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Study and Analysis of Advanced Control Algorithms on a FOPDT Model

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Abstract: This study illustrates the simulation of a simple First Order plus Delay Time (FOPDT) process model using Advanced Control algorithms. Specifically, these advanced algorithms are the IMC-based PID controller, the Model Predictive Controller (MPC) and the Proportional-Integral-Plus controller (PIP) and their performance is compared with the conventional Proportional-Integral-Derivative (PID) algorithm. The simulations took place using the Matlab Software.

Key words: FOPDT, Advanced Control algorithms, IMC-based PID, MPC, PIP, PID

INTRODUCTION

The popular control algorithm used in industry is the PID controller which has been implemented successfully in various technical fields. However, since the evolution of computers and mainly during the 1980s a number of modern and advanced control algorithms have been also developed and applied in a wide range of industrial and chemical applications. Some of them are the Internal Model-based PID controller, the Model Predictive controller and the Proportional-Integral-Plus controller. The common characteristic of the above algorithms is the presence in the controller structure an estimation of the process' model. The purpose of this study is to apply these advanced algorithms to a linear First Order Plus Delay Time (FOPDT) process model and compare their step response with the conventional PID controller.

Initially, it will be presented a brief discussion over the theoretical designing aspects of each applied algorithm. The main section of the study is devoted to the simulation results in terms of type 1 servomechanism performance of a simple FOPDT process using the above control algorithms in various practical scenarios.

Proportional-integral-derivative controller: The Proportional-Integral-Derivative (PID) Control algorithm is the most common feedback controller in industrial processes. It has been successfully implemented for over 50 years as it provides satisfactory robust performance despite the varied dynamic characteristics of a process plant (Willis, 1999).

The proper tuning of the PID controller aims a desired behavior and performance for the controlled system and refers to the proper definition of the parameters which characterize each term. Over the past, it has been proposed several tuning methods but the most popular (due to its simplicity) is the Ziegler-Nichols Tuning Method. This tuning method is based on the computation of a process's critical characteristics, i.e., critical gain K_{cr} and critical period P_{cr} (Ziegler *et al.*, 1942). Table 1 summarizes the computation of PID parameters (Astrom and Hagglund, 1995).

Table 1: Ziegler-Nichols PID tuning computation

Controller	K_P	T_I	T_D
P	$K_{cr}/2$	-	-
PI	$K_{cr}/2.2$	$P_{cr}/1.2$	-
PID	$K_{cr}/1.7$	$P_{cr}/2$	$P_{cr}/8$

IMC-based PID controller: The Internal Model Control (IMC) algorithm is based on the fact that an accurate model of the process can lead to the design of a robust controller both in terms of stability and performance (Coughanour, 1991). The basic IMC structure is shown in Fig. 1 and the controller representation for a step perturbation is described by Eq. 1:

$$G_q(s) = \frac{G_f(s)}{G_{mm}(s)} \quad (1)$$

Where:
- $G_{mm}(s)$ = The inverse minimum phase part of the process model
- $G_f(s)$ = A nth order low pass filter $1/(\lambda s+1)^n$

The filter's order is selected so that $G_q(s)$ is semi-proper and λ is a tuning parameter that affects the speed of the closed loop system and its robustness (Morari and Zafiriou, 1989).

However, there is equivalence between the classical feedback and the IMC control structure allowing the transformation of an IMC controller to the form of the well-known PID algorithm:

$$G_c(s) = \frac{G_q(s)}{1 - G_m(s)G_q(s)} \quad (2)$$

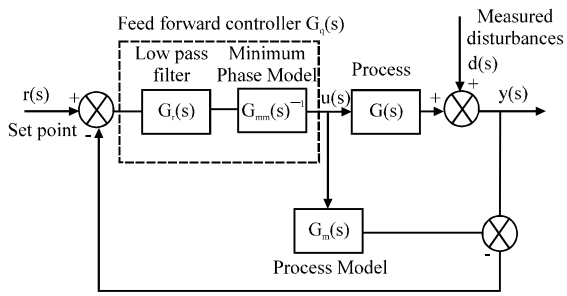

Fig. 1: IMC control structure

Table 2: IMC-based PID tuning parameters of a FOPDT process

Controller	K_P K_c	T_I	T_D	λ/θ
IMC-based PI without pade	$\frac{\tau}{\lambda}$	$\tau+\frac{\theta}{2}$	-	>1.7
IMC-based PI	$\frac{2\tau+\theta}{2\lambda}$	$\tau+\frac{\theta}{2}$	-	>1.7
IMC-based PID	$\frac{2\tau+\theta}{2\lambda+\theta}$	$\tau+\frac{\theta}{2}$	$\frac{\tau\theta}{2\tau+\theta}$	>0.8

