Research Journal of Applied Sciences 9 (7): 397-401, 2014
ISSN: 1815-932X

# Verification of Discharge Coefficient of Rectangular Side Weirs Using Shabayek Model

Temitayo Abayomi Ewemoje and Olufemi Peter Abimbola
Department of Agricultural and Environmental Engineering, Faculty of Technology,
University of Ibadan, Ibadan, Oyo State, Nigeria

**Abstract:** Farm irrigation system management depends on accurate measurement of water flow and good water delivery program. Flow over rectangular side weirs were experimentally investigated in a laboratory to verify the coefficient of discharge of side weirs for subcritical flow using Shabayek Non-Linear Combination Model. A physical model was constructed and series of experimental tests were carried out. Effects of upstream Froude number and sill height on the coefficient of discharge of the side weirs were examined at sill heights of 0.06, 0.10 and 0.14 m and weir crest lengths of 0.04, 0.055, 0.07, 0.085 and 0.10 m for each sill height. The sill heights were varied with the channel bed slope between 2.94 and 6.23%. To show the relationship of coefficient of discharge $C_d$ with both upstream Froude number $F_r$ and ratio of sill height to upstream flow depth $S/Y_1$ taken separately, linear regression analysis was used in analyzing experimental data while multiple regression analysis was used in showing the relationship of $C_d$ with both $F_r$ and $S/Y_1$ taken together. Graphical models developed showed that these relationships follow the same trend with previous research works.

**Key words:** Rectangular side weirs, subcritical flow, discharge coefficient, Froude number, sill height, Shabayek Model

## INTRODUCTION

Flow in channels with side weirs has been studied by several researchers and most of these studies were motivated by the wide use of this type of hydraulic structure in irrigation and sewerage systems; hence, there is the need to improve knowledge about its hydraulic performance characteristics (Pinheiro and Silva, 1999). Flow discharge according to Liao and Knight (2007) is an important parameter in the design of open channels and has a very significant influence on the water level or stage.

Weirs are typically installed in open channels to determine discharge. Discharge is directly related to water depth above the crotch (bottom) of the V and this distance is called head (Gupta and Gupta, 2008; LMNO Eng., 2012). This weir, also called triangular weir is typically used instead of rectangular weir under low flow conditions where rectangular weirs tend to be less accurate (Chin, 2006). The V-notch design causes small changes in discharge to have a large change in depth allowing more accurate head measurement than rectangular weirs (Munson *et al*., 1990). Side weirs are extensively used as a means of diverting excess storm waters from urban drainage systems and as water level control devices in flood control works. In irrigation engineering, side weirs of broad crests are used as head regulators of distributaries and escapes (Subramanya, 2008).

Flow over a side weir is an example of situation where spatially varied flow occurs. The concept of constant specific energy is often adopted for studying the flow characteristics of these weirs (Singh *et al*., 1994). Based on the assumption that when the Froude number is low (i.e., the flow is subcritical), the energy equation can be approximated by the stages heads equality, actual literature and many commercial packages use this concept for the treatment of a junction's internal boundary handling seeing as it is easy to implement and it avoids the solving of nonlinear equations. In the last decade, many nonlinear combining models, based on the momentum conservation through the junction have been reported. Shabayek *et al.* (2002) developed a one-dimensional theoretical model providing the necessary interior boundary equations for combining subcritical open-channel junctions. The model is based on applying the momentum principle together with mass continuity through the junction. Experimental and numerical investigations by Kesserwani *et al.* (2008) showed that even while the Froude number is subcritical, precautions have to be taken when dealing with the concept of energy heads equality.

**Corresponding Author:** Temitayo Abayomi Ewemoje, Department of Agricultural and Environmental Engineering, Faculty of Technology, University of Ibadan, Ibadan, Oyo State, Nigeria

Various studies from earlier researchers have established the relationships of $C_d$ with upstream Froude number $F_r$, ratio of sill height to upstream flow depth $S/Y_1$ and both taken together. But these studies were based on the equality model which assumes the equality of water levels or that of energy levels at the junction. The conventional concept of energy heads equality does not always ensure the conservation of momentum through the junction and is not sufficient even while the Froude number range is subcritical (Kesserwani *et al.*, 2008). Hence, there is need to experimentally investigate and verify the relationship of $C_d$ with both $F_r$ and $S/Y_1$.

