.120290	62.6984
bior1.3	0.109970	65.4762
bior1.5	0.125360	66.2698
rbio1.1	0.120290	62.6984
rbio1.3	0.083486	71.0317
rbio1.5	0.101100	73.0159

Table 9: Most accurate de-noising results for different signals

Signal	Wavelet	MSE	SNR	PRD
ECG1	db3	2.4922	192.6170	0.0065671
ECG2	rbio1.1	2.4469	192.8105	0.0065039
EEG1	db2, bior 1.3	1.1459	161.9025	0.0305020
EEG2	sym3	1.1305	161.9373	0.0304490
EMG1	bior1.5	0.0556	163.0839	0.0287530
EMG2	bior1.5	0.0563	166.1664	0.0246460

Table 10: Most accurate compression results for a sample signal

Wavelet	Metric	Values
rbio1.3	PRD	0.083486
coif3	Compression ratio	74.603200

CONCLUSION

In this study, researchers have presented a detailed analysis of the de-noising and compression process of various wavelet families and biomedical signals such as

Electrocardiogram (ECG), Electromyography (EMG) and Electroencephalography (EEG) by applying Discrete Wavelet Transform (DWT). Then, the Shift Invariant Method is used for the decomposition of noise added signals. The threshold scheme based on wavelet by means of wavelet frequency thresholding is applied after decomposition for the purpose of noise removal. By using Wavelet Reconstruction Method the original signal is reconstructed. The neural network which automatically classifies the best suitable wavelet for de-noising is used for wavelet classification which then classifies the optimized wavelet for signal de-noising. The signals are then compressed and stored in a database.

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