The resulted controller is called IMC-based PID controller and has the usual PID form Eq. 3:

$$G_c(s)=K_p\left(1+T_D s+\frac{1}{T_I s}\right) \quad (3)$$

IMC-based PID tuning advantage is the estimation of a single parameter λ instead of two (concerning the IMC-based PI controller) or three (concerning the IMC-based PID controller). The PID parameters are then computed based on that parameter (Coughanour, 1991). Though for the case of a FOPDT (Eq. 4) process model, the delay time should be approximated first by a zero-order Pade (usually) approximation (Bequette, 2003). However, the IMC-based PID Tuning Method can be summarized according tothe Table 2 (Morari and Zafiriou, 1989):

$$G(s)=\frac{k_c}{\tau s+1}e^{-\theta s} \quad (4)$$

Model predictive controller: MPC refers to a class of Advanced Control algorithms that compute a sequence of manipulated variables in order to optimize the future behavior of the controlled process. Initially, it has been developed to accomplish the specialized control needs in power plants and oil refineries. However, because its ability to handle easily constraints and MIMO systems with transport lag, it can be used in various industrial fields (Naeem, 2003).

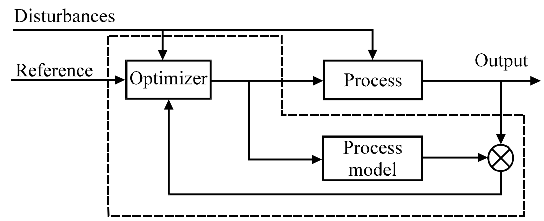

Fig. 2: MPC block diagram

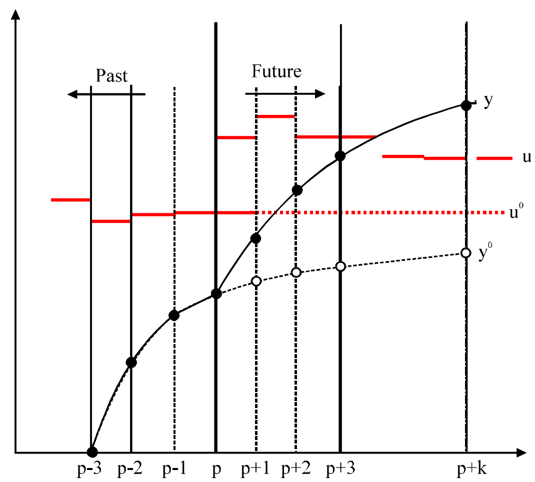

Fig. 3: Receding horizon strategy

The first Predictive Control algorithm is referred to the publication of Richalet *et al.* (1978) titled "Model Predictive Heuristic Control". However, in 1979, Cutler and Ramaker by Shell™ developed their own MPC algorithm named Dynamic Matrix Control-DMC (Cutler, 1983). Since then, a great variety of algorithms based on the MPC principle has been also developed. Their main difference is focused on the use of various plant models which is an important element of the computation of the predictive algorithm (i.e., step model, impulse model, state-space models, etc.). Figure 2 shows a typical MPC block diagram.

The main idea of the Predictive Control theory is derived from the exploitation of an internal model of the actual plant which is used to predict the future behavior of the control system over a finite time period called prediction horizon p (Fig. 3). This basic control strategy of predictive control is referred to as receding horizon strategy (Maciejowski, 2001).

Its main purpose is the calculation of a controlled output sequence y(k) that tracks optimally a reference trajectory $y^0(k)$ during m present and future control moves ($m\leq p$). Though m control moves are calculated at each sampled step only the first $\Delta\hat{u}(k)$ = $(u^0(k)-u(k))$ is implemented. At the next sampling interval, new values of the measured output are obtained. Then, the control horizon is shifted forward by one step and the above computations are repeated over the prediction horizon. In order to calculate the optimal controlled output sequence, it is used a cost function of the following form (Morari and Ricker, 1998):

$$J=\sum_{l=1}^{p}\left\|\Gamma_l^y\left[y(k+1|k)-y^0(k+1)\right]\right\|^2+\sum_{l=1}^{m}\left\|\Gamma_l^u\left[\Delta\hat{u}(k+1-1\right]\right\|^2 \quad (5)$$

where, Γ^y_1 and Γ^u_1 are weighting matrices used to penalize particular components of output and input signals, respectively at certain future intervals. The solution of the LQR control problem is resulted to a feedback proportional controller estimated as the gain matrix k solution of the well-known Riccati equation over the prediction horizon:

$$u(k)=-kx_k \quad (6)$$

PIP controller: PIP controller comprises a part of the True Digital Control (TDC) Method and can be considered as a logical extension to the conventional PI/PID controller but with inherent model predictive control action. The power of the PIP design derives from its exploitation of a specialized Non-Minimal State Space (NMSS) representation of a linear and discrete system referred as NMSS/PIP formulation (Hesketh, 1982; Young *et al*., 1987).