In the last decade, several one-dimensional theoretical junction models based on momentum principle were developed. Of all these models, Shabayek Model was chosen because it is based on applying the momentum principle together with mass continuity through the junction and also because it incorporates almost all the physical effects. Hence, the objective of this study is to experimentally investigate and verify effect of upstream Froude number and sill height on the discharge coefficient of rectangular side weirs under subcritical flow conditions using Shabayek Open-Channel Junction Model in a prismatic rectangular main channel.

**Discharge coefficient expressions:** There are many expressions to compute the discharge coefficient of side weirs. The expressions proposed by six different reseachers. Subramanya and Awasthy presented a study considering subcritical and supercritical flows. According to Ogedengbe and Ewemoje (2001), $F_1$ is the parameter having greater influence on the variation of $C_d$. Also, the parameters representing the geometrical configuration, $L/B$, $Y_1/L$ and $S/Y_1$ have low influence on the discharge coefficient. For subcritical regimes, Subramanya and Awasthy (1972) proposed:

$$C = \frac{2}{3}\left(0.611\sqrt{1-\frac{3F_1^2}{F_1^2+2}}\right) \qquad (1)$$

Experimental study in a channel with a rectangular cross-section carried out by Raju *et al.* (1979) with subcritical regime and a side weir 0.20-0.50 m long. Researchers considered the effective weir length, Le = L -0.05 m. For the smallest length used, the reach most affected by the boundary effect represents 20% of the weir length. In a longer weir such effect would have a much more reduced influence and hence the researchers proposed the following expression for broad crested weirs:

$$C = \frac{2}{3}\left(0.81\text{-}0.60F_1\right)K \qquad (2)$$

where, K is an empirical coefficient and for a sharp crested weir, K = 1. Hager (1987) refers to an expression where the discharge coefficient depends only on $F_1$, applicable whenever the channel slope and the convergence angle of the main channel walls are close to zero and in case $F_1$ do not vary significantly along the side weir:

$$C = 0.485\left[\frac{\left(2+F_1^2\right)}{\left(2+3F_1^2\right)}\right]^{\frac{1}{2}} \qquad (3)$$

The discharge coefficient of a side weir of a channel with trapezoidal cross-section was studied by Cheong (1991). However, the researcher opined that the results can be applied to rectangular channels and proposes the following expression:

$$C = 0.45 - 0.22F_1^2 \qquad (4)$$

Based on experimental results obtained by earlier researchers, Swamee (1988) presents an expression with a formulation different from the others, for the calculation of the discharge coefficient in sharp-crested weirs while Singh *et al.* (1994) proposed an expression deduced for rectangular channels and subcritical regimes in Eq. 5 and 6. For a weir to be considered sharp-crested, the thickness of the crest and side plates should be between 1 and 2 mm (Martinez *et al.*, 2005):

$$C = \frac{2}{3}\left\{1.06\left[\left(\frac{14.14S}{8.15S+Y_1}\right)^{10}+\left(\frac{Y_1}{Y_1+S}\right)^{15}\right]^{-0.1}\right\} \qquad (5)$$

$$C = 0.33 - 0.18F_1+0.49\frac{S}{Y_1} \qquad (6)$$

The expression above contradicts the conclusion presented by Subramanya and Awasthy (1972) regarding the small influence of the geometric parameters. Ogedengbe and Ewemoje (2001), based on experimental results and multiple regression analysis, presents an expression assuming stages heads equality (Eq. 7). Multiple Linear Regression Models has been selected by Patel (2007) because of its simplicity and adequacy for the estimation of low flows in water resources planning:

$$C = -0.8326+1.7536\frac{S}{Y_1}+0.0088F_1 \tag{7}$$

**Shabayek Model:** A one-dimensional theoretical combining model was developed by Shabayek *et al.* (2002). This model provides the necessary interior boundary equations for combining subcritical open-channel junctions. The main advantage of this model is that neither stages equality nor equal branches widths are assumed at the junction. This model is composed of two huge non-linear equations. The model is based on applying momentum principle together with mass continuity through the junction. This analytical approach for solving nonlinearly for the junction's upstream flow depths has the following makeup:

$$\begin{aligned}&q_u-\frac{q_u^2}{w_1Y_u}-\frac{1}{8F_d^2}\left[w_1\left(3Y_u^2-2Y_uY_L-Y_L^2\right)+q_u\left(Y_u^2+2Y_uY_L-Y_L^2-4\right)\right]-\\&\frac{1}{2F_d^2}\left(\frac{L_1S_o}{h_d}\right)(w_1Y_u+q_u)+K^*\left(\left[\frac{q_u}{w_1Y_u}\right]^2-\left[\frac{q_L}{w_2Y_L}\right]^2\right)\\&(Y_u+Y_L)(2q_Lq_u)+\frac{L_1}{B_dC_*^2}\left(1+\frac{b_d}{h_d}q_u\right)=0\end{aligned} \tag{8}$$

$$\begin{aligned}&q_L-\frac{q_L^2}{W_2Y_L}-\frac{1}{8F_d^2}\left[w_2\left(3Y_L^2-2Y_uY_L-Y_u^2\right)+q_L\left(Y_L^2+2Y_uY_L+Y_u^2-4\right)\right]-\\&\frac{1}{2F_d^2}\left(\frac{L_2S_o}{h_d}\right)(w_2Y_L+q_L)-K^*\left[\left(\frac{q_u}{w_1Y_u}\right)^2-\left(\frac{q_L}{w_2Y_L}\right)^2\right]\\&(Y_u+Y_L)(2q_Lq_u)+\frac{L_2}{B_dC_*^2}\left(1+\frac{b_d}{h_d}q_L\right)+K\frac{q_L^3}{w_2^2Y_L}=0\end{aligned} \tag{9}$$

Where the water depths at the upstream, lateral and downstream points to the junction are denoted by $h_U$, $h_L$ and $h_d$; $Y_U = h_U/h_d$ and $Y_L = h_L/h_d$ are upstream to downstream and lateral to downstream depth ratios; $q_U = Q_U/Q_d$ and $q_L = Q_L/Q_d$ are the upstream to downstream and the lateral to downstream discharge ratios.

$B_U$, $B_L$ and $B_d$ indicate widths of upstream, lateral and downstream branches at the junction; $W_1 = B_U/B_d$ and $W_2 = B_L/B_d$ are upstream to downstream and lateral to downstream width ratios; $\delta$ is the junction angle and $F_d$ is the Froude number at the downstream point of the junction; $S_0$ is the longitudinal slope of the junction and $C_*$ is the Chezy non-dimensional coefficient; $L_1$ and $L_2$ are the outer lengths of the two control volumes; $K^*$ is the interfacial shear coefficient and K is the separation zone coefficient given by:

$$K^* = -0.0015\delta + 0.3 \tag{10}$$

$$K = 0.0092\delta - 0.1855 \tag{11}$$

## MATERIALS AND METHODS

The main and lateral branches of the test facility are 6 and 1.5 m long while the branches widths and depths are 0.15 and 0.35 m, respectively with a junction angle fixed to 90°. Water flows at a constant rate from a 1.5 hp pump to the upstream section. Baffles are placed in the main channel to ensure a tranquil flow through the channels. Steady approach velocity is obtained by placing the lateral channel two-thirds of the way downwards. The flow rate downstream of the channel is determined by the head of the water which spills over the V-notch located at the downstream end of the main channel.

With a bed slope of zero, there was no downstream discharge as all the discharge flows over the side weir (i.e., $Q_d = 0$). The junction bed slope was therefore increased till there were discharges $Q_d$ and $Q_L$ at the downstream and lateral sections, respectively. Table 1 shows the inverse proportionality of the channel bed slope $S_0$ to the sill height S.

**Operational test procedure:**

i The test facility was at zero slope on the hydraulic bench and the fifteen weirs was installed one after the other. Spirit level was used to determine whether the apparatus (channels) was horizontal

ii The water pump was powered electrically and the slope of the apparatus was gradually increased till water discharges at both the lateral and downstream sections. This slope was noted. For a rectangular weir, the crest was taken as the reference point while for a V-notch, the vertex of the V serves as the indicator

iii A calibrated measuring tape was used to measure the heights of the water above the reference points. These are the heads over the weirs

iv Steps (i) to (iii) were repeated for the remaining fourteen weirs which are replica of different heights

Table 1: Variation of sill height with channel bed slope

| Sill height S (m) | Slope $S_0$ (%) |
|---|---|
| 0.06 | 6.23 |
| 0.10 | 5.29 |
| 0.14 | 2.94 |

Water was supplied to the upstream section from two 2000 L tank. Water from the nappe of the upstream V-notch was also turbulent and made to pass through a second baffle as it flows downstream. The outflow over the downstream V-notch was collected in a 120 L container and the lateral outflow was collected in another 120 L container. The two tanks are both connected to a T-junction back into the 2000 L main reservoir. Side weirs placed at the junction of the main and lateral channels was not permanent but removable so that the sill height, S and the length of the weir crest, L could be varied for each test run. Plasticine was used in holding the weirs in place and also to ensure the edges are water-tight. Sealing the edges ensures that the flow into the lateral channel can only be over the weir crest which ensures accurate measurement of water discharges. Upstream depth, $Y_1$ and downstream depth, $Y_2$ were measured at the junction.