The fact that the PIP is considered as a logical extension of the conventional PI/PID controlled can be appeared better when the process's transfer function is second order of higher or includes transport lag greater than one sampling interval. Then PIP controller includes also a dynamic feedback and input compensation introduced "automatically" by the specialized NMSS formulation of the control problem (Taylor *et al*., 2003) that in general has a numerous advantages against other advanced control structures (Taylor *et al*., 2000). Any linear discrete time and deterministic SISO ARIMAX Model can be represented by the following specialized NMSS equations:

$$x(k)=Fx(k-1)+qu(k-1)+dy_d(k) \quad (7)$$

$$y(k)=hx(k) \quad (8)$$

where, the vectors F, q, d and h comprise the parameters of the Eq. 8 (Young *et al*., 1987).

In the specialised NMSS/PIP case, the non-minimum n+m state vector x(k) consists not only in terms of the present and past sampled value of the output variable y(k) and the past sampled values of the input variable u(k) (as it happens in the conventional NMSS design) but also of the integral of error state vector z(k) introduced to ensure type 1 servomechanism performance, i.e.:

$$x(k)=\left[y(k),y(k-1),...,y(k-n+1), u(k-1),...,u(k-m+1), z(k)\right]^T \quad (9)$$

The integral of error state vector z(k) defines the difference between the reference input (setpoint) $y_0(k)$ and the sampled output y(k):

$$z(k) = z(k\text{-}1)+\left\{y_d(k)\text{-}y(k)\right\} \quad (10)$$

The control law associated with the NMSS Model results to the usual State-Variable Feedback (SVF) form:

$$u(k)=-kx(k) \quad (11)$$

where, k is the n+m SVF gain vector. The control gain vector may be easily calculated by means of a standard LQ cost function:

$$J=\frac{1}{2}\sum_{i=0}^{\infty}\left\{x(i)^TQx(i)+Ru^2(i)\right\} \quad (12)$$

Where:
Q = A n+m×n+m weighting matrix
R = A scalar input u(i) weight

It is worth noting that because of the special structure of the state vector x(k), the weighting matrix Q is defined by its diagonal elements which are directly associated with the measured variables and integral of error state vector. For example, the diagonal matrix can be defined in the following default form:

$$Q=\mathrm{diag}\begin{bmatrix}\underbrace{q_1\cdots q_n}_{qy=1/n} & \underbrace{q_{n+1}\cdots q_{n+m+1}}_{qu=1/m} & \underbrace{q_{n+m}}_{qe}\end{bmatrix} \quad (13)$$

The SVF gains are obtained by the steady-state solution of the well-known discrete time matrix Riccati equation (Astrom and Wittenmark, 1984) given the NMSS System description (F and q vectors) and the weighting matrices (Q and R):

$$k=\left[f_0 \quad f_1 \quad \cdots \quad f_{n-1} \quad g_1 \quad \cdots \quad g_{m-1} \quad -k_1\right] \quad (14)$$

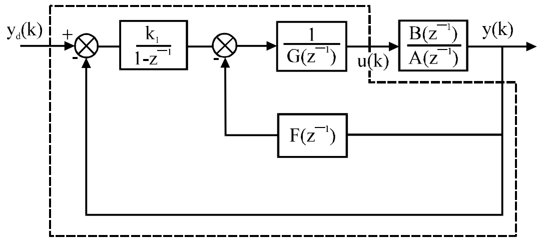

Fig. 4: PIP feedback block diagram

In a conventional feedback structure, the SVF controller can be implemented as shown in Fig. 4 where it becomes clear how the PIP can be considered as a logical extension of the conventional PI/PID algorithm, enhanced by a higher-order forward path and input/output feedback compensators $G\in\Re[z^{-1}, m\text{-}1]$ and $F\in\Re[z^{-1}, n\text{-}1]$, respectively (Taylor *et al*., 2003):

$$F(z^{-1})=f_0+f_1z^{-1}+...+f_{n-1}z^{-(n-1)} \quad (15)$$

$$G(z^{-1})=1+g_1z^{-1}+...+g_{m-1}z^{-(m-1)} \quad (16)$$

PROBLEM FORMULATION

In order to asses the practical utility of the above described advanced control algorithms, a series of implementation simulations have been conducted on a simple FOPDT process. For comparison purposes, a conventional PID controller is also designed using the Ziegler-Nichols Method.