**Computational steps:** Constant measurements used throughout the tests are:

- Width of the main channel; B = 0.15 m
- Width of the lateral channel; B = 0.15 m
- Apex angle of upstream and downstream V-notch = 45°

Computational steps for the determination of discharge coefficient $C_d$ of a rectangular weir Shabayek Model are stated as follows: from fluid head above vertex of the V-notch, discharge is calculated for both upstream and downstream sections. From upstream flow $Q_u$ and downstream flow $Q_d$, the lateral discharge $Q_L$ is computed using the mass continuity equation.

## RESULTS AND DISCUSSION

Variation of $C_d$ with ratio $S/Y_1$: The $S/Y_1$ was taken as independent variable while $C_d$ was taken as the dependent variable. The values of $C_d$ were plotted against $S/Y_1$ (Fig. 1). The regression polynomial obtained by Least-Squares Method, relating $C_d$ and $S/Y_1$ was given as:

$$C_d = -35092(S/Y_1)^5 + 145042(S/Y_1)^4 - 239122(S/Y_1)^3 + 196555(S/Y_1)^2 - 80553(S/Y_1) + 1316 \quad (12)$$

This equation is valid for $0.706 < S/Y_1 < 0.927$; with $r^2$ value of 0.53. This means that 47% of the variation in $C_d$ was due to factors other than $S/Y_1$ ratio. The relationship between $C_d$ and $S/Y_1$ supports the earlier finding by Ogedengbe and Ewemoje (2001) but contradicts the finding by Subramanya and Awasthy (1972).

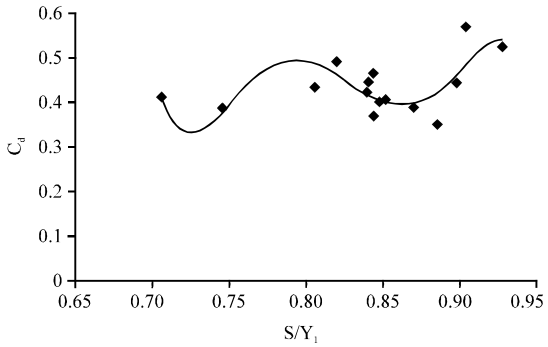

Fig. 1: Relationship between $C_d$ and $S/Y_1$

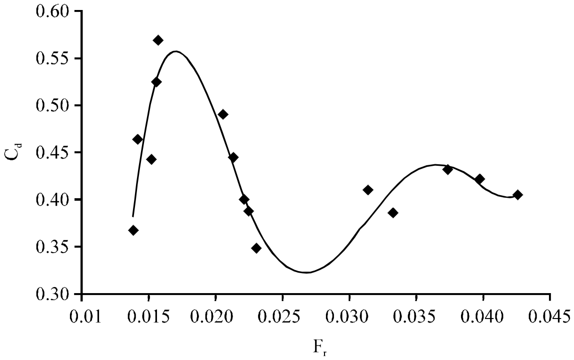

Fig. 2: Relationship between $C_d$ and $F_r$

**Variation of $C_d$ with $F_r$:** The Froude number ($F_r$) was taken as the independent variable while $C_d$ was taken as the dependent variable in the graph of $C_d$ values against $F_r$ (Fig. 2).

Regression polynomial obtained by Least-Squares Method, relating $C_d$ and $F_r$ was given as:

$$C_d = 1E{+}09F_r^5 - 2E{+}08F_r^4 + 9E{+}06F_r^3 - 262531F_r^2 + 3553.6F_r - 17.822 \quad (13)$$

This is valid for $0.014 < F_r < 0.043$ with $r^2 = 0.78$. This implies that 22.34% of the variation in $C_d$ was due to factors other than $F_r$. This relationship contradicts Subramanya and Awasthy (1972) and Singh *et al.* (1994).