The FOPDT process model is described by Eq. 17 and initially is assumed absence of plant model mismatch, inputs constraints or measured disturbances. The model selection is based on the fact that a FOPDT Model represents any typical SISO chemical process. The simulation took place using the Matlab/Simulink™ Software and the results are discussed in terms of type 1 servomechanism performance:

$$G(s)=\frac{1}{s+1}e^{-0.3s} \quad (17)$$

The next simulation scenario includes constraints in the input manipulated variables:

$$-2\leq u(t)\leq 2 \quad (18)$$

In the final simulation scenario a simple disturbance model described by Eq. 18 is also implemented in order to study the capability of each controller in disturbance rejection:

$$G_d(s)=\frac{0.8}{s+1}e^{-0.1s} \quad (19)$$

The critical characteristics for the estimation of PID parameters (Table 1) are K_{cr} = 5.64 and P_{cr} = 1.083. The IMC-based PID parameters are estimated according to Table 2 selecting λ = 0.5 and n = 1. The calculation of MPC gain matrix includes the following parameters; input weight Γ^u_1, output weight Γ^y_1, control horizon m = 10 and infinity prediction horizon. Whether the absence of measured disturbances or not the 'default' LQ weight matrices for the PIP controller are:

$$Q_{diag}=[1 \quad 0.25 \quad 0.25 \quad 0.25 \quad 1]$$

R = 0.25 (absence of measured disturbances) and:

$$Q_{diag}=[1 \quad 0.25 \quad 0.25 \quad 0.25 \quad 1 \quad 0 \quad 0]$$

R = 0.25 (presence of measured disturbances).

PROBLEM SOLUTION

With no disturbances and input constraints, the output response (Fig. 5) for the Advanced Control algorithms yields satisfactory step behavior with good set point tracking and smooth steady state approach. However, the response of the conventional PID seems to be rather disappointing as it yields a large overshoot. Figure 6 demonstrates their control action response. Mainly concerning MPC and PID algorithms, the initial sharp increase of their control action signal may not be acceptable during a practical realization of the controller in an actual industrial plant.

Figure 7 shows the output response after the introduction of input constraints defined by Eq. 17. According to the results, both PIP and IMC-based PID controllers were unaffected by the input constraints as their constrained control action response has been within the constrained limits. Although, the response of the conventional PID controller retained its large overshoot, the introduction of input constraints has optimized its smoothness. Finally, MPC maintained its satisfactory performance, although the fact that its manipulated variable has been constrained the most (Fig. 8).

Figure 9 demonstrates the output responses of the process during the introduction of measured disturbances defined by Eq. 18. According to the results, MPC controller yields the most optimal response while PIP controller sustains its performance. On the contrary IMC-based PID as well as the conventional PID yield a rather large overshoot.