**Variation of $C_d$ with $S/Y_1$ and $F_r$:** A straight-line relationship exists between each independent variable ($S/Y_1$ and $F_r$) and the dependent variable which is the value of discharge coefficient ($C_d$). This was determined using multiple regression analysis and the equation was given by:

$$C_d = 0.256\, S/Y_1 - 1.429\, F_r + 0.252 \quad (14)$$

At 95% level of significance and with sample size of 15, there is 0.05 chance of error using student t-distribution. The corresponding confidence intervals are then 0.432743±0.029982 and 0.432377±0.012629 for $C_d$ values obtained.

## CONCLUSION

The expressions analysed have been formulated based on results obtained in a small laboratory physical model which may limit the application of these expressions to a channel with larger side weirs where the boundary effects have smaller significance. Based on this study, the following conclusions are drawn: The computation of the coefficient of discharge $C_d$ was done using the two Shabayek non-linear equations to find the values of the lateral depth $h_L$. These could have been computed using the equality model or other combining models; however they are not as accurate as Shabayek Model even while the Froude number range is subcritical. A methodology for laboratory determination of coefficient of discharge $C_d$ has been described. Expressions for the variation of $C_d$ with the upstream Froude number $F_r$ and the ratio of sill height to upstream depth of flow at junction $S/Y_1$ have been proposed using multiple regression analyses for various values of sill height, length of weir crest and main channel discharge which were treated as controlled variables.

## REFERENCES

Cheong, H.F., 1991. Discharge coefficient of lateral diversion from trapezoidal channel. J. Irrig. Drainage Eng., 117: 461-475.

Chin, D.A., 2006. Water Resources Engineering. 2nd Edn., Pearson Education Inc., Pearson Prentice Hall, Upper Saddle River, New Jersey, USA.

Gupta, B.L. and A. Gupta, 2008. Water Resources Systems and Management. 2nd Edn., Standard Publishers Distributors, Delhi.

Hager, W.H., 1987. Lateral outflow of side weirs. J. Hydraul. Eng., 113: 491-504.

Kesserwani, G., R. Ghostine, J. Vazquez, R. Mose, M. Abdallah and A. Ghenaim, 2008. Simulation of subcritical flow at open-channel junction. Adv. Water Resour., 31: 287-297.

LMNO Eng., 2012. V-notch (Triangular) weir discharge calculator and equations. LMNO Engineering, Research and Software, Ltd., USA. http://www.lmnoeng.com/Weirs/vweir.htm.

Liao, H. and D.W. Knight, 2007. Analytic stage-discharge formulae for flow in straight trapezoidal open channels. Adv. Water Resour., 30: 2283-2295.

Martinez, J., J. Reca, M. Morillas and J. Lopez, 2005. Design and calibration of a compound sharp-crested weir. J. Hydraul. Eng., 131: 112-116.

Munson, B.R., D.F. Young and T.H. Okiishi, 1990. Fundamentals of Fluid Mechanics. 2nd Edn., John Wiley and Sons Inc., New York.

Ogedengbe, K. and T.A. Ewemoje, 2001. Determination of discharge coefficient of rectangular side weirs. J. Applied Sci. Technol., 1: 32-37.

Patel, J.A., 2007. Evaluation of low flow estimation techniques for ungauged catchments. Water Environ. J., 21: 41-46.

Pinheiro, A.N. and N.I. Silva, 1999. Discharge coefficient of side weirs, experimental study and comparative analysis of different formulas. Proceedings of the International Association of Hydraulic Research, 27th Congress, August, 1999, Graz, Austria, pp: 1-8.

Raju, R.K.G., S.K. Gupta and B. Prasad, 1979. Side weir in rectangular channel. J. Hydraul. Div., 105: 547-554.

Shabayek, S., P. Steffler and F. Hicks, 2002. Dynamic model for subcritical combining flows in channel junctions. J. Hydraul. Eng., 128: 821-828.

Singh, R., D. Manivannan and T. Satyanarayana, 1994. Discharge coefficient of rectangular side weirs. J. Irrig. Drain. Eng., 120: 814-819.

Subramanya, K. and S.C. Awasthy, 1972. Spatially varied flow over side weirs. J. Hydraul. Eng., 98: 1-10.

Subramanya, K., 2008. Engineering Hydrology. 3rd Edn., Tata McGraw-Hill, New Delhi.

Swamee, P.K., 1988. Generalized rectangular weir equations. J. Hydraul. Eng., 114: 945-949.