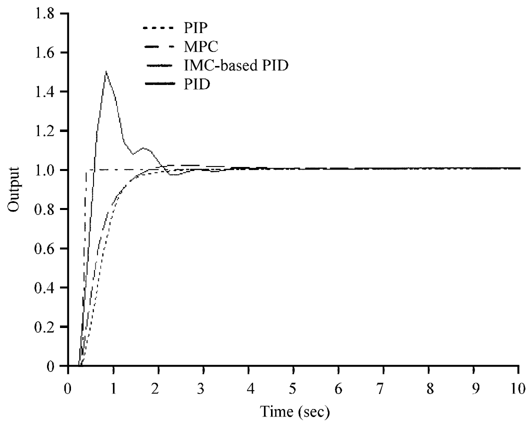

Fig. 5: Unconstrained output step response

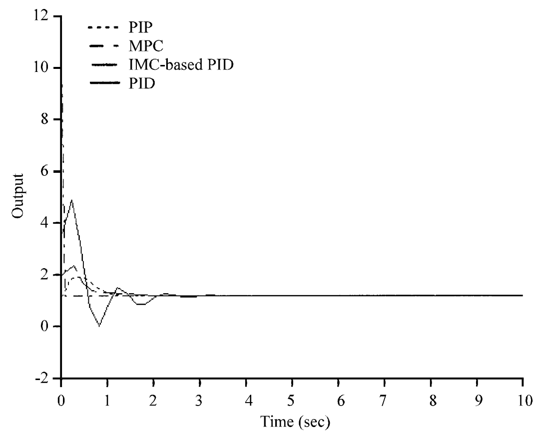

Fig. 6: Unconstrained control action step response

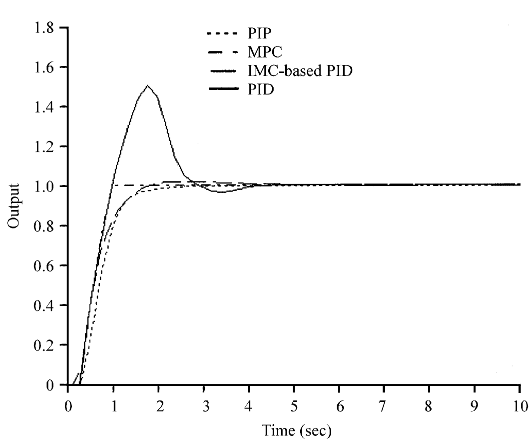

Fig. 7: Output step response with input constraints

Table 3 shows an approximate numerical evaluation of the control algorithms for each scenario. The evaluation parameters are the Overshoot (O), Rise Time (RT), Settling Time (ST), Integral Square Error (ISE), Robust stability (RS) and Robust Performance (RP).

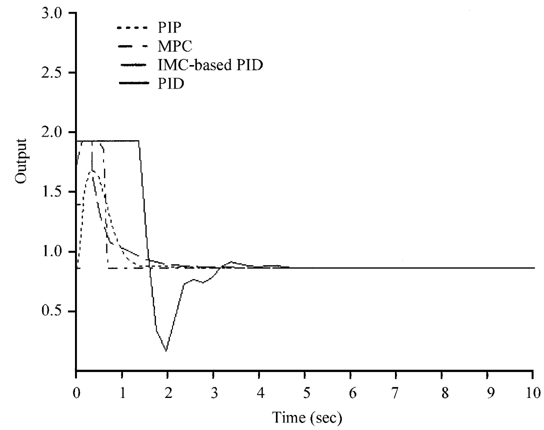

Fig. 8: Constrained control action step response

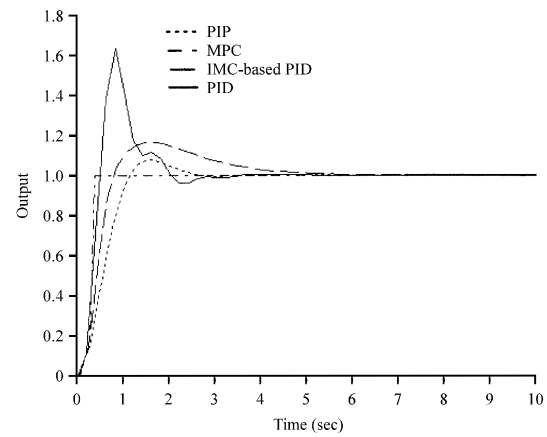

Fig. 9: Output step response with measured disturbances

Table 3: Numerical evaluation of control algorithms

Controllers	O (%)	RT	ST	ISE
Scenario 1				
PID	49.80	0.5300	1.9300	0.49
IMC-based PID	1.76	1.1800	1.3800	0.51
MPC	0.00	0.0021	0.0021	???
PIP	0.00	1.2500	1.4500	0.65
Scenario 2				
PID	50.00	0.9700	2.6600	0.67
IMC-based PID	2.00	1.2400	1.4400	0.52
MPC	0.00	0.8500	0.9500	0.97
PIP	0.00	1.2500	1.4500	0.65
Scenario 3				
PID	62.95	0.5300	1.9300	0.48
IMC-based PID	16.23	0.7800	3.3800	0.40
MPC	0.00	0.0021	0.0021	0.02
PIP	7.38	0.9500	1.9500	0.50

CONCLUSION

This study discusses the effect of three Advanced Control algorithms on a FOPDT process model in terms of type 1 servomechanism performance. These algorithms are the IMC-based PID controller, the model predictive controller and the PIP controller. After their implementation in the FOPDT process their step response was simulated using the Matlab/Simulink™ Software and compared with the conventional PID controller in various practical scenarios. Such scenarios include the implementation of input constraints or measured disturbances.

According to the simulations results, all the Advanced Control algorithms perform satisfactory step behavior with good set point tracking and smooth steady state approach. They also sustain their robustness and performance during the introduction of input constraints or measured disturbances. Surprisingly, the step response of the conventional PID controller was not as optimal as it has been expected as its overshoot exceeds any typical specification limits.

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